A Dynamic Model for Cash Flow at Risk

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Abstract

In this paper we define a new dynamic approach for measuring the Cash-Flow-at-Risk of a firm. Starting from the assumption that the balance sheet evolves according to a system of difference equations involving the most important accounting records, we define a new risk measure, tailored on our dynamic approach, which takes full advantage of its focus on the liquidity process. A numerical example based on a real case study shows the flexibility of our approach in describing distress and default events.

Keywords: Cash flow, difference equation, sensitivity analysis, Cash Flow at Risk.
JEL classification: C02, C13, G31, M40.

1 Introduction

In this paper, we introduce a new approach to measure the Cash-Flow-at-Risk (CFaR) (see e.g. RiskMetrics (1999), Andren et al. (2005), Yan et al. (2014), Stein et al. (2001a), Stein et al. (2001b)) of a non-financial firm that overcomes the limits of the previous methodologies. In order to reach this goal we proceed following three steps. First, we define a mathematical model for describing the evolution of the firm’s balance sheet, by taking into account the relevant economic dynamics of the company, with special regard to the cash flows. In the second step, we define a new risk measure, based on the CFaR concept, which takes full advantage of our formalism for the

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balance sheet representation. Finally, in the third step we give a concrete application of our approach through a case study based on real data, in which we illustrate the potentials of this new quantitative tool in providing risk management information. We now describe each step of the procedure and review the related literature.

The first point is based on the dynamic representation of the balance sheet in our approach. This gives the possibility to handle the economic information considering the entire history of a firm, thus going far beyond classic approaches for measuring CFaR that adopt static measures like EBITDA (Earnings Before Interest, Taxes, Depreciation and Amortization) and its variants (see e.g. RiskMetrics (1999), Andren et al. (2005), Stein et al. (2001a)), which are well known to be uncapable of capturing the dynamic structure of the cash flow and its relationship between the balance sheet and the macroeconomic and market variables. In fact, it is important to notice that the balance sheet is the only statement, among the firm’s financial reports, with a long-term perspective. For a review about the temporal perspective, a topic deeply investigated, see e.g. pages 91-97 in Epstein and Jermakowicz (2007). Our paper is inspired by Gentili et al. (2013), who described the evolution of the balance sheet through a dynamic model of (first order) difference equations that include the major determinants, such as accounting receivables and payables, investments and other accruals. Their framework is flexible enough to allow for any economic and financial transaction that generate a cash flow, in line with Mattessich (1961) and Dechow et al. (1998), and it can describe different types of firms, just by adding further accounting items. In our paper we extend and enhance the methodology introduced in Gentili et al. (2013) by allowing for an intertemporal analysis of the balance sheet, which may integrate the presence of exogenous variables like GDP, interest rates etc.

Following Gentili et al. (2013), we assume that the balance sheet at time \( n = 1, 2, \ldots \) can be represented as a vector \( S \), which evolves according to the following linear-affine dynamics:

\[
S_n = M_n(S_{n-1} + C_n) + F_n,
\]

where the accounting (square real) matrix \( M \) reflects the linear transition between times \( n - 1 \) and \( n \), while \( C, F \) are the vectors of economic/financial transactions in the period \([n - 1, n]\)^1. In our approach we shall investigate the particular specification where the parameters of the system are replaced by their Chisini (1929) average (see also de Finetti and Mura (1995)), that is at any time we take a particular average of past accounting values. The underlying idea is that the dynamic model evolves with parameters that reflect at any time the company results recorded by previous financial statements.

^1The presence of \( C \) is redundant in the dynamics of \( S \): however, as \( C \) denotes the purely economic transactions carried out in the reference period, it is customary in accounting to include explicitly its presence in the equation, in order to exploit the informative power of the system and distinguish the different interpretation of \( C \) and \( F \).
Before moving to the second step, it is fundamental to underline that our framework is in line with the procedures developed in literature to estimate the operational cash flow from the financial statement (see e.g. Austin and Bradbury (1995), Abu-Abbas (2014)), according to the Financial Accounting Standard Board FASB (1987). The choice of a good proxy for the operational cash flow has been largely debated in the literature. Some researchers pointed out that estimating operational cash flow from pure mechanical procedures, involving the income statement and changes in balance sheet, typically provides a poor fit of real cash flow realizations, see e.g. Austin and Bradbury (1995) and Kinnunen and Koskela (1999). For example, Kinnunen and Koskela (1999) built up a model where the estimated cash flows displayed a great dispersion in their experiment, around 72% of the sample error was greater than 10%. This bias could be explained by the complexity of the mechanical estimations while considering firm-specific events, such as acquisitions, discontinued operations and asset growth. Other sources of complexity may also arise from an imperfect mathematical correspondence between the owners’ equity and the net income for the current accounting period (see e.g. Bahnson et al. (1996)). Krishnan and Largay (2000) showed that it is possible to reduce the bias in the estimation of the cash flows, thus proving the FASB (1987) assertion that cash flow from customers and cash paid to suppliers and employees (which constitute a subset of the operational cash flow) can be determined without unduly burdensome costs directly from the informations contained in the financial statement through mechanical procedures. One possible improvement involves the information included in the disclosure note of the financial statement, which may increase the reliability of the cash flow estimation with an error around 5.5% for cash flows from customers and 8% of cash flows paid to suppliers (see e.g. Hughes et al. (2010)). Finally, Foster et al. (2012) suggested a correction to the mechanical procedure by including the information coming solely from the statement of cash flow instead of considering the whole balance sheet. Thanks to this technique, the estimation of the direct method reached around 95% of accuracy (see also Abu-Abbas (2014)).

As a second step of our procedure, we propose a new risk measure, inspired by CFaR. CFaR extends the Value-at-Risk (VaR): while the latter focuses on market risk, by forecasting changes in the overall value of an asset or portfolio, CFaR deals with variations in cash flow during a given period. In the literature, there are bottom-up and top-down approaches for calculating the probability distribution of cash flow holdings of a firm, together with the cash flow variations. The bottom-up methodologies (see e.g. RiskMetrics (1999)) consist in creating a pro-forma cash flow statement involving the relevant variables, usually macro-factors affecting the firm’s activity, and then determining the probability distribution of the cash flow variations in one or two years.

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2 Krishnan and Largay (2000) find an error on cash collected and cash paid of around resp. 1% and 4%.
Of course, the main shortcoming of this approach consists in the difficulty of creating a pro forma cash flow statement that takes into account all the accounting and macro-economic variables in a realistic and parsimonious way.

