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## **Dynamic, incentive-compatible contracting for health services**

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## Summary

This paper aims to characterise a dynamic, incentive-compatible contract for the provision of health services, allowing for both moral hazard and adverse selection. Patients' severity changes over time following a stochastic process and is private information of the provider. We characterise the optimal dynamic contract and show that it is made up of two components: a time-invariant payment, which depends on the structural characteristics of the provider, and a time-varying component, which is affected by both patient and hospital characteristics. To illustrate the characteristics of the dynamic contract and compare it with a more standard static contract, we provide a numerical exercise calibrated with data from hip replacement hospitalisations in Italy.

**Keywords:** hospital payments; dynamic mechanism design; DRG; two-part tariffs; adverse selection; moral hazard

**JELClassification:** H42, I18, D82

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# Dynamic, incentive-compatible contracting for health services

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## Abstract

This paper aims to characterise a dynamic, incentive-compatible contract for the provision of health services, allowing for both moral hazard and adverse selection. Patients' severity changes over time following a stochastic process and is private information of the provider. We characterise the optimal dynamic contract and show that it is made up of two components: a time-invariant payment, which depends on the structural characteristics of the provider, and a time-varying component, which is affected by both patient and hospital characteristics. To illustrate the characteristics of the dynamic contract and compare it with a more standard static contract, we provide a numerical exercise calibrated with data from hip replacement hospitalisations in Italy.

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## 1 Introduction

In the healthcare sector, more often than probably anywhere else, services are not paid out of pocket by consumers (patients), who are largely insured, either publicly or privately, against the related risks. Payments are often based on contracts that regulate the relationship between a provider (e.g., a hospital) and a purchaser (typically a public or private insurer). A well-known characteristic of this relationship is the presence of asymmetric information on relevant dimensions such as provider effort, case-mix complexity, and costs of provision.

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Another feature is that provider–purchaser relationships tend to be long-lasting (Chalkley and Malcomson, 1996). The first characteristic has been explored in depth in the literature studying (second-best) optimal contracts for the provision of healthcare services. However, authors have almost invariably done so by relying on static models, which prevents them from accounting for the second characteristic, i.e., the dynamic dimension of the provider–purchaser relationship.

This paper aims to characterise a second-best optimal dynamic contract between a purchaser and a provider, allowing for both moral hazard and adverse selection. In our model, the patients’ severity is private information of the provider and changes over time following a Brownian process. This process depends on parameters that are (partly) unknown to the purchaser. At each point in time, the provider selects the combination of effort (unobservable) and services (observable) based on patient severity. We characterise the optimal dynamic contract and show that it is made up of two components: a time-invariant payment, which depends on the structural characteristics of the provider, and a time-varying component, which is affected by both patient and hospital characteristics.

Starting approximately from the 1990s, a clear tendency emerged to replace *retrospective* with *prospective* systems for the reimbursement of healthcare services. In most cases, prospective payments are based on diagnosis-related groups (DRGs). In their purest form, DRG systems imply that the reimbursement made for one patient depends only on her diagnosis. Given the information advantage enjoyed by providers in their relationship with purchasers, prospective payment systems (PPSs) offer a clear advantage in terms of cost-efficiency. However, the literature has also highlighted a number of potential distortions. A first class of distortions arises because, under PPSs, the same payment is made for patients who are heterogeneous in terms of severity, which is observed by the provider but not by the purchaser. Since indexes measuring the complexity of the case mix are often unavailable or imprecise (Pettengill and Vertrees, 1982; Jencks, 1987), it is difficult for the purchaser to infer patients severity. This may lead to overprovision of treatments for low-severity (i.e., low-cost) cases and underprovision, or no provision, for high-severity cases (Ellis, 1998). PPS may also lead to underprovision of quality if higher quality requires higher costs (Dranove, 1987; Allen and Gertler, 1991; Ellis and McGuire, 1986; Brekke et al., 2015). Distortions may also arise in the allocation of treatments with different intensities to patients according to their severity (Siciliani, 2006) and in decisions concerning investment in quality (Levaggi et al., 2012, 2014; Ghandour et al., 2022). The empirical evidence on the efficiency of PPSs in terms of costs and quality is also mixed (Scanlon, 2006; Tan and Melendez-Torres, 2017).

The literature has widely explored the mechanisms and conditions allowing these limitations to be overcome while the efficiency advantages of PPSs are retained. A first result is

that efficiency can be improved through the adoption of some form of cost-sharing, whereby only part of any excess costs incurred by the provider are reimbursed (Ellis and McGuire, 1986). It is worth mentioning that several real-world DRG tariff schedules allow extra payments within a DRG under specific circumstances, e.g., a hospitalisation length exceeding a certain threshold. The implications of other adjustments to a pure fixed-payment setup based on other characteristics of the patient or the treatment provided have also been studied (see, for example, Hafsteinsdottir and Siciliani, 2010). The topic of competition as an incentive mechanism to keep quality sufficiently high even in a PPS context has probably received the greatest attention (Ma, 1994). However, with the combination of PPS and competition, an optimal allocation can be achieved only under very restrictive assumptions (Chalkley and Malcomson, 1998). Moreover, there exist clinical areas, such as emergency care, where competition simply cannot be conceived. The empirical findings on the impact of competition are also mixed with respect to both efficiency (Oliver and Mossialos, 2005; Gaynor and Town, 2011; Gravelle et al., 2014; Kessler and McClellan, 2000; Propper et al., 2008; Hunter, 2009) and equity (Dardanoni et al., 2018).

More recently, pay-for-performance has received significant attention as an approach to reducing the scope of information asymmetry (Kristensen et al., 2016). However, it is difficult to find performance measures that are both feasible and comprehensive. Circumstances have also been identified under which pay-for-performance leads to distortions in the provision of quality (Mak, 2018; Lisi et al., 2020; Arifoğlu et al., 2021).

This paper focuses on an alternative tool provided by the contract theory to address inefficiencies arising from asymmetric information: second-best, incentive-compatible contracts. In this context, De Fraja (2000) studies the optimal contract in a setting where providers differ in terms of cost efficiency, a parameter that the purchaser cannot observe. In Siciliani (2006), patients' severity is private information of the hospital, which decides whether to provide low- or high-intensity treatment. The optimal contract involves different payments for the same treatment for hospitals with different characteristics. Recent contributions to this literature include Wu et al. (2018) and Maréchal and Thomas (2021), both of which allow for moral hazard and adverse selection. However, as with all the other contributions in this strand of literature, they rely on a single-period model. This characteristic prevents the works in this strand from accounting for the implications of the long-lasting character of the relationship between the purchaser and the provider and the related dynamics.

With only a few exceptions, analyses of optimal payments for health services have been undertaken in static models. Among the exceptions, Brekke et al. (2012a) and Bisceglia et al. (2020) study dynamic price regulation in competitive settings where patients' decisions on where to be treated are driven by noncontractible quality. In discussing their results,

Bisceglia and collaborators note that ‘*a menu of incentive-compatible contracts ... could further improve welfare*’ (Bisceglia et al., 2020, p. 13).

To the best of our knowledge, ours is the first contribution to characterise an optimal, incentive-compatible contract for health services in a setting where competition does not work as an incentive mechanism. In our model, the severity of patients treated by one hospital follows a Brownian process, the realisations of which are unknown to the purchaser. In addition, we interpret the initial value of severity as a hospital-specific structural characteristic, which affects the expected severity of future patients and is also private information of the provider.<sup>1</sup>

Our continuous time approach provides benefits in terms of tractability. We fully characterise the optimal dynamic second-best contract, which entails a two-tier payment, including a negative fixed component (patient independent, provider dependent) and a positive variable component (patient dependent, provider dependent). Methodologically, we exploit the mechanism design developed by Bergemann and Strack (2015). However, an important difference between our and their setup is that whereas in Bergemann and Strack the principal is a revenue maximiser, in our paper it is a health authority that maximises consumer surplus and faces a problem of dynamic optimal regulation with both adverse selection and moral hazard.<sup>2</sup>

There are two main qualitative differences between this payment scheme and the way that DRG systems are applied in most countries: i) the fixed component of payment differs for different providers, depending on their structural characteristics;<sup>3</sup> ii) within a DRG, the adjustment of payments (*cost-sharing* in the terminology of Ellis and McGuire, 1986) depends not only on patients’ but also on providers’ characteristics. We use data on hip replacement hospitalisations in Italy to illustrate the characteristics of the optimal dynamic contract and its efficiency properties.

The remainder of the paper is organised as follows. We introduce our model in Section 2 and characterise the first- and second-best solutions in Sections 3 and 4, respectively. The characteristics of the second-best contract are discussed in Section 5. The numerical example, involving a calibration based on hip replacement data, is presented in Section 6.

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<sup>1</sup>Heterogeneity in the patient case mix across hospitals and its implications for hospital financing has been widely discussed in the literature. See, for example, Watts (1980), Sloan et al. (1983), and Söderlund et al. (1996).

<sup>2</sup>Bergemann and Strack (2015) study the optimal dynamic contract between a revenue-maximising monopolist selling a non-durable good and a privately informed buyer. This is a major contribution to the strand of literature addressing the problem of optimal dynamic contracts in dynamic settings initiated by Baron and Besanko (1984) and Laffont and Tirole (1993) in two-period settings and later extended to multiple periods (Pavan et al., 2014). See also Bergemann and Välimäki (2019) for a review of the literature and Arve and Zwart (2023) for the case of durable goods in an environment which is not time-separable.

<sup>3</sup>Note that in some systems, there may be different DRG tariffs for different types of provider. However, unlike in our case, such “types” are observable (e.g., being a teaching hospital).

Section 7 concludes.

## 2 Setup

We study an optimal contract between a purchaser (regulator: she) and a provider (hospital: he) in a continuous-time dynamic setting where patient severity, which affects the marginal productivity of effort, is stochastic. Both severity and effort are private information of the provider.

### 2.1 Patients

There is a constant flow of patients requiring a specific treatment over the time horizon defined by  $t \in [0, \infty)$ . Specifically, let  $\beta_t$  be the severity of a patient admitted at time  $t$ .<sup>4</sup>  $\beta_t$  changes over time according to a Brownian process with no drift and an upwards shift  $k \geq 0$ :

$$d\beta_t = (\beta_t - k)\sigma dB_t, \text{ with } \beta_{t=0} = \beta_0, \quad (1)$$

where  $\sigma > 0$  is the constant instantaneous volatility and  $B_t \sim N(0, t)$  is a standard Wiener process. Eq. 1 means that at any  $t$ ,  $\beta_t$  lies in the interval  $[k, \infty)$ , where the parameter  $k$  can be interpreted as the minimum level of severity such that treatment is clinically appropriate.<sup>5</sup>

By solving the above differential equation, patients' severity at each time  $t$  can be represented as (see Appendix A):

$$\beta_t \equiv \phi(t, \beta_0, \sigma, B_t) = k + \beta_0 \exp\left(-\frac{1}{2}\sigma^2 t + \sigma B_t\right). \quad (2)$$

We assume that while  $\sigma$  is public knowledge, the provider is better informed than the purchaser about  $\beta_t$ , ( $t \geq 0$ ). In particular, we assume that the initial value  $\beta_0$  is distributed over the interval  $[\beta^l \geq k, \beta^h]$ , according to the density function  $g(\beta_0)$  and the cumulative distribution function  $G(\beta_0)$ , with both  $g(\beta^l)$  and  $g(\beta^h) > 0$ . The characteristics of  $g(\beta_0)$  and  $G(\beta_0)$  are common knowledge. The function  $G(\beta_0)$  is such that  $\frac{1-G(\beta_0)}{g(\beta_0)\beta_0}$  is monotone decreasing, with  $g(\beta^l) \geq 1/\beta^l$ .<sup>6</sup>

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<sup>4</sup>Without loss of generality,  $\beta_t$  can also be interpreted as an index of the average severity of patients arriving at time  $t$ .

