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TITOLO DELLA TESI DI DOTTORATO
A micro-macro approach to commodity market analysis: risk, structural modelling and forecasting

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PREFACE

This work concerns the analysis of primary commodity markets from both a micro and macro perspective and is composed of three parts.

In the first part, we investigate how rural households in poor countries, depending for their livelihood on crop production, cope ex-ante with risk through a strategy of a diversified portfolio of crops. To this end, a portfolio model of production is set up from which structural estimates of risk preference and technology parameters are derived. The model is fit to longitudinal data from a sample of coffee producers in Ethiopia.

The second part is concerned with the modelling of commodity markets within a rational expectations approach. A special emphasis is placed on perennial crops and a world model for the cocoa market is specified, accounting for speculative stockholding. From the structural model a solved form in price and stocks is derived, using two constructed variables capturing excess supply in the short and long term. The derivation of restrictions stemming from the hypothesized rational expectations by stockholders is then illustrated. Furthermore, an equilibrium analysis of the model is carried out using the price rational expectations solution, in order to investigate the qualitative response of the system to shocks.

In the third part, the reduced form in price and stocks previously derived from a short-run version of the model is estimated using annual data on the cocoa market and the rational expectations restrictions are tested. The estimates obtained using the Generalized Method of Moments are then compared with those obtained from a restricted VAR model presenting a matching specification.

Crop choices, under risk and subsistence constraints: evidence from rural Ethiopia

1 Introduction

Rural households in developing countries are exposed to a number of shocks arising from a risky environment, including extreme weather conditions, pests, crop diseases and illnesses, and variable market conditions. Farmers are typically ill-equipped to face such shocks, mostly covariant shocks that make risk sharing agreements only partially effective (Townsend, 1994; Udry, 1995), since formal credit and insurance markets are normally missing or incomplete. Therefore households, in response to such shocks, adopt a variety of strategies (Glewwe and Hall, 1998), ex-ante to shield themselves against the shocks or to mitigate ex-post the negative effects.

These latter strategies may include: a) use of savings or sales of physical assets to smooth consumption. Livestock, being a liquidable asset, can typically play this role of buffer stock, even though the empirical evidence is not conclusive because the concern of losing a productive asset may inhibit distress sales of livestock by families (Fafchamps et al., 1998; Zimmerman and Carter, 2003); b) inter-household transfers, including informal insurance agreements within the community and transfers received, on a more or less regular basis, from better-off relatives or household members, such as remittances; c) changes in consumption patterns, such as switching to cheaper food items; d) migration or displacement of family members to look for other jobs.

Ex-ante strategies can typically include: e) diversification of income sources, balancing on-farm and off-farm activities; f) choice of a diversified crop portfolio, growing crops displaying low correlated returns; g) use of less risky technologies, for instance avoiding to purchase fertilizers (Dercon-Christiaensen, 2007); h) own production of food crops to avoid price risk and guarantee stable food supply.

Now, under survival concerns and liquidity constraints optimal portfolio strategies can bifurcate with wealthier families opting for high return-risk activities, whilst poor ones may remain stuck in low return-risk portfolios (Rosenzweig and Wolpin, 1993; Dercon, 1998). Therefore, a likely outcome is that risk induced poverty traps may emerge (Zimmerman and Carter, 2003).

These strategies are usually only partially effective, leaving a residual consumption risk (Rosenzweig and Wolpin, 2003) and come at a cost in terms of foregone consumption and profit opportunities.

For these reasons, there is a growing interest by policy makers in assessing the feasibility and effectiveness of programs aimed at providing market based risk management instruments to farmers, particularly after the withdrawal of the state from the direct control of commodity markets and the end of price stabilization schemes, following the adoption of structural adjustment programs in many developing countries. A preliminary step for the succesfull implementation of such programs is the assessment of the underlying demand for additional income insurance.