On the other side, top-down approaches can be divided into two branches: the purely top-down approaches, where the probability distribution is calculated on a dataset of cash flow data (see e.g. Stein et al. (2001a)), and the exposure based approach, where the cash flow variation at a given point in the future is obtained through a multivariate regression on some macro variables (see e.g. Andren et al. (2005), Yan et al. (2014)). The main shortcoming of the top-down methodologies lies on the fact that they cannot be supported by a reasonable data set, as financial statements are only collected annually or at most semi-annually (see Andren et al. (2005), Yan et al. (2014), Lorek and Willinger (2008), Cheng and Hollie (2007), Brown et al. (2013), Kim and Kross (2005)).

Our CFaR methodology consists in performing a balance sheet quantitative analysis that allows us to select the exogenous variables having the greatest impact on company assets and liabilities. Then, using market data on large time series, we insert in our dynamic model the evolution of such variables, so determining the probability distribution of a cash flow item at a given point in time, usually one or two years. Our approach attempts to overcome the shortcomings of the bottom-up methodology, as it does not deal only with pro-forma cash flow statements, and at the same time it tries to overcome the top-down approaches because it does not rely on pure statistical models.

Finally, as a third step, we apply the new CFaR methodology that we have just presented to a real case study. We investigate the case of high speed passenger rail transport NTV (Nuovo Trasporto Viaggiatori Spa, also known as Italo) and we discuss the descriptive capability of our approach in terms of providing information on the risk position of the firm and generating realistic and unbiased cash flows.

The paper is organised as follows: in Section 2 we introduce the dynamic framework for the balance sheet liquidity model and we compare our model with the equivalences proposed by the static approach of Abu-Abbas (2014). In Section 3 we disentangle the general model into sub-models and we analyze separately the corresponding dynamic equations, including the case where the parameters of the model are averaged and we perform a sensitivity analysis. In Section 4 we introduce and adapt the CFaR measure to our purposes. Section 5 contains a numerical illustration of the new liquidity measure adapted from a case study based on real data. Section 6 concludes.
2 The model

In order to give an intuition and introduce the reader into the topic, we begin by showing how our framework is in line with mechanical procedures developed to estimate the operational cash flow from the financial statement reports. For sake of simplicity, we compare our model with the equivalences proposed by the static approach of Abu-Abbas (2014), who represented directly the cash flow statement using the information available in the financial statement reports. Abu-Abbas (2014) considered the major operating cash components, namely the cash received by the customers, paid to suppliers and employees, the interest received and paid, including taxes. For sake of clarity, we emphasize that our approach will allow for the cash flows coming from the operational, investment and financing activities, while Abu-Abbas (2014) considers only the relations to compute the direct operational cash flow from the indirect approach.

In order to figure out the cash received from customers in a single period \([n-1, n]\) (recall that Abu-Abbas (2014) work in a static model), denoted by \(L^T_T\), Abu-Abbas (2014) proposed that the receipts from customers are computed as follows:

- Cash from Customer Collection (i.e. \(L^T_T\)) = Accounts Receivable at the beginning of the period (denoted by \(T_{n-1}\)) + Sales (denoted by \(Z_n\)) - Accounts Receivable at the end of the period (i.e. \(T_n\)).

Now in our dynamic version, we express the evolution of the accounts receivable \(T_n\) and the corresponding cash flow generated \(L^T_T\) as follows:

\[
\begin{align*}
T_n &= (1 - \eta_n)(T_{n-1} + Z_n) \\
L^T_T &= L^T_{T_{n-1}} + \eta_n(T_{n-1} + Z_n).
\end{align*}
\]

Here, \(\eta_n \geq 0\) is a fixed constant, that is it is assumed that a fixed proportion of accounts receivables are immediately settled\(^3\). That is, at the beginning of each period, accounts receivables \(T_{n-1}\) and sales \(Z_n\) are settled immediately and transformed into liquidity (i.e. in cash, or cash equivalents) with the proportion \(\eta_n\), while the remaining part, corresponding to \((1 - \eta_n)\), is postponed to the subsequent period, and transformed into accounts receivables at the ending period \(T_n\). By rearranging the first equation we get \(\eta_n = \frac{Z_n - T_n}{T_{n-1} + Z_n} - 1\). If we substitute this expression in the second equation and consider only the inflows obtained during the current accounting year, that is \(L^T_{n-1} = 0\), we get

\[\eta_n = \frac{Z_n - T_n}{T_{n-1} + Z_n} - 1.\]

\(^3\)Although the accounting standards (IAS 7 and SFAS 95) encourage the use of direct methods, we emphasize that most companies still prepare the statement of cash flow using indirect methods.

\(^4\)The percentage \(\eta_n\) of receivables settled immediately is usually computed as a weighted average of the earnings in the period \([n-1, n]\), denoted by \(F_n\), with weight \(\eta_n^1\), and the outstanding credits at the beginning of the period (that is, \(T_{n-1}\)), with weight \(\eta_n^2\). In formulas, \(\eta_n = \frac{\eta_n^1 T_{n-1} + \eta_n^2 F_n}{T_{n-1} + F_n}\). In other words, \(\eta_n\) can be written as the percentage of receivables from clients at time \(n - 1\) and the earnings from the \(n - th\) period received at time \(n\).
\[ L_T^n = T_{n-1} + Z_n - T_n, \]

which agrees with the static version in Abu-Abbas (2014).

Similarly, Abu-Abbas (2014) suggests that the payments made to suppliers, denoted by \( L_D^n \), are calculated by adding the purchases (denoted by \( G_n \)) to the difference between the accounts payable at the beginning of the period (denoted by \( D_{n-1} \)) and at the end (i.e. \( D_n \)), that is

- Cash paid to Suppliers (i.e. \( L_D^n \)) = Accounts Payables at the beginning (i.e. \( D_{n-1} \)) + Purchases at the end (i.e. \( G_n \)) - Accounts Payables at the end (i.e. \( D_n \)).

Now we put

\[
\begin{align*}
L_D^n &= L_{D,n-1} + \omega_n(D_{n-1} - G_n) \\
D_n &= (1 - \omega_n)(D_{n-1} - G_n),
\end{align*}
\]

where the fixed constant \( \omega_n \geq 0 \) represents the proportion of accounts payables that are immediately settled at the beginning of the balance\(^5\). As before, the proportion \((1 - \omega_n)\) remains in the accounts payables \( D_n \). Rearranging the second equation gives \( \omega_n = \frac{-D_n}{D_{n-1} + G_n} - 1 \). Then substituting \( \omega_n \) in the first equation and considering only the outflows obtained during one accounting year (i.e. \( L_{D,n-1} = 0 \)), yields

\[ L_D^n = D_{n-1} + G_n - D_n, \]

which also agrees with the definition of Abu-Abbas (2014). Notice that the variable \( D_n \) includes all the costs necessary to the firm’s production, like for example the wage paid to employees, while Abu-Abbas (2014) proposed a dedicated relationship for the wages:

- Employees Payments = Wages Payables at the beginning of the balance + Wages Expenses - Wages Payables at the end of the period.