<sup>5</sup>The assumption of a trendless process allows us to focus on the pure effect of the uncertainty. However, notice that, by the Markov property of Eq. 1, our general results would not be altered by using a nonzero trend for  $\beta_t$ .

<sup>6</sup>As in Arve and Zwart (2023); Skrzypacz and Toikka (2015); Buso et al. (2021), this is equivalent to assuming that the contractor's private information is represented by two stochastic processes where the one representing the initial value is constant after time zero but influences the transitions of the second process.



In what follows,  $\beta_0$  can be interpreted as a structural characteristic of the provider with respect to severity, which affects the expected severity over time via the diffusion process of Eq. 2. This allows us to account for the fact that providers treating more (less) severe patients today are also more likely to do so in the future due to their specialisation.

Two properties of Eq. 2 are particularly important for our analysis.<sup>7</sup>

1.  $\frac{\partial \phi(t, \beta_0, \sigma, B_t)}{\partial \beta_0} = \frac{\beta_t - k}{\beta_0} > 0$ , meaning that, other things being equal, a higher value of  $\beta_0$  implies higher expected severity in  $t > 0$ .<sup>8</sup>
2. The relative impact of the initial severity versus contemporaneous shock  $B_t$ , i.e.,  $\frac{\partial \phi(t, \beta_0, \sigma, B_t)}{\partial \beta_0} / \frac{\partial \phi(t, \beta_0, \sigma, B_t)}{\partial B_t} = \frac{1}{\sigma \beta_0}$ , is decreasing in  $\beta_0$ . This means that a comparatively high value of  $\beta_0$  is less informative about  $\beta_t$  because future severities are more subject to contemporary shocks  $B_t$ . This effect is similar to that of an increase in  $\sigma$ .

These properties allow us to describe the persistence over time of some nonverifiable provider characteristics, which, in our setup, are captured by the parameter  $\beta_0$ .<sup>9</sup> One may expect that, despite the variability from one period to the next in the composition of the population of patients treated, highly specialised hospitals tend to attract, on average, patients with comparatively severe conditions.

## 2.2 Providers

In line with the literature (see, e.g., Chalkley and Malcomson, 1998; Gaynor et al., 2015), we assume that a profit-maximising, risk-neutral provider receives a payment  $p_t$  from a purchaser for each patient treated. The health gain for a patient who receives treatment is indicated with  $x_t$  and depends on the combination of two variables,  $q_t$  and  $e_t$ , under the control of the provider. The first,  $q_t$ , can be interpreted as the intensity of care, and it is assumed to be observable and verifiable. In what follows, we call this component *services*. It can be interpreted as the quantity of different types of services that the patient receives. For hospital care, this measure may include the number of days of hospitalisation and/or

<sup>7</sup>See Bergemann and Strack (2015) for a discussion of these properties.

<sup>8</sup>Baron and Besanko (1984) use the term “informativeness”. For Pavan et al. (2014) and Bergemann and Strack (2015)  $\frac{\partial \phi(t, \beta_0, \sigma, B_t)}{\partial \beta_0}$  is the “impulse response” of  $\beta_0$  to  $\beta_t$ .

<sup>9</sup>For a trendless process of the type described in Eq. 1, the autocorrelation between two values of  $\beta$  is given by:

$$\rho_{s,t} = \frac{\text{cov}(\beta_s, \beta_t)}{\sqrt{V(\beta_s)}\sqrt{V(\beta_t)}} = \left( \frac{e^{\sigma^2 s} - 1}{e^{\sigma^2 t} - 1} \right)^{1/2} < 1$$

where  $s < t$ . Note that  $\rho_{t,s} \rightarrow 0$  both when  $t \rightarrow \infty$  or when  $\sigma \rightarrow \infty$ . In all other cases, we obtain a partial autocorrelation, the magnitude of which is greater the smaller the difference between  $s$  and  $t$ .

the number of diagnostic and therapeutic interventions. The other decision variable for the provider is *effort*,  $e_t$ , which, as usual, is assumed to be private information. This component includes, among others, all those clinical activities not suitable for separate reimbursement but also other dimensions, such as relational quality.

We assume that  $\frac{\partial x}{\partial q} > 0$ ,  $\frac{\partial x}{\partial e} > 0$ . Depending on the type of medical intervention, the marginal productivity of both  $q_t$  and  $e_t$ , or just one of them, may be affected by patient severity. The sign of the marginal impact of severity on the marginal productivity may also be context specific. For technologies combining high effectiveness with high risks, greater intensity of care along the observable dimension may pay more in terms of health gains for high severity patients, i.e.,  $\frac{\partial^2 x}{\partial q \partial \beta} > 0$  (the balance between the health gain and risk would be favourable only for high severity patients). On the other hand, more standard and less invasive treatments may be effective only up to a certain level of severity; in this case, we would have  $\frac{\partial^2 x}{\partial q \partial \beta} < 0$ . Similarly, for the nonverifiable input ( $e_t$ ), one may think of situations where greater severity reduces the impact of effort on health, and others where seriously ill patients can draw greater benefit from an increase in effort. *Watchful waiting*, as applied, for example, in prostate cancer, may be an example of the first, as it provides benefits mainly for patients with early-stage disease ( $\frac{\partial^2 x}{\partial e \partial \beta} < 0$ ), whereas surgery is required for more advanced stages. However, for diabetic patients, the benefits of increased monitoring are likely to be greater for patients whose disease is more severe because they are more likely to suffer from complications ( $\frac{\partial^2 x}{\partial e \partial \beta} > 0$ ).

For the sake of clarity, in the solution process, we focus on one specific case and introduce an explicit form of the production function of the health gain:<sup>10</sup>

$$x_t = q_t + \frac{1}{\beta_t} e_t. \quad (3)$$

where the marginal productivity of effort declines with patient severity.

This form has two major advantages: it keeps the optimal inputs within a bounded interval, and it allows us to interpret the severity parameter  $\beta_t$  as a measure of the rate at which the two inputs should be substituted to maintain the same level of health gain.<sup>11</sup> Although the link between  $e_t$ ,  $\beta_t$  and the contractual variable  $q_t$  available to the regulator is completely deterministic, the patient's health gain  $x_t$  is unobservable to the purchaser. This

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<sup>10</sup>This functional form implies substitutability between services and effort but no complementarity. However, under some circumstances, the two inputs may also be complements, i.e.,  $\frac{\partial^2 x}{\partial q \partial e} > 0$ . For example, Thurston and Libby (2002) find empirical evidence that capital (which is to some extent related to the provision of services in our setting) and physician labour are substitutes but that capital and labour supplied by some types of ancillary workers are complements.

<sup>11</sup>As in Maréchal and Thomas (2021), severity may be interpreted as the result of complications or comorbidities. However, in our case, severity affects the productivity of effort, whereas in Maréchal and Thomas (2021), it has an impact on costs.

can be seen as a generalisation of the problems addressed in Wu et al. (2018) and Maréchal and Thomas (2021), where a signal of patient outcomes can be observed, thus allowing for some form of pay-for-performance.

The provider receives a payment  $p_t$  for each patient and bears a monetary cost related to the provision of services,  $c(q_t)$ , and a nonmonetary cost of effort,  $\psi(e_t)$ . The provider's instant utility function can be written as:

$$u_t = p_t - c(q_t) - \psi(e_t), \quad (4)$$

where  $c(q_t)$  is assumed to be increasing and (weakly) convex (i.e.,  $\frac{\partial c}{\partial q_t} > 0$ ,  $\frac{\partial^2 c}{\partial q_t^2} \geq 0$ ) and the disutility of effort,  $\psi(e_t)$ , is an increasing and strictly convex function of  $e$  (i.e.,  $\frac{\partial \psi}{\partial e_t} > 0$ ,  $\frac{\partial^2 \psi}{\partial e_t^2} > 0$ ).

The provider's intertemporal utility is then:

$$U = E_0 \left[ \int_0^\infty e^{-rt} (p_t - c(q_t) - \psi(e_t)) dt \right]. \quad (5)$$

### 2.3 The purchaser

As is common in the literature on the regulation of health care markets (see, e.g., Brekke et al., 2012b; Levaggi et al., 2014), the purchaser is assumed to maximise consumer surplus, which is given by the difference between the (money-equivalent) health gain  $S(x_t)$  and the price  $p_t$ . The consumer surplus at time  $t$  is:

$$w_t = S(x_t) - p_t, \quad (6)$$

with  $\frac{\partial S(x_t)}{\partial x_t} > 0$  and  $\frac{\partial^2 S(x_t)}{\partial x_t^2} < 0$ , whereas the purchaser's intertemporal utility is:

$$W = E_0 \left[ \int_0^\infty e^{-rt} (S(x_t) - p_t) dt \right]. \quad (7)$$

## 3 First best

In this case, the purchaser has perfect information about the severity  $\beta_t$  and the level of effort  $e_t$ ,  $\forall t \geq 0$ . The purchaser has no incentive to raise the instant utility of the provider above his reservation value. In this context, she is able to set a payment  $p_t$  for each  $t \geq 0$  such that the provider obtains no rent, i.e.,  $u_t = 0$ ,  $\forall t \geq 0$ . We can characterise the first

best by solving:

$$\begin{aligned} \max w_t &= \max_{q_t \geq 0, e_t \geq 0, p_t \geq 0} [(S(x_t) - p_t)] \\ \text{s.t.} \quad u_t &\geq 0 \end{aligned} \tag{8}$$

which yields the first-best combination, corresponding to  $q_t^*$  and  $e_t^*$ .

To highlight the characteristics of the optimal contract and without loss of generality, whenever necessary, we adopt a linear cost function for the quantity of services,<sup>12</sup> with marginal cost equal to one, i.e.,  $c(q_t) = q_t$ , a quadratic function for the disutility of effort, i.e.,  $\psi(e_t) = \frac{e_t^2}{2}$ , and define  $S(x_t) = \frac{x_t^{1-\gamma}}{1-\gamma}$ , with  $0 < \gamma < 1$ . Finally, to avoid corner solutions, we also assume that the minimum level of severity is  $k \geq 1$  (see Appendix B).

The following proposition characterises the first best under the assumptions introduced in Section 2 and for the specific functional forms considered.

**Proposition 1** *The first-best contract is such that  $\forall t$ :*

$$p^*(\beta_t) = q^*(\beta_t) + \frac{e^*(\beta_t)^2}{2} = 1 - \frac{1}{2\beta_t^2}, \tag{9}$$

with

$$e^*(\beta_t) = \frac{1}{\beta_t}, \tag{10}$$

$$q^*(\beta_t) = 1 - \frac{1}{\beta_t^2} \tag{11}$$

*Proof:* See Appendix B.

The optimal level of effort is inversely proportional to patient severity (i.e.,  $\frac{\partial e_t^*}{\partial \beta_t} < 0$ ), while the quantity of services is increasing in severity (i.e.,  $\frac{\partial q_t^*}{\partial \beta_t} > 0$ ). This is consistent with the production function of Eq. 3, where the marginal productivity of effort decreases as severity increases. For the most severely ill patient ( $\beta \rightarrow \infty$ ), the contract is  $q^*(\infty) = 1$ ,  $e^*(\infty) = 0$  and  $p^*(\infty) = 1$ . However, for the least severely ill patient ( $\beta \rightarrow k$ ), the contract is  $q^*(k) = 1 - \frac{1}{k^2}$ ,  $e^*(k) = \frac{1}{k}$  and  $p^*(k) = 1 - \frac{1}{2k^2}$ . Finally, the payment is monotone increasing in severity (i.e.,  $\frac{\partial p_t^*}{\partial \beta_t} > 0$ ).<sup>13</sup> The results above have an intuitive explanation: for the highest severity level, it is optimal to use only services ( $q^*(\infty) = 1$ ,  $e^*(\infty) = 0$ ). For this patient, the

<sup>12</sup>For the general conditions that characterise the solution, see Appendix B.