Sarris (2002), in his study on the theory and application of commodity insurance under a fixed production structure, identifies three methodologies to assess the willingness to pay (WTP) of producers for price or income insurance: a) direct questioning of farmers and related contingent valuation techniques; b) indirect methods, combing theory and information at household level to indirectly estimate appropriate risk premiums; c) revealed preference methods, where the WTP is inferred from the analysis of the observed production and saving-investment decisions of farmers. Under the assumption that households choices are coherent with their risk attitudes, this method allows to estimate the latent demand for insurance given that households have already adopted self insurance mechanisms. The drawback is the necessity to build a general enough model to capture the households relevant choices and the heavy data requirements, even though rich household surveys are more and more available. Works using this methodology include Gautam, Hazell and Alderman (1994), Fafchamps, Udry and Czukas (1998), Kurosaki and Fafchamps (2002).

Our work belongs to this strand of literature and the methodology closely follows the study by Kurosaki and Fafchamps (2002), where risk attitudes and consumption smoothing parameters are estimated using panel data from Pakistan Punjab. Specifically, our aim is to investigate the determinants of crop portfolio decisions by farmers in a rural developing economy, under price and yield risk, where markets for credit and insurance are either incomplete or missing, assessing how risk-coping strategies of farm households, heterogeneous in risk attitudes and assets holdings, affect the composition of their crops portfolio. Therefore, we set up a portfolio model of production choices from which structural estimates of technology, risk and consumption preference parameters are derived. The model is fit to longitudinal data from a sample of coffee producers in Southern Ethiopia. The results may help verify the existence of a latent demand for crop insurance and allow to simulate the effects on welfare of alternative policies affecting the realization of shocks.

The model developed by Kurosaki and Fafchamps (2002), in addition to providing a useful benchmark for our study, correctly adresses the non-separabilty between production and consumption decisions arising from the presence of uncertainty and risk aversion. The authors find that even in developing countries with fairly well developed credit and input markets, consumption preferences do affect production decisions. As argued by Sadoulet and de Janvry (1995), in case of market failures in the output or factor markets due to transactions costs, shallow local markets or risk, the decision price faced by the household is no longer the exogenous market price but an internal shadow price. In the case of risk, whenever a complete set of state contingent securities spanning all possible sources of risks does not exist, the expected output or factor price is discounted by a markup, negative and positive respectively, reflecting the degree of risk aversion. As a consequence, separability breaks down and the production and consumption problem are to be considered jointly in terms of modeling (see also Roe and Graham-Tomasi (1986) for an application to a dynamic model with yield uncertainty).

Our work, in addition to extending Fafchamps and Kurosaki's model by increasing the dimension of the crop portfolio, represents a new empirical application to a rural context with features that makes it interesting, namely: a) poorly developed credit and input markets; b) a densely populated area with severe land constraints and a land tenure system where land sales are formally prohibited; c) the cultivation of perennial crops, such as coffee, which implies the bearing of adjustment costs that may prevent producers from readily adjusting their crop portfolio to changing market conditions, further increasing the uncertainty of investement decisions and thus the importance of risk attitudes. Such features, in particular the role of the land tenure system, have been taken into account in the specification of the risk aversion determinants, as well as the potential heterogeneity in risk preferences arising from different assets holding.

The paper is organized as follows. In Section 2 the institutional framework and the agronomic system of the area under study are described. Section 3 outlines the general theoretical model and how it is solved, while in Section 4 the empirical specification and the estimation methodology are illustrated. Finally, Section 5 presents the estimation results and Section 6 the final remarks and the possible extensions of the present work.

2 Institutional framework

Ethiopia moved toward a market based economy only in the late 1980s. After the removal from power of the emperor by the revolution in 1974, the military government of Col. Mengistu Hailemariam established a communist-inspired strong control on the economy, with the nationalization of most large private companies and the imposition of price controls and trade restrictions, notably on agricultural production. Land became state-owned and were redistributed to rural households who kept the right to cultivate it for their own benefit.

Two major events occured to the country during the 1980s, a wide-spread famine in 1984-85 and the civil war against the government forces, affecting mainly the Northern regions of Ethiopia, and leading eventually to the defeat of the Derg regime in 1991 by a coalition of rebel forces, the Ethiopian People's Revolutionary Democratic Front (EPRDF), that restored security in most areas of the country.