Of course, it is clear that this specification can be easily included in our general framework.

In order to compare our dynamic framework with the relationships, proposed by Abu-Abbas (2014), for interest paid and received and for the income tax payed, we need to introduce the evolution of the financial activities, denoted by \( B_n \), and their cash flows \( L_{B,X}^n \).

\[
\begin{align*}
L_{B,X}^n &= L_{B,X,n-1} + d_n B_{n-1} + E_n + X_n \\
B_n &= B_{n-1} - E_n.
\end{align*}
\]

\(^5\)As for the parameter \( \eta_n \), also \( \omega_n \) can be computed as a weighted average \( \omega_n = \frac{\omega_1 D_{n-1} + \omega_2 G_n}{D_{n-1} + G_n} \), with weights given by debt a time \( n-1 \) and costs faced in the \( n-1 \)th period that are liquidated at time \( n \).
Here $B_n$ stands only for the long term liability (i.e., borrowings) of the company, $E_n$ represents the principal repayment (if negative), or new borrowings (if positive), $X_n$ represents the sum of liquidity connected with the interest received, the income taxed paid and more generally it may be related with any extraordinary activity which has no mathematical formalization in our model. The term $d_n$ is negative and corresponds to the interest rate paid for financing.

Using the equation defining $L_{n}^{B,X}$, it is possible to treat the relationships, proposed by Abu-Abbas (2014), for interest paid and received and for the income tax paid:

- Interest paid ($IP_n$) = Interest payable at the beginning of balance ($Ip_{n-1}$) + Interest expenses ($Ie_n$) - Interest payables at the end of balance ($Ip_n$)
- Interest received ($IR_n$) = Interest receivable at the beginning of balance ($Ir_{n-1}$) + Interest revenue ($Ire_n$) - Interest receivable at the end of balance ($Ir_n$)
- Income tax paid ($TP_n$) = Interest receivable at the beginning of balance ($Tr_{n-1}$) + Interest revenue ($Tre_n$) - Interest receivable at the end of balance ($Tr_n$).

With the equation defining $L_{n}^{B,X}$, we can handle both the cash flow due to long term liabilities, corresponding to the term $E_n$, and the cash flow due to interest paid, through the term $d_n B_{n-1}$. Here, to keep consistency between our dynamic approach and the one of Abu-Abbas (2014), we should set $X_n = IR_n + TP_n$ and $d_n = IP_n / B_{n-1}$. More generally, it is possible to allow for changes in cash flows generated by extraordinary transactions⁶ thanks to the variable $X_n$, thus showing the flexibility and parsimony of our dynamic framework, which clearly includes as a special case the static one of Abu-Abbas (2014).

Here below we describe separately the corresponding dynamics.

Let us first consider the investing activities, denoted by $K_n$. We will denote by $L_n^K$ the portion of cash flow from the investing activities, which could include acquisitions and disposal of long term assets, such as fixed assets and investments in the form of shares and bonds etc. The corresponding dynamics are as follows:

$$\begin{align*}
L_n^K &= L_{n-1}^K - \gamma_n K_{n-1} \\
K_n &= (1 + \gamma_n) K_{n-1} - A_n.
\end{align*}$$ (5)

The constant $\gamma_n$ represents the rate of change of the fixed assets, while equal $A_n$ denotes their depreciation. In this respect, the first equation shows that $\gamma_n$ is the rate of change of liquidity due to the purchase and sale of long term productive assets, investments and intangible assets.

⁶For example, operations that change the size and composition of the equity.
Finally, let us consider the inventories, denoted by $R_n$. Note that in the model proposed by Gentili et al. (2013), inventories are also associated to an equation describing their evolution, although this quantity is not directly involved in the liquidity process. However, it is important to notice that inventories have a great information power and play a crucial role in the forecast of future cash flows. In order to show the evolution of inventories, we shall introduce the unitary average value of unsold inventories, denoted by $V_n$, and the amount of unsold inventories, denoted by $Q_n$, while $\Delta Q_n = Q_n - Q_{n-1}$.

Now we can write the difference equation satisfied by the unsold inventories:

$$R_n = \phi_n R_{n-1} + \Delta Q_n V_n,$$  \hspace{1cm} (6)

where $\phi_n$ represents the ratio between the unitary value of unsold inventories at times $n-1$ and $n$, that is

$$\phi_n = \frac{V_n}{V_{n-1}}.$$  \hspace{1cm} (7)

Typically, $\phi_n$ (or $V_n$ and $V_{n-1}$) depends on both business decisions and the market trend, while $\Delta Q_n$ can be considered as exogenous as it depends on the production cycle.

Now we have all the ingredients to write the difference equation describing the evolution of the entire cash flow process $L_n$, involving the previous quantities:

$$L_n = L_0 + L_n^T + L_n^R + L_n^K + L_n^D + L_n^{B,X},$$

with initial conditions $L_0^T = L_0^R = L_0^K = L_0^D = L_0^{B,X} = 0$, where for sake of clarity we recall the single terms:

- $L_n^T$ denotes the cash flow coming from the outstanding trade receivables $T$, typically generating positive cash flows;
- $L_n^R$ denotes the cash flow coming from the inventories: as we assumed no direct impact on the cash flow process it turns out that $L_n^R$ satisfies the trivial equation $L_n^R = 0$, i.e. it does not generate any cash flow;
- $L_n^K$ denotes the cash flow coming from the fluctuations of the balance sheet properties, which can be positive or negative;
- $L_n^D$ denotes the cash flow coming from the variation of the payables to suppliers, which typically generates negative cash flows (for example in the case of non-performing loans);
- $L_n^{B,X}$ denoted the cash flow coming from the financial activities and it can generate positive or negative cash flows.
We get

\[
\begin{align*}
L_n &= L_{n-1} + \eta_n(T_{n-1} + Z_n) - \gamma_n K_{n-1} + \omega_n(D_{n-1} - G_n) + d_n B_{n-1} + E_n + X_n \\
T_n &= (1 - \eta_n)(T_{n-1} + Z_n) \\
D_n &= (1 - \omega_n)(D_{n-1} - G_n) \\
B_n &= B_{n-1} - E_n \\
K_n &= (1 + \gamma_n)K_{n-1} - A_n \\
R_n &= \phi_n R_{n-1} + \Delta Q_n V_n,
\end{align*}
\]

which is easily seen to be of the form (1), where the balance sheet vector is

\[S_n = (L_n, T_n, D_n, B_n, K_n, R_n)^\top,\] (8)

and the transition matrix \(M_n\) given by

\[
M_n = \begin{pmatrix}
1 & \eta_n & \omega_n & d_n & -\gamma_n & 0 \\
0 & 1 - \eta_n & 0 & 0 & 0 & 0 \\
0 & 0 & 1 - \omega_n & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 + \gamma_n & 0 \\
0 & 0 & 0 & 0 & 0 & \phi_n
\end{pmatrix}
\]

once the transaction vector \(C\) is defined as \(C_n = (0, Z_n, -G_n, 0, 0, 0)^\top\) and the vector \(F_n\) is defined by \(F_n = (E_n + X_n, 0, 0, -E_n, -A_n, (\Delta Q_n)V_n)^\top\).