<sup>13</sup>The assumption that  $k \geq 1$  ensures that  $q_t^* \geq 0$ ,  $\forall t$ . However, even with  $0 < k < 1$ , the optimal contract can be characterised by  $q_t^* = 0$ ,  $e_t^* = \beta_t^{\frac{\gamma-1}{\gamma+1}} < \infty$  (see Appendix B).

(marginal) cost is equal to 1, which is also the first-best optimal price. For the other patients, it is optimal to exert some effort ( $e^* > 0$ ). The optimal payment in this case is determined by subtracting from the cost of the most severe type ( $\beta \rightarrow \infty$ ) the savings in terms of service provision that are enabled because effort also contributes to health achievement.

In addition,  $q_t^*$  and  $e_t^*$  contribute to the achievement of a constant health gain  $x_t$  across patients and through time, i.e.,  $x_t^* = 1, \forall t \geq 0$  (see Appendix B). Therefore, the FB scheme does not create any trade off between health gains and costs.

## 4 Second best

Eq. 2 implies that, even if the purchaser knows  $\beta_0$ , this information would not be sufficient to infer the ex post values of  $\beta_t$ . To solve the intertemporal asymmetric information problem, the purchaser needs to define a mechanism to induce the provider to truthfully report  $\beta_t$  ( $\forall t \geq 0$ ). The timing of the direct mechanism is the following. The purchaser defines a payment flow for each  $t \geq 0$ , as a function of  $\beta_t$ , to which he can commit. In every period, the provider observes  $\beta_t$  and reveals the severity level to the purchaser. Based on the value of  $\beta_t$  revealed, the purchaser makes a payment consistent with the contract initially offered to the provider, who decides on the level of services and effort.

This complex intertemporal adverse selection problem can be solved by exploiting the procedure proposed by Bergemann and Strack (2015).<sup>14</sup> By the properties of the stochastic process Eq. 2, the purchaser’s problem can be divided into two subproblems:

1. For any given initial value of  $\beta_0$ , it is optimal for the purchaser to commit in each  $t > 0$  to the repetition of static contracts. Thus, by the revelation principle, the analysis can be restricted to a direct truthful revelation mechanism where the provider reports  $\beta_t$ .
2. Since each realisation  $\beta_t$  depends on the initial value  $\beta_0$  and contemporaneous shock  $B_t$ , i.e.,  $\beta_t \equiv \phi(\beta_0, B_t)$ , at  $t = 0$ , the purchaser’s problem reduces to induce the provider to report the initial value  $\beta_0$  truthfully.<sup>15</sup>

By the separability of the problem, two instruments are sufficient to obtain a truthful revelation of the two unknowns,  $\beta_t$  ( $t > 0$ ) and  $\beta_0$ . In the next sections, we show that the optimal contract that makes the provider reveal the severity takes the form of a “two-part tariff” scheme, made up of an annuitized fixed part  $F(\beta_0)$  and a time-varying part  $V(\beta_0, \beta_t)$ . Therefore, we can redefine the payment as  $p(\beta_0, \beta_t) = F(\beta_0) + V(\beta_0, \beta_t)$ .

<sup>14</sup>The authors derive an allocative mechanism with respect to a class of deviations defined as “consistent deviations”. See Bergemann and Strack (2015)[p. 826].

<sup>15</sup>Hereafter, we drop the dependence on time in  $\phi(\beta_0, B_t)$  when this does not cause confusion.

## 4.1 Incentive-compatibility conditions

As usual, it is convenient to work backwards, starting from  $t > 0$ . Let us assume that the purchaser has obtained a truthful report of  $\beta_0$  by appropriately defining the payment  $F(\beta_0)$ . The provider's intertemporal utility becomes the sum over time of single standard problems. In particular, since, by Eq. 3, we can write  $e(\beta_0, \beta_t, \hat{\beta}_t) = \beta_t(x(\beta_0, \hat{\beta}_t) - q(\beta_0, \hat{\beta}_t))$ , where  $\hat{\beta}_t$  is the value reported by the provider at time  $t$ , by substituting into Eqs. 4 and 5, we obtain:

$$U(\beta_0, \hat{\beta}_t, \beta_t) = E_0 \left[ \int_0^\infty e^{-rt} \left[ V(\beta_0, \hat{\beta}_t) - c(q(\beta_0, \hat{\beta}_t)) - \psi(\beta_t(x(\beta_0, \hat{\beta}_t) - q(\beta_0, \hat{\beta}_t))) + F(\beta_0) \right] dt \right]. \quad (12)$$

The flow of transfers  $V(\beta_0, \hat{\beta}_t)$  can be used to make truthful reporting of  $\hat{\beta}_t = \beta_t, \forall t > 0$ . Let us define

$$\tilde{u}(\beta_0, \beta_t) = V(\beta_0, \hat{\beta}_t) - c(q(\beta_0, \hat{\beta}_t)) - \psi(\beta_t(x(\beta_0, \hat{\beta}_t) - q(\beta_0, \hat{\beta}_t))). \quad (13)$$

The necessary and sufficient conditions for incentive compatibility that entice the provider to truthfully reveal  $\beta_t, \forall t > 0$ , are (see Appendix C):

$$\frac{d\tilde{u}(\beta_0, \beta_t)}{d\beta_t} = -\psi'(e(\beta_0, \beta_t))(x(\beta_0, \beta_t) - q(\beta_0, \beta_t)) \quad (14)$$

and

$$\frac{d(x(\beta_0, \beta_t) - q(\beta_0, \beta_t))}{d\beta_t} < 0, \quad (15)$$

with  $e(\beta_0, \beta_t) \geq 0$ , and  $q(\beta_0, \beta_t) \geq 0, \forall t > 0$ .

Once  $V(\beta_0, \beta_t)$  has been determined, we have to characterise the fixed part  $F(\beta_0)$ . It is straightforward to show that the purchaser's problem boils down to a standard adverse selection problem. The provider's intertemporal utility becomes:

$$U(\beta_0, \hat{\beta}_0) = E_0 \left[ \int_0^\infty e^{-rt} \left[ V(\hat{\beta}_0, \beta_t) - c(q(\hat{\beta}_0, \beta_t)) - \psi(\phi(\beta_0, B_t)[x(\hat{\beta}_0, \beta_t) - q(\hat{\beta}_0, \beta_t)]) + F(\hat{\beta}_0) \right] dt \right] \quad (16)$$

where  $F(\hat{\beta}_0)$  is such that it is optimal for the provider to truthfully report the initial level, i.e.,  $\hat{\beta}_0 = \beta_0$ . The following condition is necessary and sufficient for incentive compatibility (see Appendix C):

$$\frac{dU(\beta_0)}{d\beta_0} = -E_0 \left[ \int_0^\infty e^{-rt} (\psi'(\phi(\beta_0, B_t))(x(\beta_0, \beta_t) - q(\beta_0, \beta_t))) \frac{\partial \phi(\beta_0, B_t)}{\partial \beta_0} (x(\beta_0, \beta_t) - q(\beta_0, \beta_t)) dt \right], \quad (17)$$

whereas the second-order sufficient condition is:

$$\frac{d(x(\beta_0, \beta_t) - q(\beta_0, \beta_t))}{d\beta_0} \leq 0. \quad (18)$$

## 4.2 Time-varying payment

Let us start from the problem at  $t > 0$ . Given  $\beta_0$ , the purchaser maximises the following objective function:

$$\max_{q_t \geq 0, e_t \geq 0, p_t \geq 0} \int_{\beta^l}^{\beta^h} \left\{ E_0 \left[ \int_0^\infty e^{-rt} (S(x_t) - F(\beta_0) - V(\beta_0, \beta_t)) dt \right] \right\} g(\beta_0) d\beta_0. \quad (19)$$

subject to Eqs. 3, 14, and 15 and the intertemporal participation constraint:

$$U(\beta_0) \geq 0. \quad (20)$$

Since  $F(\beta_0)$  does not depend on  $\beta_t$  ( $t > 0$ ), we have a standard adverse selection problem under asymmetric information (Baron and Myerson, 1982; Laffont and Tirole, 1993), the general solution to which is presented in Appendix D. The following proposition characterises the second-best contract for  $t > 0$  under the assumptions introduced in Section 2 and for the specific functional forms considered.

**Proposition 2** *Under the above assumptions, the purchaser offers,  $\forall t > 0$ , a severity-contingent payment:*

$$V^{**}(\beta_0, \beta_t) = q^{**}(\beta_0, \beta_t) + \frac{[e^{**}(\beta_0, \beta_t)]^2}{2} + \int_{\beta_t}^\infty \frac{[e^{**}(\beta_0, z)]^2}{z} dz, \quad (21)$$

with

$$e_t^{**} = \left[ \beta_t + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} (\beta_t - k) \right]^{-1} < e_t^* \quad (22)$$

$$q_t^{**} = 1 - \frac{1}{\beta_t} \left[ \beta_t + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} (\beta_t - k) \right]^{-1} > q_t^*. \quad (23)$$

*Proof:* See Appendix D.

For any given  $\beta_0$ , the optimal effort is decreasing in  $\beta_t$  (i.e.,  $\frac{\partial e_t^{**}}{\partial \beta_t} < 0$ ), while optimal services are increasing in  $\beta_t$  (i.e.,  $\frac{\partial q_t^{**}}{\partial \beta_t} > 0$ ). Additionally, in the second best,  $q_t$  and  $e_t$  are combined to ensure a constant health gain, i.e.,  $x_t^{**} = 1$ .

Finally, substituting (21) into (5) gives:

$$U(\beta_0) = E_0 \left[ \int_0^\infty e^{-rt} \left[ \int_{\beta_t}^\infty \frac{[e^{**}(\beta_0, z)]^2}{z} dz + F(\beta_0) \right] dt \right]. \quad (24)$$

### 4.3 Two-part tariff

Since, by construction,  $V^{**}(\beta_0, \beta_t)$ , is such that the provider reports his true type  $\beta_t$  based on his initial report  $\beta_0$ , this transforms the reporting problem at  $t = 0$  into the static design problem for the truthful revelation of  $\beta_0$  (Eqs. 16–18). By applying the Envelope theorem to Eq. 17, we obtain:

$$U(\beta_0) = \int_{\beta_0}^{\beta^h} E_0 \left[ \int_0^\infty e^{-rt} \left[ \frac{e^{**}(y, \beta_t)^2}{\beta_t} \frac{\partial \phi(y, B_t)}{\partial y} \right] dt \right] dy. \quad (25)$$

Thus, by using Eqs. 24 and 25, we can solve for  $F(\beta_0)$ .

Under the assumptions introduced in Section 2 and for the specific functional forms considered, the next proposition characterises the second-best optimal payment.

**Proposition 3** *The second-best optimal payment by the regulator is made up of a fixed and a time-varying component. The fixed component is given by:*

$$F^{**}(\beta_0) = rE_0 \left[ \int_0^\infty e^{-rt} \left[ \int_{\beta_0}^{\beta^h} \frac{[e^{**}(y, \phi(y, B_t))]^2}{\phi(y, B_t)} \frac{\phi(y, B_t) - k}{y} dy - \int_{\beta_t}^\infty \frac{[e^{**}(\beta_0, z)]^2}{z} dz \right] dt \right]. \quad (26)$$

whereas the severity-contingent payment is described by Eq. 21.

*Proof:* See Appendix D.

## 5 Characteristics of the two-part tariff

### 5.1 Provider behaviour

The second-best optimal levels of effort and services are characterised by Eqs. 22 and 23, respectively. It is immediately clear that  $e_t^{**}$  and  $q_t^{**}$  are such that  $x_t^{**} = 1, \forall \beta_t$ . At each point in time, it is optimal to equalise health outcomes across patients with different levels of severity. Moreover, patient outcomes are as in the first best. From a policy point of view, this is an interesting result because it preserves equity across patients<sup>16</sup>

<sup>16</sup>Notice that, in some of the related literature, the independence of the final health outcome from severity is an assumption rather than a result (see, e.g., Siciliani, 2006).