Actually, since 1988 the economic crisis had led to some economic reforms toward a more liberalized economy, including the abolition of high rural taxes and trade restrictions on food crops. In 1992, further measures, including a large devaluation, were taken and these reforms became part of a wider structural adjustment program sponsored by the International Monetary Fund and the World Bank in 1994.

Despite a series of land tenure reforms during the 1990s, driven by strong population pressure and leading to the removal of many restrictions on rental and sharecropping, land remains state-onwed and individuals are given only use rights. Although several studies (Fafchamps and Pender, 2001) have highlighted how the operation of land lease markets may guarantee an efficient use of variable inputs, land tenure insecurity and limited transfer rights may hinder long-term investments, such as planting or replanting of perennial crops (Dercon et al, 2007).

These features of the Ethiopian land tenure system are particularly relevant to our study as the economy of the two villages under investigation, Adado and Aze Deboa, belongs to the so called enset-based agricultural system, characterized by the widespread cultivation of enset, a semi-permanent food crop, together with other crops such as coffee, a tree crop[1].

3 Theoretical model

3.1 Assumptions

In this section we outline the theoretical model and the main assumptions made. Some of them are justified by the specific characteristics of this rural economy and the focus on risk of the present study, others respond mainly to the opportunity to keep the model close to the empirical specification, which is partly driven by data availability. Therefore: a) labour market is not modeled, as off-farm employment is rare in the area and occurs mainly through labour sharing agreements. Thus, we are not distinguishing between family and hired labour, which is nonetheless employed at harvest time; b) labour, as other joint inputs, is assumed not allocatable to different crops, hence it is applied in somewhat constant proportions to cultivated land; c) land market is not modeled, since as explained in the previous section, transactions are prohibited by law (yet land rental is active); d) credit and liquidity constraints are not included, even though in-kind working capital loans (fertilizers, pesticides) from cooperatives under government guarantees are available; e) saving or dissaving occur only in the form of livestock purchases, while on-farm inventories of harvested crops or other assets accumulation is assumed negligible. In fact, livestock holding is a feasible store of wealth, a reversible and liquid investment in this rural economy.

3.2 Objective function

We assume that households maximize welfare defined, over a finite time horizon T, as the expected value of an intertemporally additive utility function in the consumption of a basket of goods c_t, which may include staple crops, animal and other non-food products, conditional on a vector of household characteristics z_t^h, such as the number of household members, the education of the household head and so forth. Instantaneous utility $u(.)$ is well behaved, with $u'(.) > 0$ and $u''(.) < 0$, and agents discount future utilities over time according to the discount factor $\beta = 1/(1+\delta)$, where δ is the time preference rate

$$V_t = E_t \sum_{\tau=0}^{T} \beta^{\tau} u(c_{t+\tau}, z_{t+\tau}^h). \qquad (1)$$

[1] More precisely, Adado can be classified as belonging to the enset sub-system where enset is the predominant staple food crop, typical of Sidama and Gurage ethnic groups, while in Aze Deboa it is a co-staple together with cereals and tuber crops, as it is common among Hadiya, Wolayita and other SNNPR groups.

We are thus moving whithin the expected utility framework, without dealing explicitely with downside risk (Kimball, 1990) and thus focusing in the statistical model on the first two moments of the stochastic processes governing the random variables. Moreover, we do not disentangle risk aversion and elasticity of intertemporal substitution (Epstein and Zin, 1989).

Since utility is additively separable, given total consumption expenditure and current consumption prices p_t^r, we can solve for c_t to get the indirect utility function $v(c_t, p_t^r)$ with the usual properties.

3.3 Household income

Total household income y_t is given by farm profits, either from cropping π_t^a or livestock rearing π_t^b, as well as from other revenues π_t^w assumed to be exogenous, such as wage from off-farm work, profits from self-employment and remittances

$$y_t = \pi_t^a + \pi_t^b + \pi_t^w. \tag{2}$$

Profits from livestock rearing are given by

$$\pi_t^b = p_t^b q_t^b - w_t^b x_t^b \tag{3}$$

where $q_t^b = F^b(B_t)$ is the quantity of livestock products, B_t is the herding stock at the beginning of period t, expressed in tropical livestock units, p_t^b and w_t^b are vectors of output and input prices, respectively, and x_t^b are the quantities of inputs used in livestock breeding.