It is easy to show that the recursive system for the vector \(S_n\) can be solved in compact form by using the product of the transition matrices \(M_n\). Such representation is useful in view of the implementation of the model:

\[S_n = \left(\prod_{h=0}^{n-1} M_{n-h}\right) (S_0 + C_1) + \sum_{j=2}^{n} \left(\prod_{h=0}^{n-j} M_{n-h}\right) (F_{j-1} + C_j) + F_n.\] (10)

In conclusion, we showed that our dynamic model, which takes inspiration from Gentili et al. (2013), is in line with the approaches that aim at computing indirectly the operational cash flow, with in addition the possibility to consider the cash provided by the financial and investing activities. We emphasize that the model links together all business components in a dynamic perspective, so that we get a good proxy of the cash flow generated by characteristic management, investment management, finance and treasury management. What is more, the model can be modified or expanded by introducing, if needed, additional equations, including different dynamic characteristics, according to the particular business of the firm considered.
3 Sub-Models and Parameter Averaging

The decomposition (2), describing the time evolution of the cash flow process, suggests the possibility to consider separately each class of sub-models for the cash flow. In this section we will solve the system of equations for each sub-model. Then, we will focus on the special case where all the parameters in the accounting matrix $M$ in (1) and the major exogenous items (such as $\Delta Q, Z$ and $X$) are constant, i.e. we consider the system (8) in the following specification:

\[
\begin{align*}
L_n &= L_{n-1} + \eta(T_{n-1} + Z) - \gamma K_{n-1} + \omega(D_{n-1} - G) + dB_{n-1} + E + X \\
T_n &= (1 - \eta)(T_{n-1} + Z) \\
D_n &= (1 - \omega)(D_{n-1} - G) \\
B_n &= B_{n-1} - E \\
K_n &= (1 + \gamma)K_{n-1} - A \\
R_n &= R_{n-1} + \Delta Q \cdot V,
\end{align*}
\]

with a given initial condition $(L_0, T_0, D_0, B_0, K_0, R_0)^7$.

In the case of constant parameters, Formula (10) becomes

\[
S_n = M^n \cdot C_0 + \sum_{h=1}^{n} M^h \cdot C \cdot E + \sum_{h=0}^{n-1} M^h \cdot F.
\]

Solving the system in the constant parameter case is useful in order to introduce the main contribution of the paper, namely the interpretation of the constant parameters as averages in the sense of Chisini (1929). That is, we look for the average parameters that can be obtained by equating the constant-parameter system with the result coming from the general model with general parameters. Notice that the average parameters obviously depend on the choice for the time interval $[0, n]$, like any moving average. This interpretation is very useful because it allows us to estimate at any time the (constant) parameter set that is coherent with the past accounting data. What is more, our equations represent the evolution of both the accounting items and the business structure of the firm, so that the average parameters of the transition matrix describe the (idiosyncratic) structure of the firm, which are affected by shocks in the economic/financial factors. This will allow us to introduce a new measure of cash flow at risk.

\footnote{Note that in the steady state scenario we are considering, we have $\phi = 1$ directly from its definition (7).}
3.1 Sub-Model 1: Cash Received from Customers

In order to solve the system (2), let us consider the first equation which admits the following solution:

$$T_n = \prod_{i=0}^{n-1} (1 - \eta_{n-i})T_0 + \sum_{h=0}^{n-1} \left( \prod_{i=0}^{h} (1 - \eta_{n-i})Z_{n-h} \right),$$

(12)

while the dynamics of $L^T_n$ gives

$$L^T_n = \sum_{h=0}^{n-1} \eta_{n-h}Z_{n-h} + \sum_{j=0}^{n-1} \eta_{n-j}T_{n-1-j}.$$  

(13)

We plug the expression (12) into (13) and we arrive to the closed form solution for the operating cash flows $L^T_n$:

$$L^T_n = \sum_{h=0}^{n-1} \eta_{n-h}Z_{n-h} + \sum_{j=0}^{n-1} \eta_{n-j}T_{n-1-j}.$$  

(13)

Let us now consider the constant parameters case associated to (11), for which the first sub-model (2) becomes:

$$\begin{cases}
L_n^T = T_0(1 - (1 - \eta)^n) + Z \left( n - (1 - \eta) \left( \frac{1 - (1 - \eta)^n}{\eta} \right) \right) \\
T_n = (1 - \eta)^nT_0 + Z(1 - \eta) \frac{(1 - (1 - \eta)^n)}{\eta},
\end{cases}$$

while the recursive relation (14) reads

$$L_n^T + T_n - (L_{n-1}^T + T_{n-1}) = Z,$$

(14)

from which we deduce

$$L_n^T + T_n - (L_0^T + T_0) = nZ.$$  

(15)

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8We use hereafter the convention that $\prod_{i=a}^{b} = 1$ and $\sum_{i=a}^{b} = 0$ whenever $a > b$, then for $j = n - 1$ we have $\prod_{i=0}^{n-2-j} (1 - \eta_{n-1-i-j}) = 1$ and $\sum_{h=0}^{n-1-j} (\prod_{i=0}^{h} (1 - \eta_{n-1-i-j}))Z_{n-1-i-j} = 0$. Also, note that for the positive core business system, it is easily seen that summing up the equations we obtain

$$L_n^T + T_n - (L_{n-1}^T + T_{n-1}) = Z_n,$$

(14)

from which we deduce

$$L_n^T + T_n - (L_0^T + T_0) = \sum_{i=1}^{n} Z_i,$$

(15)

meaning that, between two periods, the difference in value in the positive core business is due only to earning from sales, in line with any basic principle of cash flow statement.
We now apply the definition of mean given by Chisini (1929) to the constant parameter setting. According to the average in the sense of Chisini, we are looking for the constant sales process \( Z \) and the constant parameter \( \eta \) that replicate, at a given fixed time horizon \([0, n]\), the behavior of the sub-model with general parameter \( \eta_n \) and general sales process \( Z_n \). Note that this assumption is not equivalent to the stationarity of the system, but rather that we reason in terms of a somehow moving average. As we can also equate the differences of the sub-models, we use the recursive relation (16) and we arrive to the following equation:

\[
 nZ = \sum_{i=1}^{n} Z_i
\]

from which we can then define the Chisini mean value \( Z \) as

\[
 Z = \frac{\sum_{i=1}^{n} Z_i}{n}.
\]