Additionally, consistent with the first-best solution is the fact that effort is decreasing in  $\beta_t$  whereas services are increasing (i.e.,  $\frac{\partial e_t^{**}}{\partial \beta_t} < 0$ ,  $\frac{\partial q_t^{**}}{\partial \beta_t} > 0$ ; see Appendix D). However, due to information asymmetry, the combination of inputs is distorted in favour of the observable component:  $e_t^{**} \leq e_t^*$  and  $q_t^{**} \geq q_t^*$ . Equality holds only for the lowest level of severity: if  $\beta_t = k$ ,  $e^{**}(\beta_0, k) = e^*(k)$  and  $q^{**}(\beta_0, k) = q^*(k)$ . As usual, it is optimal to pay rents that induce the first-best combination of effort and services as  $\beta_t$  approaches  $k$ , as this corresponds to the case where effort is most productive.

As  $\beta_t$  increases, the substitution of effort with services grows more rapidly in the second than in the first best as a result of information asymmetry. However, for  $\beta_t \rightarrow \infty$ ,  $e^{**}(\beta_0, \infty) = e^*(\infty) = 0$  and  $q^{**}(\beta_0, \infty) = q^*(\infty) = 1$ , meaning that distortions tend to shrink above a certain level of severity. This is because with  $\beta_t \rightarrow \infty$ , the marginal productivity of effort goes to zero, meaning that it is efficient to use only the verifiable input. As a result, distortions are largest for intermediate values of severity.

It is also interesting to study the impact of  $\beta_0$  on effort and services. By totally differentiating Eqs. 22 and 23, we obtain:

$$\frac{de_t^{**}}{d\beta_0} = \frac{\partial e_t^{**}}{\partial \beta_0} + \frac{\partial e_t^{**}}{\partial \beta_t} \frac{\beta_t - k}{\beta_0} < 0, \quad (27)$$

$$\frac{dq_t^{**}}{d\beta_0} = \frac{\partial q_t^{**}}{\partial \beta_0} + \frac{\partial q_t^{**}}{\partial \beta_t} \frac{\beta_t - k}{\beta_0} > 0. \quad (28)$$

Thus,  $\beta_0$  affects  $e_t^{**}$  and  $q_t^{**}$  directly through information rents and indirectly by affecting the expected value of  $\beta_t$ . The latter is given by the measure of informativeness  $\frac{\partial \phi(\beta_0, B_t)}{\partial \beta_0} = \frac{\beta_t - k}{\beta_0} > 0$ , which captures the impact on  $\beta_t$  of a marginal change in  $\beta_0$ . Overall, *ceteris paribus*, a higher level of severity in  $t = 0$  leads to a substitution of services for effort.

## 5.2 Payment

To highlight the characteristics of the second-best optimal contract, it is convenient to write the full payment scheme, including both the variable and the fixed components:

$$\begin{aligned} p^{**}(\beta_0, \beta_t) &= F^{**}(\beta_0) + V^{**}(\beta_0, \beta_t) \\ &= rE_0 \left[ \int_0^\infty e^{-rt} \left[ \int_{\beta_0}^{\beta_t} \frac{[e^{**}(y, \phi(y, B_t))]^2}{\phi(y, B_t)} \frac{\phi(y, B_t) - k}{y} dy - \int_{\beta_t}^\infty \frac{[e^{**}(\beta_0, z)]^2}{z} dz \right] dt \right] \\ &\quad + q^{**}(\beta_0, \beta_t) + \frac{[e^{**}(\beta_0, \beta_t)]^2}{2} + \int_{\beta_t}^\infty \frac{[e^{**}(\beta_0, z)]^2}{z} dz \end{aligned} \quad (29)$$

The second and third lines on the RHS of Eq. 29 correspond, respectively, to the fixed

and variable components of the second-best price. The variable component is composed of the costs incurred to provide the second-best level of services and effort,  $\left(q^{**}(\beta_0, \beta_t) + \frac{[e^{**}(\beta_0, \beta_t)]^2}{2}\right)$ , and the information rent associated with truthful revelation of  $\beta_t$  at  $t > 0$ ,  $\left(\int_{\beta_t}^{\infty} \frac{[e^{**}(\beta_0, z)]^2}{z} dz\right)$ . Notice, however, that the same term related to the information rent also appears, with negative sign, in the fixed component (second term in the second line). That is, since  $\beta_0$  is informative, to some extent, about future levels of severity  $\left(\frac{\phi(\beta_0, B_t) - k}{\beta_0} = \frac{\partial \phi(\beta_0, B_t)}{\partial \beta_0}\right)$  knowing it allows the purchaser to reduce the payment to the hospital of an amount that, in expected terms, equals the rent that will be paid in the future to induce truthful revelation of  $\beta_t$ . Thus, in defining the annuitized amount of the fixed component, the purchaser subtracts from the amount related to the informative content of  $\beta_0$  (first term in the second line), the amount corresponding to the expected value at  $t = 0$  of information rents to be paid in subsequent periods. This leads to a negative value of the fixed component (see Appendix F).

In Appendix F we also show that the fixed component is increasing (decreasing in absolute value) in  $\beta_0$ . This is again the result of the combination of two effects going in opposite directions. A first effect arises because a comparatively high value of  $\beta_0$  leads to higher values of  $\beta_t$ . Therefore, the higher the revealed value of  $\beta_0$  is, the lower are the information rents that the regulator expects to pay in the future, and, therefore, the higher is the fixed part. However, Eq. 29 shows that there is a second effect with the opposite sign: the value of the first term of the second line of Eq. 29 decreases when  $\beta_0$  is comparatively high because a higher value of  $\beta_0$  is less informative in terms of the ability to predict  $\beta_t$ . Overall, we show that the former effect prevails.

### 5.3 Distortions over time

Finally, we study how distortions implied by the second-best contract change over time. To do so, we consider the ratio  $\frac{e_t^*}{e_t^{**}} = 1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} \frac{\beta_t - k}{\beta_t}$  as a proxy for the (relative) size of the distortion at time  $t$ . Taking the expectation at time  $t = 0$ , it can be shown that (see Appendix E):

$$E_0 \left( \frac{e_t^*}{e_t^{**}} \right) = 1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} \left\{ 1 - \left[ \left( \frac{k}{\beta_0} \right)^{-3} e^{-3\sigma^2 kt} + 2k \left( 1 - e^{-3\sigma^2 kt} \right) \right]^{-1/3} \right\}, \quad (30)$$

where  $\left[ \left( \frac{k}{\beta_0} \right)^{-3} e^{-3\sigma^2 kt} + 2k \left( 1 - e^{-3\sigma^2 kt} \right) \right]^{-1/3}$  is the expected impact of the initial shock  $\beta_0$  on the distortion at time  $t$ . Taking the limit of Eq. 30, we obtain:

$$\lim_{t \rightarrow \infty} E_0 \left[ \left( \frac{e_t^*}{e_t^{**}} \right) \right] = 1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} [1 - (2k)^{-1/3}], \quad (31)$$

which defines the convergence value for the relative distortion. As  $k \geq 1$  implies  $[1 - (2k)^{-1/3}] < 1$ , this reflects the fact that although, as time goes on, the effect of the initial severity  $\beta_0$  on  $\beta_t$  reduces, it never disappears.

Moreover, taking the derivative of Eq. 30 with respect to  $\beta_0$ , it is easy to show that  $\frac{\partial E_0 \left( \frac{e_t^*}{e_t^{**}} \right)}{\partial \beta_0} > 0$ . In words, as a high value of  $\beta_0$  is not very informative about future values of  $\beta_t$ , the higher the revealed value of  $\beta_0$ , the greater are the information rents that the purchaser expects to pay in the future, and therefore, the higher are the expected distortions in the use of inputs. This suggests that hospitals with high initial values of patient complexity may experience comparatively large distortions.

## 6 A case study

In this section, we provide a numerical illustration of the characteristics of the second-best optimal contract by calibrating the model using data on hip replacement from a sample of Italian hospitals. In particular, we rely on hospital discharge records for the years between 2010 and 2016 of 225 hospitals from five of the largest Italian regions (Emilia-Romagna, Lombardy, Piedmont, Veneto, Tuscany). The dataset includes 678,094 observations, which are used to define the parameters of the stochastic process described in Eq. 1.

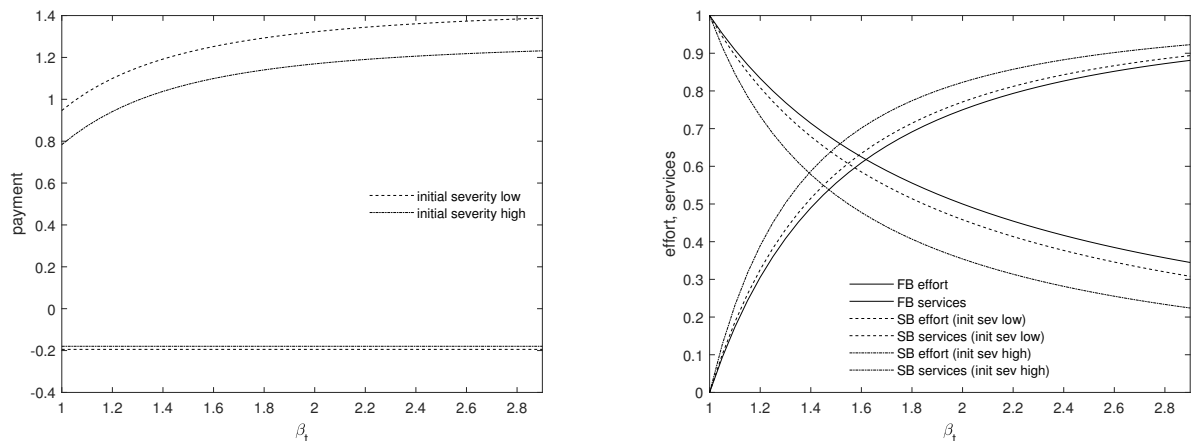
Since none of the variables included in the dataset can be directly interpreted as a measure of patient severity, the first step is to construct one. To this end, we follow an approach frequently adopted in the literature, whereby information on secondary diagnoses is used to obtain the Charlson comorbidity index (Charlson et al., 1987).<sup>17</sup> Considerable evidence shows that this index is strongly correlated with patient outcomes and healthcare costs (see, e.g., Johnson et al., 2015; Ofori-Asenso et al., 2018; Whitmore et al., 2014), which makes it a suitable proxy for severity in our setting.

The value of the index in our sample ranges between 0 and 12. To make it consistent with the fact that in our baseline model  $k \geq 1$ , we rescale the values of the Charlson index by adding one unit to the originally calculated values and use the rescaled values to proxy the value of  $\beta_t$ .

Another required adjustment is discretization of the stochastic process. The discrete-

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<sup>17</sup>For each episode, a value of the Charlson index was computed with the Stata routine developed by Stagg (2017).



(a) Variable and fixed component of payment as a function of  $\beta_t$  for different values of  $\beta_0$ . (b) Effort and services as a function of  $\beta_t$  for different values of  $\beta_0$ . Functions increasing (decreasing) in  $\beta_t$  refer to services (effort).