Livestock evolve according to the following law of motion

$$B_{t+1} = B_t + b_t + n_t \tag{4}$$

where b_t is the variation of livestock in the period from purchases and sales, with $-b_t \leq B_t$, and n_t is any exogenous change due to births or losses.

Net revenues from crop cultivation are expressed as

$$\pi_t^a = \sum_{j=1}^{S} (p_t^j q_t^j - w_{t-1}^j x_{t-1}^j) \tag{5}$$

where

$$q_t^j = F^j(L_{t-1}^j, x_{t-1}^j; z_t^q, \epsilon_t) \qquad j = 1, ..., S \tag{6}$$

is the stochastic production function for crop j, giving the maximum output attainable using land L_{t-1}^j and other variable inputs x_{t-1}^j, conditional on farm specific characteristics z_t^q and production shocks ϵ_t, such as adverse weather conditions, pests and crop diseases; p_t^j is the output price and w_{t-1}^j a vector of input prices used in crop j cultivation.

In the case of perennial crops, such as coffee, we have not included the tree stock does in the production function, assuming that homogeneously planted land is a good proxy for it. As argued by Akiyama and Trivedi (1987), by focusing exclusively on land allocation we are admittedly overlooking some of the features typical of perennial crops, namely: a) the existence of a biological gestation lag between planting and obtaining yield during which supply conditions may change; b) the bearing of adjustment costs related to the removal and planting of trees; c) the fact that the productivity of trees varies systematically with the age; d) the heterogeneous nature of the tree stock, since age-yield profile and productive life depend on technical change and hence are not invariant with respect to the date (*vintage*) of the investment. In a future development of this work we will adress some of these issues.

3.4 Constraints

Land available to the household, $\overline{L}_t$, is allocated between S crops

$$\sum_{j=1}^{S} L_t^j = \overline{L}_t \quad \text{or} \quad \sum_{j=1}^{S} \theta_t^j = 1 \tag{7}$$

where θ_t^j is the share of cultivated land allocated to the j^{th} crop category and must sum to one. Since land is available in limited amount, increases in the share allocated to a crop occur mainly by substituting other cultivations.

We further introduce an agronomic constraint

$$g(\theta_t^1, ..., \theta_t^{S-1}) = 0 \tag{8}$$

where $g(.)$ describes the agronomic interactions between the $S-1$ free crops. It is non-increasing in the arguments and concave.

The annual budget constraint is given by

$$p_t^c c_t = y_t - p_t^B b_t \tag{9}$$

where p_t^c and p_t^B are price vectors of consumption goods and live animals, respectively, and y_t is total income. As anticipated above, we are implicitly assuming that livestock is the only accumulable asset in which savings can be invested in this rural economy.

3.5 Timing of economic decisions

Time is divided into discrete intervals during which shocks occur and decisions are taken by households:

1. at the beginning of the crop year t income from harvested crop production, cattle rearing and other activities is observed, as well as consumption prices;
2. total income (y_t) can be spent on consumption items (c_t) or saved in the form of livestock purchases (b_t);

3. available land ($\overline{L}_t$) is allocated to S competing crops;

4. households observe the prices of variable inputs (fertilizers, pesticides) and choose the amount to use in production (x_t^j).

3.6 Bellman equation

The model is set in discrete time and therefore we can make use of dynamic programming techniques. The dynamic optimization problem of the household can thus be written in Bellman equation form, comprising the following control variables (θ_t^j, x_t^j, b_t) and state variables ($y_t, B_t, p_t, \epsilon_t$)

$$\begin{aligned} &V_t(y_t, B_t, p_t, \epsilon_t) \\ &\qquad = \max_{\theta^j, b, x^j} \left\{ v(y_t, b_t, x_t^j, p_t) + \beta E_t V_{t+1}(y_{t+1}, B_{t+1}, p_{t+1}, \epsilon_{t+1}) \right\} \end{aligned} \tag{10}$$

for $t < T$, subject to (8), the non-negativity constraints $\theta_t^j, x_t^j, B_t \geq 0$ and the transversality condition