We plug this value in the equation defining the receivables \( T_n \) and we deduce the corresponding average value for \( \eta \) that replicates the value of the cash flow in the sub-model:

\[
 T_n = (1 - \eta)^n T_0 + Z(1 - \eta)(\frac{1 - (1 - \eta)^n}{\eta}).
\]

This is a polynomial equation in \( \eta \) that can be solved by using standard numerical procedures.

From an accounting point of view, the average parameters represent the portion of sales and receivables turned into cash and they can be considered as the best proxy for the firm business structure.

### 3.2 Sub-Model 2: Cash Paid to Suppliers and Employees

Consider the system (3), describing the impact of the costs in both current and future cash flow. From the dynamics of the accounts payables \( D_n \) we get

\[
 D_n = \prod_{i=0}^{n-1} (1 - \omega_{n-i}) D_0 - \sum_{h=0}^{n-1} \left( \prod_{i=0}^{h} (1 - \omega_{n-i}) \right) G_{n-h},
\]

while for the corresponding cash flow \( L_n^D \) we have

\[
 L_n^D = \sum_{j=0}^{n-1} \omega_{n-j} D_{n-1-j} - \sum_{h=0}^{n-1} \omega_{n-h} G_{n-h}.
\]

---

9For sake of notational simplicity we skip the obvious dependence of \( Z \) and \( \eta \) on the time window \( n \).
Replacing the expression of \( D_n \) gives \(^{10}\)

\[
L^D_n = \sum_{j=0}^{n-1} \omega_{n-j} \left[ \prod_{i=0}^{n-2-j} (1 - \omega_{n-1-i-j})D_0 - \sum_{h=0}^{n-2-j} \prod_{i=0}^{h} (1 - \omega_{n-1-j-i})G_{n-1-h-j} \right] - \sum_{h=0}^{n-1} \omega_{n-h}G_{n-h}.
\]

In the constant parameter case, we get

\[
L^D_n = (1 - (1 - \omega)^n)D_0 - \left( n - (1 - \omega)\left(\frac{1 - (1 - \omega)^n}{\omega}\right) \right) G,
\]

which leads to \(^{11}\)

\[
D_n = (1 - \omega)^n D_0 - (1 - \omega)\left(\frac{1 - (1 - \omega)^n}{\omega}\right) G. \tag{19}
\]

We now apply the definition of mean given by Chisini to the constant parameter setting. From (18) it follows immediately

\[
nG = \sum_{i=1}^{n} G_i
\]

that is

\[
G = \frac{\sum_{i=1}^{n} G_i}{n}.
\]

Again, we replace this value into the expression (19) giving \( D_n \) and obtain the following polynomial equation defining the average parameter \( \omega \):

\[
D_n = (1 - \omega)^n D_0 - (1 - \omega)\left(\frac{1 - (1 - \omega)^n}{\omega}\right) G.
\]

From an accounting point of view, the average parameters represents the portion of cost and payables turned into cash.

\(^{10}\)The recursive relation driving the system (3) for \( L^D_n \) is

\[
L^D_n + D_n - (L^D_{n-1} - D_{n-1}) = -G_n, \tag{17}
\]

that is

\[
L^D_n + D_n - (L^D_0 + D_0) = -\sum_{i=1}^{n} G_i,
\]

which is also in line with the basic principles of cash flow statement.

\(^{11}\)The recursive relation (17) becomes

\[
L^D_n + D_n - (L^D_{n-1} - D_{n-1}) = -G,
\]

from which it follows

\[
L^D_n + T_n - (L^D_0 + D_0) = -nG. \tag{18}
\]
3.3 Sub-Model 3: Financing Cash Flows

Consider the system (4), related to the cash flow coming from the financial business and the total non operating income/expenses.

From the dynamics of $B_n$, we obtain

$$B_n = B_0 - \sum_{h=1}^{n} E_h,$$

and

$$L_{n}^{B,X} = \sum_{i=1}^{n} d_i B_{i-1} + \sum_{i=1}^{n} (E_i + X_i).$$

Replacing in $L_{n}^{B,X}$ the expression of $B_n$ yields

$$L_{n}^{B,X} = \sum_{i=1}^{n} d_i (B_0 - \sum_{j=1}^{i-1} E_j) + \sum_{i=1}^{n} (E_i + X_i).$$

When parameters are constant, we get

$$\begin{cases}
    L_{n}^{B,X} = n d B_0 - d \frac{n(n-1)}{2} E + n (E + X), \\
    B_n = B_0 - n E.
\end{cases} \tag{20}$$

Now we perform the parameter averaging according to Chisini. Assuming that extraordinary financial activities are equi-distributed along time, i.e.

$$n X = \sum_{i=1}^{n} X_i,$$

and using relations (20), we obtain the following average parameters:

$$E = \frac{B_0 - B_n}{n},$$

$$d = \frac{2(L_{n}^{B,X} + B_n - B_0 - n X)}{(n + 1) B_0 + (n - 1) B_n}.$$  

The previous average parameters represent the portion of interest expenses, movements in long term liabilities, income taxes paid and any other activity that can be converted into cash and for which no specific dynamics is considered in the model.
3.4 Sub-Model 4: Investing Cash Flows

Consider the cash flow \( L^K_n \) coming from the fluctuations of the balance sheet properties, given in system (5), which can be positive or negative.