Figure 1: Characteristics of the second-best optimal contract and the role of  $\beta_0$ .

time approximation of Eq. 1 can be written as:

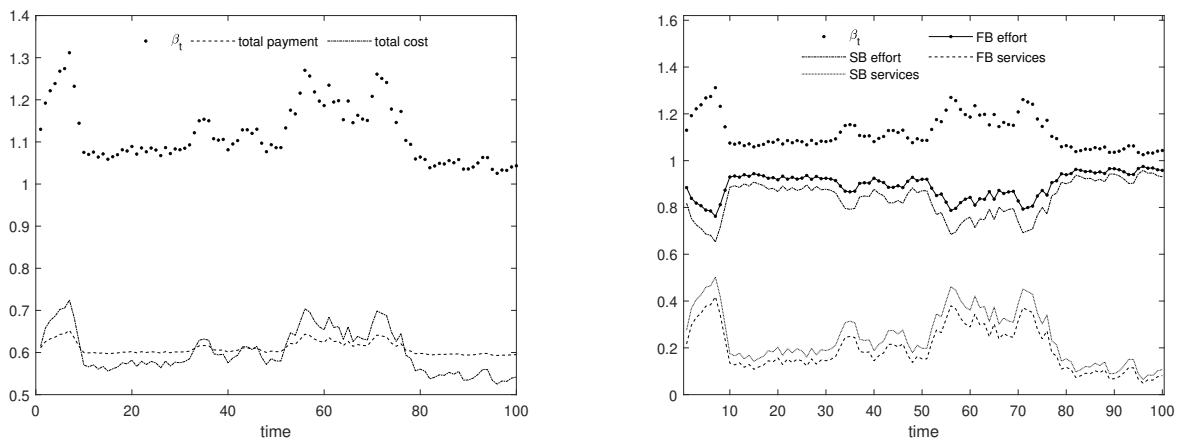
$$\frac{\beta_{t+1} - \beta_t}{\beta_t - k} = \sigma(B_{t+1} - B_t), \quad (32)$$

where we can estimate  $\sigma$  by computing the standard deviation of the sampling distribution of the left-hand side of Eq. 32. In particular, for each hospital, we compute the percentage change in the complexity index from one patient to the next. We take the mean across institutions of the standard deviations of the percentage changes over time as our proxy of  $\sigma$ . This leads us to a calibrated value of  $\sigma = 20\%$ .

The distribution of the initial value of severity is assumed to be uniform between 1 and 1.7, where the upper extreme corresponds to the highest among the mean values of the adjusted severity index computed for each hospital over the first month of the first year of observation.<sup>18</sup> The fixed component of payment (Eq. 26) involves an expected value that cannot be computed analytically. We compute it using a Monte Carlo simulation with 1000 replications over 500 periods, using a discount rate of 5% (per year).

A distinguishing characteristic of our model is that, for a given value of  $\beta_t$ , the variable component of payment also depends on  $\beta_0$ . Figure 1(a) shows the values of the two payment components for different values of  $\beta_t$  (horizontal axis) and for different values of initial severity:  $\beta_0$  close to the lower and upper extremes of the range assumed for this parameter. A high value of  $\beta_0$  is associated with a lower variable payment for a given  $\beta_t$  and a higher (smaller in absolute value) fixed payment. Figure 1(b) shows the resulting levels of effort

<sup>18</sup>We consider the first month and not the first patient for each institution to avoid the impact of outliers.



(a) Total payment (sum of variable and fixed component) and total cost.

(b) First- and second-best effort and services.

Figure 2: Characteristics of the second-best optimal contract over time for a specific path of  $\beta_t$ .

and services and provides a comparison with the first best. The lower value of rents paid when  $\beta_0$  is high, resulting from the lower variable payment shown in Figure 1(a), widens the distance between the first and the second best (see Figure 1(b)). This implies a stronger tendency to favour the observable (services) against the unobservable (effort) dimension.

For illustrative purposes, Figure 2 shows what happens over time for one specific path of  $\beta_t$  among those generated according to the stochastic process of Eq. 1 with the parameter values described above. We also set  $\beta_0 = 1.13$ , which is the average value of the severity index for the whole sample in the first period. Figure 2(a) shows how the total payment changes over time as patients' severity, indicated by dots in the upper part of the figure, changes and compares it with the total cost of treatment provision. When the value of  $\beta_t$  is high, total costs are high. However, only a fraction of the increase in costs is covered by an increase in payment. In our simulation, the average (over all replications and all periods) elasticity of payment with respect to total cost is 0.2.

This second-best optimal contract departs from a fixed payment per case as under a pure form of DRG since it also includes a cost-sharing component (Ellis and McGuire, 1986). Another key difference is that the amount of cost-sharing and the fixed component of payment depend on the structural characteristic,  $\beta_0$  (see Figure 1(a)). As a result of these characteristics, the total payment is lower (higher) than total costs when  $\beta_t$  is comparatively high (low).

Figure 2(b) shows how second-best effort and services vary over time as  $\beta_t$  changes and compares them with the first-best levels. Consistent with our theoretical findings, when  $\beta_t$  grows, providers substitute effort with services. This also leads to larger inefficiencies, as

shown by the widening gap between the first- and second-best levels. When  $\beta_t$  is low, the gap is reduced due to the higher value of information rents paid.

Finally, we use the present case study to illustrate the efficiency gains that an optimal dynamic contract allows by comparing it against a static benchmark. We take as a benchmark an incentive-compatible contract that is designed in a static framework and repeated in each period and thus fails to account for the stochastic process that defines the evolution of  $\beta_t$  over time. We assume that the purchaser knows the density  $g(\beta_t)$  and the cumulative distribution function  $G(\beta_t)$ , as well as  $\beta^l$  and  $\beta^h > 0$ , with  $\beta_t \in [\beta^l \geq k, \beta^h]$ .

We show in Appendix G that, under these assumptions, the optimal static contract can be characterised as follows:

$$p^{***}(\beta_t) = q^{***}(\beta_t) + \frac{(e^{***}(\beta_t))^2}{2} + \int_{\beta_t}^{\beta^h} \frac{(e^{***}(\beta_t))^2}{z} dz, \quad (33)$$

with

$$e_t^{***} = \left[ \beta_t + 2 \frac{G(\beta_t)}{g(\beta_t)} \right]^{-1} \quad (34)$$

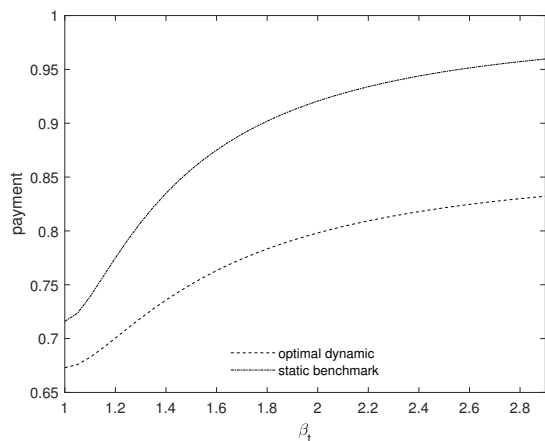
$$q_t^{***} = 1 - \frac{1}{\beta_t} \left[ \beta_t + 2 \frac{G(\beta_t)}{g(\beta_t)} \right]^{-1}. \quad (35)$$

As for the other contracts we previously considered, the combination of services and effort is such to ensure  $x_t^{***} = 1, \forall \beta_t$ . To ease the comparison of the two types of contracts, in the numerical exercise, we assume that  $\beta_t$  ( $t \geq 0$ ) is uniformly distributed between 1 and 2.9, where the upper extreme corresponds to the 99<sup>th</sup> percentile of the distribution of the values of  $\beta_t$  previously simulated for the solution of the optimal dynamic contract.

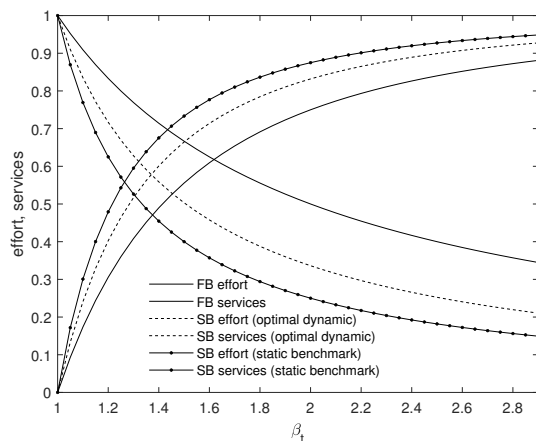
Figure 3 provides a comparison of this optimal static contract and the optimal dynamic contract discussed in the previous sections. Figure 3(a) shows that the total payment under the optimal dynamic contract is lower than that under the static benchmark for the whole range of values of  $\beta_t$  considered. Moreover, as shown in Figure 3(b), effort is higher and the level of services lower for the optimal dynamic contract than for the static benchmark over the whole range of values of  $\beta_t$  considered. If we recall that in both cases  $x_t = 1 \forall \beta_t$ , the optimal dynamic contract dominates the static benchmark.

## 7 Conclusion

From the end of the 1980s, a clear tendency emerged in the organisation of healthcare systems to separate the role of providers of services from that of purchasers. A major



(a) Payment as a function of  $\beta_t$  (payment for the optimal dynamic contract is the sum of the fixed and variable component).



(b) Effort and services as a function of  $\beta_t$ .

Figure 3: Comparison of the optimal dynamic and optimal static contracts.

challenge in the design of efficient contracts between the two parties is the existence of an information advantage for the provider on several dimensions. This paper characterises an optimal dynamic contract allowing for both moral hazard and adverse selection in a setting where patient disease severity evolves stochastically over time. This provides a tool to incentivise the provision of effort and healthcare quality in settings where competition does not work as an incentive mechanism. Moreover, it allows us to account for the long-lasting nature of the relationship between the purchaser and the provider in a setting where patients' severity changes over time and providers are heterogeneous with respect to the expected complexity of their case mix.

The optimal dynamic contract entails a two-part tariff made up of an annuitized fixed payment and a time-varying component. At the initial time, the provider chooses from a menu of contracts that entail different combinations of fixed and variable payments. The fixed payment is designed to induce truthful revelation of the provider's structural characteristic, which affects the expectations of future patients' severity. Over time, the variable payments are affected by both current patients' severity and the provider's structural characteristic. A provider whose structural characteristic is such that future patients are expected to be comparatively severely ill will receive a higher fixed component but, for a given level of patient complexity, a lower variable payment. An interesting result concerning the equity dimension is that in our model, patients' severity affects the combination of unobservable effort and observable services that they receive but not the health gain, which is the same irrespective of initial severity.

The dependency of the variable component on severity is a departure from the pure

model of prospective payment, but it is in line with previous results from the literature (Ellis and McGuire, 1986; Chalkley and Malcomson, 2002) and with refinements of the DRG system that have been widely adopted in several countries (Busse et al., 2013). However, our results show that, for the optimal dynamic contract, adjustments to payments based on patient severity (*cost-sharing*) should not be uniform for all providers, as they typically are, but should also depend on hospitals' structural characteristics. We used a case study based on data of hip replacement hospitalisations in Italy to illustrate the properties of the optimal dynamic contract and the efficiency gains achieved with respect to an optimal static contract.

The mechanism that we study is direct. This naturally raises questions on the implementability of the contract. The most straightforward way to operationalise it would be to make payment contingent on the level of services provided, i.e., the observable dimension. This indirect mechanism would be feasible, provided that the function that maps severity to the second-best optimal provision of services is invertible. At the initial time, the provider could be given the opportunity to choose from a menu of contracts, each of which defines the amount of a fixed payment and a relationship between the amount of (observable) services provided in each period and the variable component of payment. Higher fixed payments would be associated with lower levels of reimbursement for a given amount of services provided.

There are several directions in which our model could be extended. For example, we assume that effort and services are substitutes in the health production function, and it would be interesting to investigate the impact of assuming some degree of complementarity. Concerning the constraints under which our second-best optimal contract is defined, we consider an intertemporal participation constraint for the provider, but the constraint could also be assumed to be binding in every period.

We hope that our findings will help increase interest in the development of further investigations of dynamic contracts for the provision of healthcare services, which seem to be understudied in light of the long-lasting character of purchaser–provider relationships.