$$B_{T+1} = \overline{B}_q$$

where $\overline{B}_q$ is the livestock bequeathed to offsprings at the end of period T.
Since we are assuming that $v(.)$ and $V(.)$ are twice continuously differentiable, exploiting the Envelope Theorem, we differentiate the Bellman equation to get

$$\frac{\partial V_t}{\partial \theta_t^j} : \beta E_t \left[\frac{\partial V_{t+1}}{\partial y_{t+1}} p_{t+1}^j \frac{\partial F^j}{\partial \theta_t^j} \right] + \mu_{jt} - \lambda_t \frac{\partial g}{\partial \theta_t^j} = 0 \quad \forall j \tag{11}$$

$$\frac{\partial V_t}{\partial b_t} : -p_t^B \frac{\partial v_t}{\partial b_t} + \beta E_t \left[\frac{\partial V_{t+1}}{\partial y_{t+1}} p_{t+1}^B \frac{\partial F^b}{\partial B_{t+1}} \right] + \mu_t^b = 0 \tag{12}$$

$$\frac{\partial V_t}{\partial x_t^j} : -w_t^j \frac{\partial v_t}{\partial x_t^j} + \beta E_t \left[\frac{\partial V_{t+1}}{\partial y_{t+1}} p_{t+1}^j \frac{\partial F^j}{\partial x_t^j} \right] + \mu_{jt}^x = 0 \quad \forall j. \tag{13}$$

3.7 Crop portfolio

Staple crops in developing countries are always cultivated by small farmers to achieve food self-sufficiency (Fafchamps, 1992). In fact, food markets are often thin and isolated, resulting in prices that are volatile and highly correlated with farmers' own production patterns. On the other hand, cash crops provide a means to relax the household's liquidity constraint because formal credit markets are often absent.

We divide crops into four categories: 1) coffee (θ_t^1), the main cash crop; 2) enset (θ_t^2), the main staple crop, usually not traded; 3) cereals (θ_t^3), such as wheat or teff, often sold for cash; 4) other crops (θ_t^4), including pulses, chat and other tree crops.

The aggregation of crops into different categories has been done on the basis of the relative importance in terms of allocated land and output and, in a portfolio strategy, by looking at the correlation of returns. For instance, in the case of cereals, visual inspection and cointegration analysis using regional producer prices have confirmed that they could be safely aggregated into a single category.

As to consumption items, suitable aggregates have been identified using observed households expenditure shares.

Table 1: Mean crop shares by land quartiles (Aze Deboa and Adado, 1999)

quartile	**coffee**	**enset**	**cereals**	**trees nec**	**other**	**land (*ha*)**
			Aze Deboa			
1	.211	.222	.431	.0346	.101	.395
2	.187	.14	.473	.0324	.167	.799
3	.148	.133	.526	.0476	.145	1.48
4	.348	.116	.34	.0811	.116	2.16
Total	.188	.163	.47	.037	.142	.845
			Adado			
1	.444	.484	.0104	.024	.0375	.0435
2	.473	.478	0	.0239	.0255	.101
3	.46	.459	.0437	.0292	.00773	.226
4	.41	.483	.00482	.0433	.0588	1.5
Total	.446	.476	.0147	.0299	.033	.457

Source: ERHS (round 5)

3.8 Sources of risk and expectations

As it is typical of agricultural production, decisions are taken before uncertainty about future output and price is solved and therefore the way expectations are formed has to be considered. In what follows, we do not make specific assumptions on the probability distribution governing the stochastic processes of the relevant variables, focusing instead on their certainty equivalents.

Specifically, we assume that each year farmers form expectations on next year net revenues per hectare for each crop category, their variability and cross correlation by looking at past years realizations. Therefore, we are moving within an adaptive expectations framework and we are jointly considering price and yield risk. This is clearly a drawback of the study, due to price data availability issues.