To solve this system, we develop the equation for \( K_n \):

\[
K_n = \prod_{i=0}^{n-1} (1 + \gamma_{n-i})K_0 - \sum_{i=1}^{n-1} \prod_{j=0}^{i-1} (1 + \gamma_{n-j})A_{n-i} - A_n,
\]

while the dynamics for \( L^K_n \) gives

\[
L^K_n = -\gamma_1 K_0 - \sum_{i=1}^{n-1} \gamma_{i+1} K_i.
\]

Using the (natural) convention that for \( i = 1 \) we have \( \sum_{i=1}^{n-1} (\prod_{j=0}^{i-1} (1 + \gamma_{n-j})A_{n-h}) = 0 \), we get

\[
L^K_n = -\gamma_1 K_0 - \sum_{i=1}^{n-1} \gamma_{i+1} \left[ \prod_{h=0}^{i-1} (1 + \gamma_{i-h})K_0 - \sum_{h=1}^{i-1} (\prod_{j=0}^{h-1} (1 + \gamma_{i-j})A_{n-h}) - A_i \right].
\]

In the constant parameter case, we obtain

\[
\begin{align*}
L^K_n &= K_0 (1 - (1 + \gamma)^n) - A \left( n + \frac{1-(1+\gamma)^n}{\gamma} \right) \\
K_n &= (1 + \gamma)^n K_0 + \frac{1-(1+\gamma)^n}{\gamma} A.
\end{align*}
\]

We now perform the parameter averaging according to Chisini. From the expression (21), giving \( L^K_n \), we get immediately

\[
nA = \sum_{i=1}^{n} A_i,
\]

that is

\[
A = \frac{\sum_{i=1}^{n} A_i}{n}.
\]

---

12 We use the convention that for \( n = 1 \) we have \( \sum_{i=1}^{n-1} (\prod_{j=0}^{i} (1 + \gamma_{n-j})A_{n-i}) = 0 \) and \( \sum_{i=1}^{n-1} \gamma_{i+1} K_i = 0 \).

13 The recursive relation reads

\[
L^K_n + K_n - (L^K_{n-1} + K_{n-1}) = -A,
\]

which leads to

\[
L^K_n + K_n - (L^K_0 + K_0) = -nA.
\]

(21)
and replacing this value in the expression (22) defining $K_n$, we get

$$K_n = (1 + \gamma)^n K_0 + \frac{1 - (1 + \gamma)^n}{\gamma} A,$$

which is a polynomial implicitly giving the average parameter $\gamma$.

The previous parameters represent the changes in the company’s cash position resulting from investment gains or losses, and changes resulting from amounts spent in investments in capital assets, such as plant and equipment.

### 3.5 Sub-Model 5: Inventories

Although unsold inventories have no direct impact on the cash flow process, it is important to notice that they can affect the possible future borrowings. In fact, unsold inventories can be considered as additional guarantees for external financings. We shall investigate this crucial aspect in future research. For the moment, we limit to associate a trivial sub-model to the part of cash flow coming from the inventories.

$$\begin{align*}
L_n^R & = 0 \\
R_n & = \phi_n R_{n-1} + \Delta Q_n V_n,
\end{align*}$$  \hspace{1cm} (22)

which admits the following solution:

$$\begin{align*}
L_n^R & = L_0^R, \\
R_n & = \prod_{i=0}^{n-1} (\phi_{n-i}) R_0 + \sum_{h=1}^{n-1} \left[ \prod_{i=0}^{h-1} (\phi_{n-i}) \Delta Q_{n-h} V_{n-h} \right] + \Delta Q_n V_n.
\end{align*}$$

In the constant parameter case, we have

$$R_n = R_0 + n\Delta Q \cdot V$$  \hspace{1cm} (23)

and, of course, $L_n^R = L_0^R$.

Finally, we perform the parameter averaging according to Chisini. From (23) and assuming that unsold inventories are equi-distributed along time, i.e.

$$\Delta Q = \frac{\sum_{i=1}^{n} \Delta Q_i}{n}$$

we can determine the average value for $V$:

$$V = \frac{R_n - R_0}{\sum_{i=1}^{n} \Delta Q_i}.$$  \hspace{1cm} (24)
In conclusion, we provided explicit expressions for the averaged parameters replicating the behavior of the balance sheet at a fixed time $n$, in terms of observable quantities involving past financial statements. In other terms, the average parameters represent the best proxy for the firm business structure.

4 Cash Flow at Risk

Cash Flow at Risk (CFaR) determines the maximum shortfall of cash the firm is willing to tolerate with a given confidence level (Andren et al. (2005)) and is calculated in a rather similar way as VaR, but on cash flow rather than asset value. It was introduced at the end of the 90’s in order to create a risk measure for industrial companies capable to reflects in a single value the firm’s risk tolerance (Yan et al. (2014)). However, this new risk measure did not achieve popularity among researchers and practitioners because it was viewed to be equivalent to VaR, especially from financial institutions, which seemed to be not particularly interested in the cash flow at risk, especially given the stock market conditions before 2007. This assumption was proved to be false during the last financial crisis where thin markets and the presence of liquidity risk made possible that a well-capitalized bank would face bankruptcy because illiquid markets would not allow transferring marketable securities into cash in time (Yan et al. (2014)). In this perspective a poignant debate started over the liquidity issue and CFaR was be seen for the first time not only a generic tool in order to evaluate the firm’s risk tolerance but as a possible useful approach in terms of measuring liquidity risk, especially for non financial company because the VaR completely ignores the risks of the company’s underlying commercial cash flow (see Andren et al. (2005)). In addition, as it could see in the following paragraphs, some CFaR approaches permit to evaluate the cash flow at risk distinguishing between value-adding risk factors, that is, risks generated by the company activities where the firm has a comparative advantage to handle them and not value-adding risk, that is, risks where the firm has no convenience in being exposed (Andren et al. (2005), Merton (2005)).

4.1 Methods for computing Cash-Flow-at-Risk

The CFaR calculation mainly requires a forecast of the probability distribution of the cash flow at some future point in time. There are three different approaches proposed so far to evaluate such distribution. The first approach, employed by RiskMetrics (1999), in line with Montecarlo VaR, is constituted by a family of bottom-up methodologies and consists in building a pro-forma cash flow statement assuming that production volumes, prices, and costs are the key factors that determine the future cash flows. The distribution of the conditional value of cash flow can be calculated by random prices and rates generating their own variance-covariance matrix. The main shortcomings of this approach are: its inability to capture different macroeconomic effects and the
difficulty to build a pro forma cash flow statement capable to take into account of all the accounting variables that affect the future cash flows (Yan (2014), Andren et al. (2005)). The second approach is constituted by a family of top-down methodologies, (see e.g. Stein et al. (2001a)). They are based on the assumption that total cash flow distribution is the ultimate variable of interest (Yan (2014)) so that the volatility is estimated by the historical cash flows of a company (when such data exist), or from data taken from clustering of similar firms. This method shares the same shortcomings of the rating models, because a stand-alone firm’s dataset is too small to be statistically significant, while when it uses the clustering techniques, in order to overcome the aforementioned problem, the dataset obtained is not representative of the specific firm that we want to analyze. Given the limitations of both bottom-up and top-down methods, Andren et al. (2005) proposed a third approach, called Exposure-Based CFaR. This approach estimates, through a multivariate regression, called Exposure Model, the firm’s cash flow sensitivity to the non-value-adding risk factors which could specifically affect the company’s liquidity. Then, assuming a particular distribution for the risk factors involved, it would be possible to evaluate the cash flow distribution as the result of the insertion into the Exposure Model of the simulated random sample of the risk factors. The main shortcoming of this methodology relies on the fact that the Model Exposure, based only on a statistical approach, with difficulty could show the cash flow sensitivity to the risk factors.