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## A Properties of the diffusion process

Defining  $y_t = \ln(\beta_t - k)$  by Ito's lemma, we obtain  $d\ln(\beta_t - k) = -\frac{1}{2}\sigma^2 dt + \sigma dB_t$ . Integrating both sides between 0 and  $t$ , we obtain:

$$\beta_t - k = \exp(y_0) \exp\left(-\frac{1}{2}\sigma^2 t + \sigma B_t\right), \quad \text{with } B_0 = 0$$

. Finally, defining  $\exp(y_0) = \beta_0$ , Eq. 2 is obtained. If  $B_t \rightarrow \infty$ , then  $\beta_t - k \rightarrow \infty$ . In contrast, if  $B_t \rightarrow -\infty$ , we obtain  $\beta_t - k \rightarrow 0$ . Thus,  $\beta_0$  must also lie in the interval  $[k, \infty)$ .

## B First-best solutions

From the participation constraint,  $p_t = c(q_t) + \psi(e_t)$ . Substituting into Eq. 8, we obtain:

$$S(x_t) - p_t = S\left(q_t + \frac{1}{\beta_t}e_t\right) - c(q_t) - \psi(e_t). \quad (36)$$

As  $c'(q_t) \geq 0$ , the purchaser should maximise Eq. 36 under the constraint that  $q_t \geq 0$ . We form the Lagrangian:

$$\mathcal{L}(q_t, e_t, \lambda) = S\left(q_t + \frac{1}{\beta_t}e_t\right) - c(q_t) - \psi(e_t) - \lambda(-q_t)$$

and the first-order conditions for the optimal solutions are:

$$S'\left(q_t + \frac{1}{\beta_t}e_t\right) - c'(q_t) + \lambda = 0 \quad (37)$$

$$S'\left(q_t + \frac{1}{\beta_t}e_t\right)\frac{1}{\beta_t} - \psi'(e_t) = 0 \quad (38)$$

and

$$q_t \geq 0; \quad \lambda q_t = 0; \quad \lambda \geq 0.$$

In the case of the specific functional forms introduced in Section 3 and  $k = 1$  (i.e.,  $\beta_t \geq 1$ ), we have that  $\lambda = 0$ , and the solution is characterised as in Proposition 1. If we let  $\beta_t$  be smaller than one (i.e.,  $k < 1$ ), the solution is:

$$e_t^* = (\beta_t)^{\frac{\gamma-1}{\gamma+1}} \quad (39)$$

$$q_t^* = 0 \quad (40)$$

$$\lambda = 1 - (\beta_t)^{\frac{2\gamma}{\gamma+1}} < 1 \quad (41)$$

$$x_t^* = (\beta_t)^{-\frac{2}{\gamma+1}} \quad (42)$$

$$p_t^* = (\beta_t)^{2\frac{\gamma-1}{\gamma+1}}. \quad (43)$$

## C Incentive-compatibility conditions

Starting from the definition of Eq. 13,<sup>19</sup> the FOC for the optimal report  $\hat{\beta}$  is:

$$\frac{\partial \tilde{u}(\beta, \hat{\beta})}{\partial \hat{\beta}} = \frac{dV(\hat{\beta})}{d\hat{\beta}} - c'(q(\hat{\beta})) \frac{dq(\hat{\beta})}{d\hat{\beta}} - \psi'(e(\beta, \hat{\beta}))\beta \left( \frac{dx(\hat{\beta})}{d\hat{\beta}} - \frac{dq(\hat{\beta})}{d\hat{\beta}} \right) = 0. \quad (44)$$

For the truthful revelation to be an optimal response, it must be the case that:

$$\left. \frac{\partial \tilde{u}(\beta, \hat{\beta})}{\partial \hat{\beta}} \right|_{\hat{\beta}=\beta} = 0. \quad (45)$$

Further, the local second-order condition (SOC) is:

$$\begin{aligned} \frac{\partial^2 \tilde{u}(\beta, \hat{\beta})}{\partial \hat{\beta}^2} &= \frac{d^2V(\hat{\beta})}{d\hat{\beta}^2} - c''(q(\hat{\beta})) \frac{dq(\hat{\beta})}{d\hat{\beta}} - c'(q(\hat{\beta})) \frac{d^2q(\hat{\beta})}{d\hat{\beta}^2} \\ &\quad - \psi''(e(\beta, \hat{\beta}))\beta \left( \frac{dx(\hat{\beta})}{d\hat{\beta}} - \frac{dq(\hat{\beta})}{d\hat{\beta}} \right) - \psi'(e(\beta, \hat{\beta}))\beta \left( \frac{d^2x(\hat{\beta})}{d\hat{\beta}^2} - \frac{d^2q(\hat{\beta})}{d\hat{\beta}^2} \right) \Bigg|_{\hat{\beta}=\beta} \leq 0. \end{aligned} \quad (46)$$

Noting that for  $\hat{\beta} = \beta$  Eq. 44 is an identity, we can totally differentiate to obtain:

$$\begin{aligned} \frac{d^2V(\hat{\beta})}{d\hat{\beta}^2} - c'' \frac{dq(\hat{\beta})}{d\hat{\beta}} - c' \frac{d^2q(\hat{\beta})}{d\hat{\beta}^2} - \psi''(\cdot)\beta \left( \frac{dx(\hat{\beta})}{d\hat{\beta}} - \frac{dq(\hat{\beta})}{d\hat{\beta}} \right) - \psi'(\cdot)\beta \left( \frac{d^2x(\hat{\beta})}{d\hat{\beta}^2} - \frac{d^2q(\hat{\beta})}{d\hat{\beta}^2} \right) + \\ - \psi'(\cdot) \left( \frac{dx(\hat{\beta})}{d\hat{\beta}} - \frac{dq(\hat{\beta})}{d\hat{\beta}} \right) \Bigg|_{\hat{\beta}=\beta} = 0. \end{aligned} \quad (47)$$

By replacing Eq. 47 into 46, we find:

$$\left. \frac{\partial^2 \tilde{u}(\beta, \hat{\beta})}{\partial \hat{\beta}^2} \right|_{\hat{\beta}=\beta} = \psi' \left( \frac{dx(\beta)}{d\beta} - \frac{dq(\beta)}{d\beta} \right) \leq 0, \quad (48)$$

which holds if the inequality in Eq. 15 of the main text is satisfied. Let us now consider the problem at  $t = 0$ . The provider's intertemporal utility can be written as in Eq. 16, where  $\hat{\beta}_0$  is the provider's report at  $t = 0$  and, from Eq. 45,  $\phi(\hat{\beta}_0, B_t)$  is the report at  $t > 0$ . Since Eq. 45 is satisfied, the FOC for the optimal report at  $t = 0$  reduces to:

$$\begin{aligned} \frac{\partial U(\beta_0, \hat{\beta}_0)}{\partial \hat{\beta}_0} &= E_0 \left[ \int_0^\infty e^{-rt} \left[ \frac{\partial V(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0} - c'(q(\hat{\beta}_0, \beta_t)) \frac{\partial q(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0} \right. \right. \\ &\quad \left. \left. - \psi'(\phi(\beta_0, B_t))(x(\hat{\beta}_0, \beta_t) - q(\hat{\beta}_0, \beta_t))\phi(\beta_0, B_t) \left( \frac{\partial x(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0} - \frac{\partial q(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0} \right) + \frac{\partial F(\hat{\beta}_0)}{\partial \hat{\beta}_0} \right] dt. \right] = 0 \\ &= E_0 \left[ \int_0^\infty e^{-rt} \left[ \frac{\partial V(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0} - \frac{\partial q(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0} \right. \right. \\ &\quad \left. \left. - e(\hat{\beta}_0, \phi(\beta_0, B_t))\phi(\beta_0, B_t) \left( \frac{\partial x(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0} - \frac{\partial q(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0} \right) + \frac{\partial F(\hat{\beta}_0)}{\partial \hat{\beta}_0} \right] dt \right] = 0 \end{aligned} \quad (49)$$

<sup>19</sup>To simplify the notation, we omit  $\beta_0$  and the time dependence of  $\beta_t$ .

For the truthful revelation to be an optimal response, it must be the case that  $\hat{\beta}_0 = \beta_0$ , i.e.,

$$\left. \frac{\partial U(\beta_0, \hat{\beta}_0)}{\partial \hat{\beta}_0} \right|_{\hat{\beta}_0 = \beta_0} = 0.$$

Again, the local SOC is:

$$\frac{\partial^2 U(\beta_0, \hat{\beta}_0)}{\partial \hat{\beta}_0^2} = E_0 \left[ \int_0^\infty e^{-rt} \left[ \frac{\partial^2 V(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0^2} - \frac{\partial^2 q(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0^2} - \frac{\partial e(\hat{\beta}_0, \phi(\beta_0, B_t))}{\partial \hat{\beta}_0} \phi(\beta_0, B_t) \left( \frac{\partial x(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0} - \frac{\partial q(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0} \right) \right. \right. \\ \left. \left. - e(\hat{\beta}_0, \phi(\beta_0, B_t)) \phi(\beta_0, B_t) \left( \frac{\partial^2 x(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0^2} - \frac{\partial^2 q(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0^2} \right) + \frac{\partial^2 F(\hat{\beta}_0)}{\partial \hat{\beta}_0^2} \right] dt \right] \Big|_{\hat{\beta}_0 = \beta_0} \leq 0. \quad (50)$$

By totally differentiating Eq. 49, we obtain:

$$E_0 \left[ \int_0^\infty e^{-rt} \left[ \frac{\partial^2 V(\hat{\beta}_0, \beta_t)}{\partial^2 \hat{\beta}_0} - \frac{\partial^2 q(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0^2} - \frac{\partial e(\hat{\beta}_0, \phi(\beta_0, B_t))}{\partial \hat{\beta}_0} \phi(\beta_0, B_t) \left( \frac{\partial x(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0} - \frac{\partial q(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0} \right) \right. \right. \\ \left. \left. - e(\hat{\beta}_0, \phi(\beta_0, B_t)) \left( \frac{\partial \phi(\beta_0, B_t)}{\partial \beta_0} \left( \frac{\partial x(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0} - \frac{\partial q(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0} \right) + \phi(\beta_0, B_t) \left( \frac{\partial^2 x(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0^2} - \frac{\partial^2 q(\hat{\beta}_0, \beta_t)}{\partial \hat{\beta}_0^2} \right) \right) + \frac{\partial^2 F(\hat{\beta}_0)}{\partial \hat{\beta}_0^2} \right] dt. \right] \Big|_{\hat{\beta}_0 = \beta_0} \quad (51)$$

Finally, by substituting Eq. 51 into 50, we obtain:

$$\left. \frac{\partial^2 U(\beta_0, \hat{\beta}_0)}{\partial \hat{\beta}_0^2} \right|_{\hat{\beta}_0 = \beta_0} = E_0 \left[ \int_0^\infty e^{-rt} \left[ e(\beta_0, \phi(\beta_0, B_t)) \frac{\partial \phi(\beta_0, B_t)}{\partial \beta_0} \left( \frac{\partial x(\beta_0, \beta_t)}{\partial \beta_0} - \frac{\partial q(\beta_0, \beta_t)}{\partial \beta_0} \right) \right] dt \right] \leq 0. \quad (52)$$

A sufficient condition for the condition of Eq. 52 to hold is provided by Eq. 18 in the main text.