Using data on regional yields and prices from external sources and household survey data on farm specific factors affecting yields, Kurosaki and Fafchamps

Table 2: Mean crop shares by land quartiles (Aze Deboa and Adado, 2004)

quartile	**coffee**	**enset**	**cereals**	**trees nec**	**other**	**land** (*ha*)
			Aze Deboa			
1	.152	.175	.268	.163	.203	.34
2	.117	.117	.382	.128	.205	.646
3	.083	.104	.329	.181	.283	1.23
4	.106	.0838	.391	.183	.199	43.4
Total	.114	.12	.354	.149	.221	4.2
			Adado			
1	.45	.457	.0231	.0302	.0396	.424
2	.419	.44	.0185	.117	.00556	.703
3	.438	.392	.0688	.0357	.0658	1.05
4	.472	.43	.0276	.0318	.039	5.19
Total	.446	.429	.0374	.0446	.0428	1.49

Source: ERHS (round 6)

(2002) manage to disentangle and model separately the covariant and idiosyncratic components of risk. This approach has the clear advantage of exploiting, in a rational expectations framework, the full information set specific to each household in modelling expectations (Kurosaki, 1997).

Therefore, we use information on returns from round 1 through 4 to calculate moments up to the second order for the year 1999, updated using data from round 5 to construct expectations for the year 2004 (Table 3).[2]

Consumption prices by category are calculated as indexes by using unit values and household expenditure shares as weights from survey data.

Table 4 provide figures on the mean variances-covariances of farm profits computed over the period 1994-1997. Table 5 presents instead the mean covariances of profits with consumption prices calculated using the same period.

[2]The simple process of expectations formation assigns the same weight to past realizations, which can be acceptable given the short period considered. Wherever farm specific data on net revenues are not available, average values computed for the respective land size category are used instead.

Figure 1: Monthly producer prices for food and cash crops (SSNPR region)

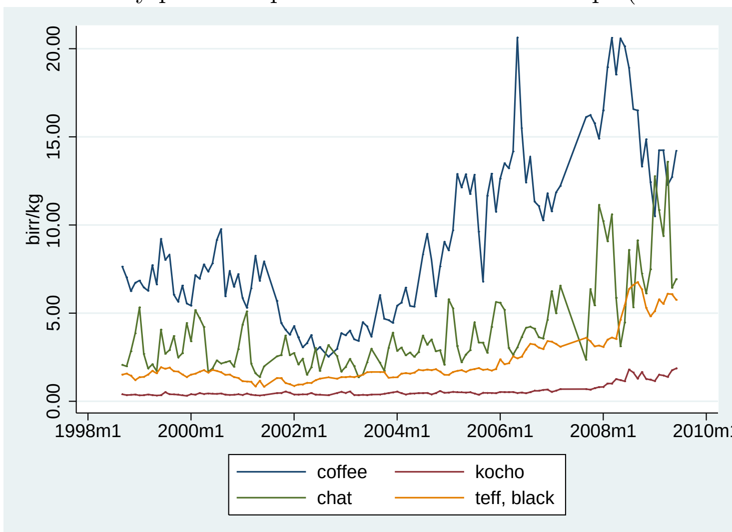

Table 3: Expectations on profits and consumption prices, 1994-1997

Item	**Aze Deboa**		**Adado**	
	mean	**sd**	**mean**	**sd**
Profits (000 birr/ha)				
Coffee	37.680	32.970	214.201	410.573
Enset	10.925	19.492	105.759	228.551
Cereals	6.831	7.650	16.755	75.843
Other trees	4.781	6.178	5.594	10.654
Other crops	5.423	5.981	20.225	40.400
Livestock prod.	3.816	10.927	4.099	3.451
Consumption price indexes				
Cereals	1.107	0.019	2.717	0.236
Enset	0.972	0.092	0.455	0.024
Livestock prod.	28.239	4.859	5.501	0.009
Other food prod.	8.149	0.460	4.033	0.086
Non-food prod.	32.585	21.745	12.844	15.270

Source: Authors'calculation using ERHS data (round 1-4)

