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Editorial

Innovations in modeling and numerical methods for evolutionary PDEs: theory and applications☆

The numerical simulation of complex physical systems plays an instrumental role in modern scientific and engineering advancements. Within this framework, evolutionary partial differential equations (PDEs) are of interest in a wide range of physically relevant situations, such as computational fluid and solid mechanics, oceanography, plasma physics, materials science, and computational astrophysics. However, these types of PDEs are characterized by significant computational difficulties due to the vast range of involved spatial and temporal scales, as well as the complex mathematical structure underlying their behavior. Consequently, their solution remains a major challenge today and represents the central theme of the International HONOM cycle of conferences on *High-order nonlinear numerical methods for evolutionary PDEs: theory and applications*.

The tenth edition of this event (https://www.elenagaburro.it/honom2024.html) was held in Chania, Crete Island, Greece, in September 2024, gathering more than 80 international experts in the field, and the research presented there serves as the main source of contributions for this special issue of Computers & Fluids (CAF), which contains 36 original papers. The HONOM conference series started in Trento, Italy, in 2005, initiated by R. Abgrall, M. Dumbser, C.-D. Munz, and E. F. Toro. Following the initial event, the conference was hosted in Trento again in 2007, 2009, 2011, and 2015. Subsequently, HONOM was organized in Bordeaux, France, in 2013, Stuttgart, Germany, in 2017, Madrid, Spain, in 2019, and Braga, Portugal, in 2022. Therefore, HONOM 2024 in Crete represents a major milestone, celebrating nearly two decades of scientific excellence and collaboration within this community.

In this context, the papers collected in our special issue present the *design*, *analysis*, and *application* of advanced mathematical *models* and numerical *algorithms* for the robust and effective solution of evolutionary PDEs. Particular interest is dedicated to modern high-order schemes on Cartesian, unstructured, or moving meshes, with a specific emphasis on Finite Volume (FV), Finite Difference (FD), discontinuous Galerkin (DG), and Active Flux approaches.

In particular, Active Flux methods have seen significant growth over the last two years, serving as a major theme during the conference and yielding numerous contributions within this volume. Structurally, these approaches incorporate both cell-averaged values and explicit point values at cell interfaces as independent degrees of freedom, evolving them simultaneously to achieve high accuracy and bypass standard Riemann solvers. Here, these core principles are extended in several directions. For instance, source terms are directly embedded within the numerical fluxes to enable general steady-state preservation and path-conservative, well-balanced properties for shallow water equations [1,2] and Euler equations with gravitation [3]. High-order WENO FD extensions are also presented using a global-flux quadrature approach [4]. In addition a stability analysis reveals where point values need to be placed [5] on 2D Cartesian meshes. Concurrently, the method is conceptualized as a dual formulation on overlapping meshes to achieve high-order accuracy within the FV framework without the use of Riemann solvers [6].

At the same time, several contributions are instead dedicated to the development of *multidimensional* Riemann solvers and generalized Riemann problem (GRP) solvers, which appear to be essential for capturing complex wave structures with high resolution and robustness [7,8].

The special issue also addresses theoretical investigations on the properties of well-known Riemann solvers [9], Patankar-type methods (for which a Lax–Wendroff theorem has been proposed) [10], relaxation and bound preserving methods [11,12], and an in-depth analysis of the mechanisms to suppress spurious modes in low Mach number simulations [13]. A strong focus is also placed on entropy stable numerical methods analyzed in [14,15] and accompanied by a wide range of numerical results obtained using Discontinuous Galerkin methods. Furthermore, to complement these DG methods, a study on symmetry-preserving neural indicators for discontinuities detection has also been proposed [16].

Next, a highly relevant topic is represented by investigations on Lagrangian and Arbitrary Lagrangian Eulerian (ALE) methods on general moving grids, including quasi-Lagrangian Voronoi approximations [17], the study of velocity-gradient artificial viscosity terms [18], and the addition of well-balancing capabilities to a high-order DG code on moving polygonal meshes with topology changes, which allows the study of Keplerian disks and frontogenesis with an excellent level of resolution [19].

These methodological advancements are paired with sophisticated mesh optimization and computational strategies. These include dynamic *hp*-adaptation for turbulent multi-component flows, immersed boundaries, high-fidelity 2D interface reconstruction, accelerated do-

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main decomposition for near-wall turbulence, and GPU-accelerated field evaluation on high-order meshes [20–25]. Next, to achieve very high orders of accuracy, advanced polynomial reconstructions are introduced, focusing on AENO-C and WENO-DK variants within the ADER framework [26,27].

Finally, these formulations are validated against challenging applications and multi-physics scenarios. These encompass incompressible magnetohydrodynamics (MHD) via differential forms [28], mimetic differences and pseudo-symplectic methods for Hamiltonian and Navier–Stokes equations [29,30], Hybrid High-Order (HHO) formulations for incompressible flows [31], and corner singularities in cavity flows [32]. Moreover, the collection covers explicit kinetic schemes on non-uniform meshes [33], ImEx schemes for low Froude shallow water regimes [34], Eulerian modeling of multicomponent elastic materials [35], and dispersive effects in fluid flows interacting with moving solid boundaries [36].

This volume therefore covers a wide range of relevant topics in the numerical modeling of evolutionary PDEs, demonstrating how members of our community collaborate to provide increasingly competitive and compelling solutions. Thus, we would like to thank all the authors for their excellent contributions. Our gratitude also goes to the anonymous reviewers, the conference participants, and the current and previous Editors-in-Chief of *Computers & Fluids*, Paola Cinnella and Pierre Sagaut, for making the proposal, management, and ultimate publication of this special issue possible.

In memoriam

This CAF special issue is dedicated to the memory of Prof. Arturo Hidalgo López (*July 03rd 1966 - †August 26th 2024) of the Universidad Politecnica de Madrid, organizer of HONOM 2019 and active participant in many other editions of HONOM. Our thoughts and wishes go to his wife Lourdes and his sister María Jesús, whom he left behind.

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