## D Optimal dynamic contract

The standard approach to solving Eq. 19 is to ignore, for the moment, the second-order conditions of Eqs. 15 and 18 and to solve the relaxed problem. By the Envelope theorem (Milgrom and Segal, 2002, Theorems 1 and 2), Eq. 17 implies that:

$$U(\beta_0) = \int_{\beta_0}^{\beta^h} E_0 \left[ \int_0^\infty e^{-rt} \left[ \psi'(\phi(y, B_t)(x(y, \beta_t) - q(y, \beta_t))) \frac{\partial \phi(y, B_t)}{\partial y} (x(y, \beta_t) - q(y, \beta_t)) \right] dt \right] dy \\ = \int_{\beta_0}^{\beta^h} E_0 \left[ \int_0^\infty e^{-rt} \left[ \psi'(e(y, \beta_t)) \frac{1}{\beta_t} e(y, \beta_t) \frac{\partial \phi(y, B_t)}{\partial y} \right] dt \right] dy, \quad (53)$$

where the highest  $\beta_0$  obtains zero utility, i.e.,  $U(\beta^h) = 0$ . By using integration by part, we find:

$$\int_{\beta^l}^{\beta^h} U(\beta_0) g(\beta_0) d\beta_0 = \int_{\beta^l}^{\beta^h} \left\{ \int_{\beta_0}^{\beta^h} E_0 \left[ \int_0^\infty e^{-rt} \left[ \psi'(e(y, \beta_t)) \frac{1}{\beta_t} e(y, \beta_t) \frac{\partial \phi(y, B_t)}{\partial y} \right] dt \right] dy \right\} g(\beta_0) d\beta_0 \\ = \int_{\beta^l}^{\beta^h} E_0 \left[ \int_0^\infty e^{-rt} \left[ \psi'(e(y, \beta_t)) \frac{1}{\beta_t} e(y, \beta_t) \frac{\partial \phi(y, B_t)}{\partial y} \right] dt \right] \frac{G(\beta_0)}{g(\beta_0)} g(\beta_0) d\beta_0. \quad (54)$$



By using Eq. 4, we obtain:

$$\begin{aligned}
& \int_{\beta^l}^{\beta^h} \left\{ E_0 \left[ \int_0^\infty e^{-rt} p_t dt \right] \right\} g(\beta_0) d\beta_0 \\
&= \int_{\beta^l}^{\beta^h} U(\beta_0) g(\beta_0) d\beta_0 + \int_{\beta^l}^{\beta^h} \left\{ E_0 \left[ \int_0^\infty e^{-rt} [c(q(\beta_0, \beta_t)) + \psi(e(\beta_0, \beta_t))] dt \right] \right\} g(\beta_0) d\beta_0 \\
&= \int_{\beta^l}^{\beta^h} E_0 \left[ \int_0^\infty e^{-rt} \left[ \psi'(\beta_t(x(\beta_0, \beta_t) - q(\beta_0, \beta_t))) \frac{1}{\beta_t} e(\beta_0, \beta_t) \frac{\partial \phi(\beta_0, B_t)}{\partial \beta_0} \right] dt \right] \frac{G(\beta_0)}{g(\beta_0)} g(\beta_0) d\beta_0 \\
&+ \int_{\beta^l}^{\beta^h} \left\{ E_0 \left[ \int_0^\infty e^{-rt} [c(q(\beta_0, \beta_t)) + \psi(e(\beta_0, \beta_t))] dt \right] \right\} g(\beta_0) d\beta_0 \\
&= \int_{\beta^l}^{\beta^h} \left\{ E_0 \left[ \int_0^\infty e^{-rt} \left[ c(q(\beta_0, \beta_t)) + \psi(e(\beta_0, \beta_t)) + \psi'(e(\beta_0, \beta_t)) \frac{1}{\beta_t} e(\beta_0, \beta_t) \frac{\partial \phi(\beta_0, B_t)}{\partial \beta_0} \right] dt \right] \right\} g(\beta_0) d\beta_0
\end{aligned} \tag{55}$$

Replacing Eq. 55 into the objective function (Eq. 19), allows us to write:

$$\max_{q(\beta_0, \beta_t), e(\beta_0, \beta_t)} \int_{\beta^l}^{\beta^h} \left\{ E_0 \left[ \int_0^\infty e^{-rt} \underbrace{\left[ S(x(\beta_0, \beta_t)) - \psi'(e(\beta_0, \beta_t)) e(\beta_0, \beta_t) \frac{G(\beta_0)}{g(\beta_0)\beta_0} \frac{(\beta_t - k)}{\beta_t} \right]}_{\text{Virtual Surplus at time } t > 0} - c(q(\beta_0, \beta_t)) - \psi(e(\beta_0, \beta_t))] dt \right] \right\} g(\beta_0) d\beta_0, \tag{56}$$

where  $\frac{\partial \phi(\beta_0, B_t)}{\partial \beta_0} = \frac{\beta_t - k}{\beta_0}$ . Differentiating Eq.56 with respect to  $q_t$  and  $e_t$  gives the first-order conditions:

$$S'(q(\beta_0, \beta_t) + \frac{1}{\beta_t} e(\beta_0, \beta_t)) - c'(q(\beta_0, \beta_t)) = 0 \tag{57}$$

and

$$\begin{aligned}
& S'(q(\beta_0, \beta_t) + \frac{1}{\beta_t} e(\beta_0, \beta_t)) \frac{1}{\beta_t} - \psi'(e(\beta_0, \beta_t)) - \\
& - (\psi''(e(\beta_0, \beta_t)) e(\beta_0, \beta_t) + \psi'(e(\beta_0, \beta_t))) \frac{G(\beta_0)}{g(\beta_0)\beta_0} \frac{(\beta_t - k)}{\beta_t} = 0.
\end{aligned} \tag{58}$$

Using our specific functional forms leads to the contract described by Proposition 2. Finally, as  $x_t^{**} - q_t^{**} = \frac{1}{\beta_t} \left[ \beta_t + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} (\beta_t - k) \right]^{-1}$ , it is easy to prove that the second-order conditions (Eqs. 15 and 18) are always satisfied. In addition, for any given  $\beta_0$ , the optimal effort is monotone decreasing in  $\beta_t$ , while the treatments are monotone increasing in  $\beta_t$ :

$$\frac{\partial e_t^{**}}{\partial \beta_t} = - \left[ \beta_t + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} (\beta_t - k) \right]^{-2} \left[ 1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} \right] < 0 \tag{59}$$

$$\frac{\partial q_t^{**}}{\partial \beta_t} = \frac{1}{\beta_t^2} \left[ \beta_t + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} (\beta_t - k) \right]^{-1} + \frac{1}{\beta_t} \left[ \beta_t + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} (\beta_t - k) \right]^{-2} \left[ 1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} \right] > 0. \tag{60}$$

Let us now determine the variable component of payment. For each time  $t > 0$ , by integrating Eq. 14, we can write:

$$\tilde{u}(\beta_0, \infty) - u(\beta_0, \beta_t) = - \int_{\beta_t}^{\infty} \psi'(e^{**}(\beta_0, z))(x^{**}(\beta_0, z) - q^{**}(\beta_0, z))dz, \quad (61)$$

where  $\tilde{u}(\beta_0, \infty) = 0$ . By substituting Eq. 61 into Eq. 4, we obtain:

$$\begin{aligned} V^{**}(\beta_0, \beta_t) &= c(q^{**}(\beta_0, \beta_t)) + \psi(e^{**}(\beta_0, \beta_t)) + \\ &+ \int_{\beta_t}^{\infty} \psi'(e^{**}(\beta_0, z))(x^{**}(\beta_0, z) - q^{**}(\beta_0, z))dz \quad \text{for } \beta_t \in [k, \infty) \end{aligned} \quad (62)$$

By using our specific functional forms, the variable component of payment is defined as in Eq. 21 of the main text. Concerning the comparative statics:

$$\begin{aligned} \frac{\partial V^{**}(\beta_0, \beta_t)}{\partial \beta_t} &= \frac{1}{\beta_t} \left[ \beta_t + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} (\beta_t - k) \right]^{-1} \left[ \frac{1}{\beta_t} - \left[ \beta_t + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} (\beta_t - k) \right]^{-1} \right] \\ &+ \frac{1}{\beta_t} \left[ \beta_t + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} (\beta_t - k) \right]^{-2} \left[ 1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} \right] \left[ \frac{1}{\beta_t} - \left[ \beta_t + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} (\beta_t - k) \right]^{-1} \right], \end{aligned} \quad (63)$$

which is positive for all values of  $\beta_0$  in the admissible range. The above results allow us to write the firm's intertemporal utility as:

$$U(\beta_0; \beta_t) = E_0 \left[ \int_0^{\infty} e^{-rt} \left[ \int_{\beta_t}^{\infty} \psi'(e^{**}(\beta_0, z))(x^{**}(\beta_0, z) - q^{**}(\beta_0, z))dz \right] dt + e^{-rt} F(\beta_0) \right] \quad (64)$$

We next turn to the second problem (Eqs. 16–18). Since, by  $V^{**}(\beta_0, \beta_t)$ , the provider will report his type  $\beta_t$  truthfully,  $\forall t > 0$ , independent of his initial report  $\beta_0$ , the value (Eq. 16) can be rewritten as:

$$\begin{aligned} U(\beta_0, \hat{\beta}_0) & \quad (65) \\ &= E_0 \left[ \int_0^{\infty} e^{-rt} \left[ V(\hat{\beta}_0, \beta_t) - c(q(\hat{\beta}_0, \beta_t)) - \psi(\phi(\beta_0, B_t)(x(\hat{\beta}_0, \beta_t) - q(\hat{\beta}_0, \beta_t))) + F(\hat{\beta}_0) \right] dt \right] \end{aligned}$$

where  $\beta_0$  is the true initial shock and  $\hat{\beta}_0$  is the one reported. In addition, as it is optimal to report  $\beta_t$  truthfully, we have:

$$\begin{aligned} & \frac{\partial [V^{**}(\hat{\beta}_0, \beta_t) - c(q^{**}(\hat{\beta}_0, \beta_t)) - \psi(\beta_t(x^{**}(\hat{\beta}_0, \beta_t) - q^{**}(\hat{\beta}_0, \beta_t)))]}{\partial \beta_t} \quad (66) \\ &= -\psi'(\phi(\beta_0, B_t)(x^{**}(\hat{\beta}_0, \beta_t) - q^{**}(\hat{\beta}_0, \beta_t)))(x(\hat{\beta}_0, \beta_t) - q(\hat{\beta}_0, \beta_t)). \end{aligned}$$

Thus, recalling that  $\beta_t = \phi(\beta_0, B_t)$ , the derivative of Eq. 65 with respect to the initial shock  $\beta_0$  gives:

$$\frac{dU(\beta_0)}{d\beta_0} = -E_0 \left[ \int_0^{\infty} e^{-rt} \left[ \psi'(e^{**}(\beta_0, \beta_t)) \frac{e^{**}(\beta_0, \beta_t)}{\beta_t} \frac{\partial \phi(\beta_0, B_t)}{\partial \beta_0} \right] dt \right]. \quad (67)$$

However, the integral of Eq. 67 is simply Eq. 86, i.e.,

$$U(\beta_0) = \int_{\beta_0}^{\beta^h} E_0 \left[ \int_0^\infty e^{-rt} (\psi'(e(y, \beta_t)) \frac{1}{\beta_t} e(y, \beta_t) \frac{\partial \phi(y, B_t)}{\partial y}) dt \right] dy$$

Finally, by Eqs. 86 and 64, we are able to determine the fixed transfer  $F(\beta_0)$ , i.e.,

$$\begin{aligned} U(\beta_0) &= \int_{\beta_0}^{\beta^h} E_0 \left[ \int_0^\infty e^{-rt} (\psi'(e^{**}(y, \beta_t)) \frac{e^{**}(y, \beta_t)}{\beta_t} \frac{\partial \phi(y, B_t)}{\partial \beta_0}) dt \right] dy \\ &= E_0 \left[ \int_0^\infty e^{-rt} \left[ \int_{\beta_t}^\infty \psi'(e^{**}(\beta_0, z)) \frac{e^{**}(\beta_0, z)}{z} dz \right] dt + e^{-rt} F(\beta_0) \right] = U(\beta_0; \beta_t) \end{aligned} \quad (68)$$