Table 4: Mean variances-covariances of farm profits, 1994-1997

	Coffee	**Enset**	**Cereals**	**Livestock prod.**
		Aze Deboa		
Coffee	3035.618	152.767	-5.473	58.784
	(13299.098)	(1016.053)	(145.856)	(324.698)
Enset		1389.908	19.191	13.136
		(507.222)	(84.6459)	(67.677)
Cereals			176.915	8.285
			(823.278)	(44.904)
Livestock prod.				393.568
				(956.199)
		Adado		
Coffee	316538.413	23407.14	-912.822	20.996
	(1163854.305)	(85219.801)	(39598.394)	(1638.1)
Enset		92333.932	4808.996	-96.496
		(352920.38)	(33933.81)	(808.934)
Cereals			4443.208	-46.02
			(9986.192)	(220.228)
Livestock prod.				42.803
				(168.306)

Source: Authors'calculations using ERHS data (round 1-4)

Note: The figures are given in 000 birr/ha. Standard deviations are reported in brackets.

Table 5: Mean covariances of profits and consumption prices, 1994-1997

	Cereals	**Enset**	**Livestock prod.**	**Other food**	**Non food**
			Aze Deboa		
Coffee	-0.641	3.974	197.798	-13.568	0.86
	(1.779)	(6.192)	(773.603)	(138.288)	(641.468)
Enset	-0.254	-4.455	128.866	-23.118	-15.191
	(1.802)	(14.94)	(618.928)	(62.264)	(245.787)
Cereals	0.159	-0.457	-23.858	-5.526	-1.387
	(0.692)	(1.885)	(145.245)	(15.735)	(281.966)
			Adado		
Coffee	166.588	18.394	5.714	57.758	1629.073
	(677.033)	(64.061)	(25.739)	(248.987)	(21767.899)
Enset	-62.547	-3.707	-2.853	-25.73	14.366
	(262.85)	(20.99)	(11.082)	(102.752)	(1565.178)
Cereals	-24.879	-1.733	-1.078	-9.908	46.382
	(147.192)	(10.662)	(6.294)	(58.145)	(861.319)

Source: Authors'calculation using ERHS data (round 1-4)

Note: Standard deviations are reported in brackets

4 Empirical specification

In this section we turn to the empirical specification of the model. We make further assumptions to ease the empirical tractability and proceed assigning suitable functional forms. We then perform a linearization that allows us to solve the expectations without using integration techniques, getting first and second moments of the random variables.

Therefore, we make the following assumptions: a) no investment decisions concerning livestock holding are taken. Inventories of crops or cash in hand are thus the only possible forms of saving, though paying no returns; b) a simplified production function is adopted, where variable inputs are attributed proportionally to the output value of different crops, according to the $(M \times 1)$ coefficients vector κ_j, where M is the number of inputs; c) we consider only interior solutions for the control variables; d) the residual crop category from the adding-up land constraint is negligible in terms of output value and is not considered. We are thus focusing exclusively on equation (11) of the first order conditions in what follows.

Net revenues from cropping can be expressed as

$$\pi^a_{t+1} = \sum_{j=1}^{S} (p^j_{t+1}\xi^j_{t+1} - w^j_t \kappa_j)\theta^j_t \overline{L}_t \tag{14}$$

where ξ^j_{t+1} are random yields per hectare of cultivated land.

The Bellman equation reduces to

$$V_t(y_t, p_t, \epsilon_t) = \max_{\theta^j} \; v(y_t, p_t) + \beta E_t V_{t+1}\left(\sum_{j=1}^{S}(p^j_{t+1}\xi^j_{t+1} - w^j_t \kappa_j)\theta^j_t \overline{L}_t + \pi^b_{t+1} + \pi^w_{t+1}\right) \quad (15)$$

for $t < T$, subject to (8) and the non-negativity constraints $\theta^j_t \geq 0$.

Assuming interior solutions ($\mu_{jt} = 0$), we can differentiate with respect to the crop shares to get the first order conditions

$$\beta E_t \left[\frac{\partial V}{\partial y}(p^j_{t+1}\xi^j_{t+1} - w^j_t \kappa_j)\overline{L}_t\right] - \lambda_t \frac{\partial g_t}{\partial \theta^j_t} = 0 \qquad \text{for } j = 1, ..., S-1 \quad (16)$$

and combining the above FOCs we have

$$E_t \left[V_y \left(\pi_{jt} - \frac{g_{jt}}{g_{S-1t}}\pi_{S-1t}\right)\right] = 0 \qquad \text{for } j = 1, ..., S-2 \quad (17)$$

where $V_y \equiv \partial V / \partial y$, while π_{jt} and g_{jt} denote the partial derivatives of π_t and $g(.)$ with respect to θ^j_t, respectively.