4.2 Our CFaR Approach

The new methodology we are introducing here tries to overcome the limitation of the Exposure-Based CFaR approach substituting the statistical Model Exposure with the average representation, previously described. This model, in line with the procedures developed in literature to estimate the operational cash flow, allows us to determine the evolution of the firm’s balance sheet with special regard to the cash flow, determining a more reliable picture of the cash flow sensitivity to the risk factors. Our methodology can be summarized in three steps. The first step of our approach consists in estimating, from the past financial statements, a set of average parameters and variables that describe the dynamics of the main accounting items, according to the dynamic definition of the liquidity process of the company. We consider this averaging procedure not only capable to make the accounting data more mathematically tractable but also fine in order to describe the business model structure of the company implicit in the accounting data. In this respect, thanks to our dynamic model we are improving the CFaR approach of Andren et al. (2005), as we are able to explain endogenously such sensibilities instead of using a regression analysis on external (macro or microeconomic) risk factors. The second step consists in forecasting the dynamics of the macroeconomic variables that mainly affect the financial statement, by simulating multiple paths. At this stage, we limit ourselves to consider just no adding-value variables, but it could be possible to imagine a more complete scenario analysis involving also adding-value
variables like for example sales, earnings, etc.. Finally, we feed the dynamics (1) with the average parameters (using the matrix (11) together with the vectors $F, C$ and $S_{n-1}$ given by Formula (10)), except for macroeconomic variables that have to be simulated, thus for every path we compute the value of the cash flow. With this technique we will define the distribution of Cash Flow at Risk. At this point, we can evaluate the CFaR as any risk measure such as VaR or expected shortfall by introducing a particular confidence level.

5 NTV Case Study

In order to illustrate our methodology in a concrete case, in this section we present a case study on NTV - Nuovo Trasporto Viaggiatori S.p.A., a high-speed passenger rail transport also known as Italo S.p.A. In 2006 the rail transport deregulation process opened the door for a group of Italian entrepreneurs to set up Italo, which represents the first private company to operate in the high-speed passenger rail transport market within the European Union. Since its first year of operation, Italo was greatly appreciated for the quality of its service and has undoubtedly raised the standards of its market. However, in 2014 the company faced a financial distress, mainly due to its high operative costs and high levels of short-term debt. In order to avoid bankruptcy, in agreement with the lending banks, Italo decided to restruct the financial debt by extending the maturities of the liabilities and negociating a lower spread on the interest rate. This operation gave good results, permitting to the firm to recover its financial health, to increase its net profit margin and continue its fixed investment growth, in particular in terms of new equipment. In 2017 the company was able to raise capital by issuing a floating rate secure bond for 550 mln euros. At the end of 2017 the company decided to redeem the current bonds, in order to raise investment capital by offering its stock to the public. This operation was financed by a new bullet loan, with maturity 2023 an notional about 710 mln euros.

We now proceed as explained in the previous paragraphs and we are going to examine the financial statements of Italo, concerning the fiscal years from 2013 to 2017, in order to define a sequence of average parameters which represent the business structure of the firm. We then analyze the financial structure and we look for the non-value-adding variables which mostly affect its cash flow and we evaluate the cash flow at risk distribution.

The first step consists in putting together all relevant quantities and finding the average parameters that identify the business structure of the firm.

- Account Receivables (sum of the Trade Accounts Receivable) $T_n$

---

14 See e.g. https://www.italotreño.it/it.
15 Between 2015 and 2017, NTV bought 17 new trains.
16 All the data refer to the firm annual Financial Statements published and available online.
- Revenues $Z_n$ (sum of the annual operating revenue)
- Account Payables (sum of the Trade Accounts and Other Payables) $D_n$
- Costs (sum of annual costs for Purchases, Service expenses, Employee salaries, other operative costs) $G_n$
- Financial Debt (sum of the long-term and short-term liabilities) $B_n$
- Principal repayments $E_n$
- Tangible and Financial fixed assets (sum of all investing activities such as fixed assets, operating lease) $K_n$
- Amortization (sum of annual Depreciation of investment assets) $A_n$
- Inventory assets (sum of raw goods, in-progress goods and finished goods) $R_n$
- Cash and Cash equivalence (sum of the Bank and post office accounts, Cheques, Cash on hand) $L_n$
- Interest paid $d_n$
- Non operating Income/Expenses, Prepayments, Changes in equity, Changes in Provisions, Cash Flow from Financing Activities, Financial Income $X_n$

Here below we present the values of the five fiscal years.

<table>
<thead>
<tr>
<th>Years</th>
<th>2017</th>
<th>2016</th>
<th>2015</th>
<th>2014</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_n$</td>
<td>267.122.535</td>
<td>127.430.642</td>
<td>145.607.179</td>
<td>39.150.953</td>
<td>51.634.873</td>
</tr>
<tr>
<td>$T_n$</td>
<td>31.402.387</td>
<td>34.971.035</td>
<td>37.051.910</td>
<td>34.041.240</td>
<td>41.588.519</td>
</tr>
<tr>
<td>$D_n$</td>
<td>199.375.339</td>
<td>100.248.413</td>
<td>106.629.851</td>
<td>88.301.299</td>
<td>119.266.080</td>
</tr>
<tr>
<td>$B_n$</td>
<td>710.331.179</td>
<td>673.834.869</td>
<td>686.389.055</td>
<td>671.048.613</td>
<td>661.327.755</td>
</tr>
<tr>
<td>$K_n$</td>
<td>743.726.054</td>
<td>654.672.705</td>
<td>623.481.947</td>
<td>626.915.901</td>
<td>672.982.771</td>
</tr>
<tr>
<td>$R_n$</td>
<td>5.577.119</td>
<td>2.108.785</td>
<td>2.123.171</td>
<td>2.590.052</td>
<td>2.762.472</td>
</tr>
<tr>
<td>$Z_n$</td>
<td>454.943.354</td>
<td>379.760.727</td>
<td>320.803.672</td>
<td>267.848.738</td>
<td></td>
</tr>
<tr>
<td>$G_n$</td>
<td>312.938.107</td>
<td>272.281.563</td>
<td>256.594.417</td>
<td>281.689.724</td>
<td></td>
</tr>
<tr>
<td>$A_n$</td>
<td>28.987.262</td>
<td>38.413.133</td>
<td>39.924.668</td>
<td>43.845.931</td>
<td></td>
</tr>
<tr>
<td>$d_n$</td>
<td>12.191.696</td>
<td>10.141.859</td>
<td>5.327.827</td>
<td>6.964.302</td>
<td></td>
</tr>
<tr>
<td>$X_n$</td>
<td>27.816.478</td>
<td>40.476.517</td>
<td>65.137.727</td>
<td>22.450.489</td>
<td></td>
</tr>
</tbody>
</table>

Using the formulae presented in Section 3 we find the average parameters that replicate the accounting items of the period 2013 - 2017\(^\text{17}\):

\(^\text{17}\)The term $V_n$ here embeds the value of $\Delta Q_n$. 