Solving for  $F(\beta_0)$ , the annuitized fixed payment is:

$$F^{**}(\beta_0) = r \left\{ \begin{array}{l} \int_{\beta_0}^{\beta^h} E_0 \left[ \int_0^\infty e^{-rt} (\psi'(e^{**}(y, \beta_t)) \frac{e^{**}(y, \beta_t)}{\beta_t} \frac{\beta_t - k}{y}) dt \right] dy \\ - E_0 \left[ \int_0^\infty e^{-rt} \left[ \int_{\beta_t}^\infty \psi'(e^{**}(\beta_0, z)) \frac{e^{**}(\beta_0, z)}{z} dz \right] dt \right] \end{array} \right\} \quad (69)$$

In the case of the functional forms proposed, Eq. 69 reduces to:

$$F^{**}(\beta_0) = r E_0 \left[ \int_0^\infty e^{-rt} \left[ \int_{\beta_0}^{\beta^h} \frac{[e^{**}(y, \beta_t)]^2}{\beta_t} \frac{\beta_t - k}{y} dy - \int_{\beta_t}^\infty \frac{[e^{**}(\beta_0, z)]^2}{z} dz \right] dt \right]. \quad (70)$$

## E Analysis of distortions

The ratio  $\frac{e_t^*}{e_t^{**}}$  can be written as

$$\frac{e_t^*}{e_t^{**}} = 1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} \frac{\beta_t - k}{\beta_t}. \quad (71)$$

Taking the expectation at  $t = 0$ :

$$E_0 \left[ \frac{e_t^*}{e_t^{**}} \right] = 1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} \left[ 1 - E_0 \left[ \frac{k}{\beta_t} \right] \right] \quad (72)$$

To evaluate  $E_0 \left[ \frac{k}{\beta_t} \right]$ , we define  $y_t = \frac{k}{\beta_t}$ . Using Ito's lemma (and dropping time subscripts):

$$dy = k[-\beta^{-2}d\beta + \beta^{-3}\sigma^2(\beta - k)^2dt] \quad (73)$$

As  $E_0(d\beta) = 0$ , taking the expected value and defining  $y' = \frac{E_0(dy)}{dt}$ , we are able to reduce the deterministic part of Eq. 73 to the nonlinear differential equation

$$y' = \sigma^2 k y + \sigma^2 k^3 y^3 - 2\sigma^2 k^2 y^2,$$

which depends on  $k$ . For  $k = 0$ , the solution is straightforward and equal to  $1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0}$ ; when  $k > 0$ , we need to transform  $y'$  into a nonhomogeneous nonlinear equation. Dividing first by  $y^3$  and then by substituting the variable  $z = y^{-1/2}$ , where  $z' = -\frac{1}{2}y^{-3/2}y'$ , we obtain:

$$z' + az + bz^{-\frac{1}{2}} = c, \quad (74)$$

where  $a = 2k\sigma^2$ ,  $b = -4\sigma^2k^2$ ,  $c = -2\sigma^2k^3$ . We first solve the homogeneous setting  $z = uv$ :

$$v(u' + au) + uv' + b(uv)^{-\frac{1}{2}} = 0,$$

where the first part is equal to  $u = u(0)e^{-at}$  and the second one is  $v = \left[ v(0)^{\frac{3}{2}} - \frac{b}{a}u(0)^{-\frac{3}{2}} \left[ e^{\frac{3}{2}at} - 1 \right] \right]^{2/3}$ . By combining the different components, we find the solution of the homogeneous:

$$y = \left[ y(0)^{-3}e^{-3\sigma^2kt} + 2k \left[ 1 - e^{-3\sigma^2kt} \right] \right]^{-1/3}. \quad (75)$$

The easiest particular solution of the full equation is  $y = m$ . Then, we obtain:

$$m = \frac{1 \pm \sqrt{2}}{k} \quad \text{and} \quad m = 0.$$

Selecting  $m = 0$ , the general solution becomes:

$$E_0 \left[ \frac{k}{\beta_t} \right] = \left[ \left( \frac{k}{\beta_0} \right)^{-3} e^{-3\sigma^2kt} + 2k \left[ 1 - e^{-3\sigma^2kt} \right] \right]^{-1/3}, \quad (76)$$

which can be plugged into Eq. 72 to obtain Eq. 30 of the main text. Taking the derivative with respect to  $t$ , we find:

$$\frac{\partial E_0 \left[ \frac{k}{\beta_t} \right]}{\partial t} = \frac{\sigma^2 k^2 (3k^4 - \beta_0^3)}{\left[ \left( \frac{k}{\beta_0} \right)^{-3} e^{-3\sigma^2kt} + 2k \left[ 1 - e^{-3\sigma^2kt} \right] \right]^{4/3}} e^{-3\sigma^2kt}. \quad (77)$$

Finally, taking the limit for  $t \rightarrow \infty$ , the relative distortion converges to:

$$\lim_{t \rightarrow \infty} E_0 \left[ \left( \frac{e_t^*}{e_t^{**}} \right) \right] = 1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} [1 - (2k)^{-1/3}]. \quad (78)$$

## F Characteristics of the fixed component of payment

Let us consider the fixed component of payment

$$rE_0 \left[ \int_0^\infty e^{-rt} \left[ \int_{\beta_0}^{\beta_t} \frac{[e^{**}(y, \phi(y, B_t))]^2 \phi(y, B_t) - k}{\phi(y, B_t) y} dy - \int_{\beta_t}^\infty \frac{[e^{**}(\beta_0, z)]^2}{z} dz \right] dt \right]. \quad (79)$$

It is worth noting that for any given value of  $e^{**}(y, \phi(y, B_t))$  and  $e^{**}(\beta_0, z)$ , Eq. (79) is, in absolute value, larger with  $k > 0$  than with  $k = 0$ . Now, setting  $k = 0$ , the second term inside the square brackets of Eq. 79 becomes:

$$\begin{aligned} \int_{\beta_t}^\infty \frac{[e^{**}(\beta_0, z)]^2}{z} dz &= \int_{\beta_t}^\infty \frac{1}{z^3} \frac{1}{\left[ 1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} \right]^2} dz \\ &= \frac{1}{\left[ 1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} \right]^2} \int_{\beta_t}^\infty z^{-3} dz = \frac{1}{2} \frac{1}{\beta_t^2 \left[ 1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0} \right]^2}. \end{aligned}$$

and the expected value is:

$$\begin{aligned} E_0 \left[ \int_{\beta_t}^{\infty} \frac{[e^{**}(\beta_0, z)]^2}{z} dz \right] &= \frac{1}{2} \frac{1}{\left[1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0}\right]^2} E_0[\beta_t^{-2}] \\ &= \frac{1}{2} \frac{1}{\left[1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0}\right]^2} \beta_0^{-2} e^{3\sigma^2 t}, \end{aligned} \quad (80)$$

where  $E_0[\beta_t^{-2}] = 3\sigma^2 \beta^{-2} dt$ . Similarly, the expected value of the first term of Eq. 79 becomes:

$$\begin{aligned} E_0 \left[ \int_{\beta_0}^{\beta^h} \frac{[e^{**}(y, \phi(y, B_t))]^2}{y} dy \right] &= \int_{\beta_0}^{\beta^h} E_0 \left( \frac{1}{\phi(y, B_t)^2} \right) \frac{1}{y} \frac{1}{\left[1 + 2 \frac{G(y)}{g(y)y}\right]^2} dy \\ &= \int_{\beta_0}^{\beta^h} e^{3\sigma^2 t} \frac{1}{y^3} \frac{1}{\left[1 + 2 \frac{G(y)}{g(y)y}\right]^2} dy. \end{aligned} \quad (81)$$

Substituting Eqs. 80 and 81 into Eq. 79 and rearranging, we obtain:

$$\begin{aligned} &\frac{r}{r - 3\sigma^2} \left[ \int_{\beta_0}^{\beta^h} \frac{1}{y^3} \frac{1}{\left[1 + 2 \frac{G(y)}{g(y)y}\right]^2} dy - \frac{1}{2\beta_0^2} \frac{1}{\left[1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0}\right]^2} \right] \\ &= \frac{r}{r - 3\sigma^2} \left[ \int_{\beta_0}^{\beta^h} \frac{1}{y^3} \left( \frac{1}{\left[1 + 2 \frac{G(y)}{g(y)y}\right]^2} - \frac{1}{\left[1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0}\right]^2} \right) dy - \frac{1}{2(\beta^h)^2} \frac{1}{\left[1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0}\right]^2} \right], \end{aligned} \quad (82)$$

where the last equality follows from

$$\frac{1}{2\beta_0^2} \frac{1}{\left[1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0}\right]^2} = \frac{1}{\left[1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0}\right]^2} \int_{\beta_0}^{\beta^h} \frac{1}{y^3} dy + \frac{1}{2(\beta^h)^2} \frac{1}{\left[1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0}\right]^2}.$$

Since  $\frac{G(y)}{g(y)y}$  is increasing in  $y$ , the term  $\left( \frac{1}{\left[1 + 2 \frac{G(y)}{g(y)y}\right]^2} - \frac{1}{\left[1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0}\right]^2} \right)$  is negative  $\forall y \in [\beta_0, \beta^h]$ , which implies that Eq. 79 is also negative. Finally, taking the derivative with respect to  $\beta_0$ , we obtain:

$$\frac{2}{(\beta^h)^2} \frac{\frac{d \frac{G(\beta_0)}{g(\beta_0)\beta_0}}{d\beta_0}}{\left[1 + 2 \frac{G(\beta_0)}{g(\beta_0)\beta_0}\right]^3} > 0.$$

## G Characteristics of the optimal static contract

The hospital utility function at time  $t$  is:

$$u(p(\hat{\beta}_t), q(\hat{\beta}_t), e(\beta_t, \hat{\beta}_t)) = p(\hat{\beta}_t) - c(q(\hat{\beta}_t)) - \psi(\beta_t(x(\hat{\beta}_t) - q(\hat{\beta}_t))), \quad (83)$$

which requires a transfer  $p(\hat{\beta}_t)$  to make truthful reporting of  $\hat{\beta}_t = \beta_t$  optimal. The necessary and sufficient conditions for incentive compatibility that induce the provider to truthfully reveal  $\beta_t$  are:

$$\frac{du(\beta_t)}{d\beta_t} = -\psi'(e(\beta_t))(x(\beta_t) - q(\beta_t)), \quad (84)$$

and

$$\frac{d(x(\beta_t) - q(\beta_t))}{d\beta_t} < 0, \quad (85)$$

with  $e(\beta_t) \geq 0$ ,  $q(\beta_t) \geq 0$  and  $p(\beta_t) \geq 0$ . By the Envelope theorem (Milgrom and Segal, 2002, Theorems 1 and 2), we obtain:

$$u(\beta_t) = \int_{\beta_t}^{\beta^h} \psi'(e(y)) \frac{1}{y} e(y) dy, \quad (86)$$

Where, as usual,  $u(\beta^h) = 0$ . The optimal static contract needs to satisfy the following conditions:

$$S' \left( q(\beta_t) + \frac{1}{\beta_t} e(\beta_t) \right) - c'(q(\beta_t)) = 0, \quad (87)$$

$$S' \left( q(\beta_t) + \frac{1}{\beta_t} e(\beta_t) \right) \frac{1}{\beta_t} - \psi'(e(\beta_t)) - (\psi''(e(\beta_t))e(\beta_t) + \psi'(e(\beta_t))) \frac{G(\beta_t)}{g(\beta_t)\beta_t} = 0. \quad (88)$$

By substituting the specific functional forms used for the optimal dynamic contract into the FOCs, we obtain the static benchmark that was characterised in Section 6. It is easy to prove that the second-order conditions are also satisfied.

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