20
We emphasize that the previous average parameters lead to an error less than 1% in the overall cash flow given by system (8) in the period from 2013 to 2017.

The second step consists in identifying the most important non-value-adding variables that affect the cash flow. By looking at the historical series of the financial statement, we realize a strong impact of the financial debt on the cash flow. We conclude that the main non-value-adding macro variable that could affect the cash flow is the interest rate on financial debt. In fact, the total cost of production can be absorbed by the train fare, while the interest rate paid on the debt cannot be handled similarly. In order to evaluate the probability distribution of the interest rate in the future we decided to use simple approach, namely we consider a Gaussian perturbation around the forward curve of the spot interest rate. The standard deviation of the Gaussian distribution reflects the historical volatility of the interest rate.

At this point, we have all the inputs in order to evaluate the CFaR distribution. We emphasize that within our approach we are not limited to compute the CFaR for a maturity of just one year as typically done in the literature. On the contrary, we can push forward our computation according to the distribution that we assume to be stationary. This is particularly interesting for firms like NTV, for which it is important to analyze the capability to respect the bullet loan in 2023. For sake of awareness, we evaluate both the cash flow at risk to the one-year horizon and to four years.

Here below the results for the CFaR in one year (2018) for different confidence levels:

\[
\begin{align*}
    CFaR_{1\text{year}} (99\%) &\approx 260.176.141 \\
    CFaR_{1\text{year}} (95\%) &\approx 260.492.836 \\
    CFaR_{1\text{year}} (90\%) &\approx 260.662.576
\end{align*}
\]

\(\eta \approx 0.9189\) \hspace{1cm} \(V \approx 27.191\)

\(\omega \approx 0.4182\) \hspace{1cm} \(Z \approx 355.839.122\)

\(\gamma \approx -0.0795\) \hspace{1cm} \(G \approx 280.875.952\)

\(d \approx 0.01092\) \hspace{1cm} \(E \approx 12.250.856\)

\(\phi \approx 1\) \hspace{1cm} \(A \approx 37.792.748\)

\(d \approx 0.01092\) \hspace{1cm} \(X \approx 18.732.044\)

\(\eta \approx 0.9189\) \hspace{1cm} \(V \approx 27.191\)

\(\omega \approx 0.4182\) \hspace{1cm} \(Z \approx 355.839.122\)

\(\gamma \approx -0.0795\) \hspace{1cm} \(G \approx 280.875.952\)

\(d \approx 0.01092\) \hspace{1cm} \(E \approx 12.250.856\)

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\(d \approx 0.01092\) \hspace{1cm} \(X \approx 18.732.044\)

\(\eta \approx 0.9189\) \hspace{1cm} \(V \approx 27.191\)

\(\omega \approx 0.4182\) \hspace{1cm} \(Z \approx 355.839.122\)

\(\gamma \approx -0.0795\) \hspace{1cm} \(G \approx 280.875.952\)

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\(d \approx 0.01092\) \hspace{1cm} \(X \approx 18.732.044\)

The debt equity ratio was around 5.42 in 2016 and 3.81 in 2017.

\(\text{Note that the debt equity ratio was around 5.42 in 2016 and 3.81 in 2017.}\)

\(\text{One could find natural to introduce a dynamic model for the interest rate, for example a Vasicek (Gaussian) dynamics or similar. However, the particular period considered in our case study would lead to some difficulties in the estimation of the parameters of the interest rate process. In fact, in the last years historical time series displays a strong non stationary behavior, so that the informative power of forecasting would be very low due when using a mean reverting diffusive model as the Vasiceck one.}\)
In order to evaluate the distribution at four years horizon, we project the system by using the solution in Formula (10), and we consider the volatility estimated according to the same time window.

\[
\text{CFaR}_{4\text{years}} (99\%) \approx 397.939.087 \\
\text{CFaR}_{4\text{years}} (95\%) \approx 417.870.474 \\
\text{CFaR}_{4\text{years}} (90\%) \approx 434.476.746
\]

The previous results show a positive and increasing cash flow both on a one year time horizon and a four years horizon. In addition, it is possible to state how the company, in a couple of years, is potentially able to completely recover, finding also a good growth rate both in revenues and in cash flow. However, the liquidity generated, considering also other percentiles, is not enough to pay back the bullet loan of 710 mln euros. This shows how the company’s problem is bound to his structure more than the non-value adding variable. In fact, the inability to repay the loan is mostly due to the low level of revenues. Thus, it is difficult to think that the company would be able to survive without a partial refinancing of the debt at the end date. Of course, the permanent introduction of new trains (2019-2020) as declared by the company could increase the revenues as well as the cash flow level, but this is just a possibility.

In conclusion, we highlight that within our approach can consider all type of variables, value-adding as well as non-value-adding, so that we can consider a stress test analysis of the business structure.

6 Conclusion

We introduced a general framework that allows to compute a liquidity risk measure for non listed industrial firms, namely the Cash Flow at Risk, just by using the information...
coming from the balance sheets and some related macroeconomic variables. The mathematical model describes the dynamics of the overall liquidity, specifically allowing to overcome some limits present in previous techniques for the measurement of Cash Flow at Risk, and it can also be useful in other areas of risk management. The case study based on real data highlights the great flexibility of the approach, which leads to clear and straight conclusions on the future financial stability of the firm. In particular, the case study shows that, although the financial and restructuring operations of the company performed well, the average expected growth in terms of turnover is not sufficient to guarantee the correct repayment of the bullet loan. Therefore, the company, before the expiry date, will face a high probability to provide a new restructuring operation of the debt or an increase in the risk capital. In conclusion, our CFaR measure is able to capture the fragility of the financial structure in its full evidence and it helps in predicting distress and default.

References