In spite of its local validity, we take a first order approximation of $\partial V / \partial y$ around the expected values of income $\bar{y}$ and prices $\bar{p}$, as in Fafchamps (1992)

$$V_y \approx \bar{V}_y + \sum_{r=1}^{R} \bar{V}_{yp^r}(p^r - \bar{p}^r) + \bar{V}_{yy}(y - \bar{y}) \quad (18)$$

where $\bar{V}_y$ stands for $V_y(\bar{y}, \bar{p})$. After taking expectations and rearranging, equation (17) can be rewritten as

$$\bar{V}_y \left[E_t[\pi_{jt}] - \frac{g_{jt}}{g_{St}}E_t[\pi_{St}] + \sum_{r=1}^{R}\frac{\bar{V}_{yp^r}}{\bar{V}_y}E_t\left[(p^r - \bar{p}^r)\left(\pi_{jt} - \frac{g_{jt}}{g_{S-1t}}\pi_{S-1t}\right)\right] + \frac{\bar{V}_{yy}}{\bar{V}_y}E_t\left[(y - \bar{y})\left(\pi_{jt} - \frac{g_{jt}}{g_{S-1t}}\pi_{S-1t}\right)\right]\right] = 0 \qquad \text{for } j = 1, ..., S-2. \quad (19)$$

As in Kurosaki and Fafchamps (2002), to make (19) empirically tractable we assign V the following power form

$$V(y_t, p^r_t) = \frac{1}{1 - \Psi_t}\left[\frac{y_t - \sum_{r=1}^{R} p^r_t \gamma^r}{\prod_{r=1}^{R}(p^r_t)^{\beta^r}}\right]^{1-\Psi_t} \quad (20)$$

where Ψ_t is a relative risk aversion coefficient with respect to income after necessary consumption $\sum_r \gamma^r$ has been satisfied.

The standard Arrow-Pratt coefficient of relative risk aversion is obtained after adjusting for necessary consumption through the transformation

$$R_t = \Psi_t \frac{E[y_t]}{E[y_t - \sum_{r=1}^{R} p^r_t \gamma^r]}. \quad (21)$$

We assume that the concavity of V depends on household's ability to bear risk and thus we parameterize Ψ_t as

$$\Psi_t = \psi_0 + \sum_h \psi_h z_t^h \qquad h = 1, ..., H \tag{22}$$

where z_t^h include household's assets, such land and livestock owned, demographic characteristics and proxies for human capital, such as the education of the household head, and institutional features like the land tenure system.

The CPI in the indirect utility function is calculated as a geometric average, where β^r is the expenditure share of the r^{th} good in the linear expenditure system

$$p_t^r c_t^r = p_t^r \gamma^r + \beta^r (y_t - \sum_{r=1}^{R} p_t^r \gamma^r) \qquad \text{for } r = 1, ..., R. \tag{23}$$

Finally, as in Chavas and Holt (1996), we assign a quadratic form to the technology constraint $g(\theta_t^1, ..., \theta_t^{S-1})$ to take into account agronomic constraints between the $S-1$ free land shares, indipendent of past crop choices, such as crop complementarities and water requirements

$$\theta_t^1 - \alpha_0 - \sum_{j=1}^{S-2} [\alpha_{2j-1} \theta_t^{j+1} + \alpha_{2j} (\theta_t^{j+1})^2] = 0. \tag{24}$$

The structural model comprises the first order conditions for land allocation (19), the technology constraint (24) and the demand equations (23)

$$E_t \left[V_y \left(\pi_{jt} - \frac{g_{jt}}{g_{S-1t}} \pi_{S-1t} \right) \right] = \upsilon_t^j \qquad \text{for } j = 1, ..., S-2 \tag{25}$$

$$\theta_t^1 - \alpha_0 - \sum_{j=1}^{S-2} [\alpha_{2j-1} \theta_t^{j+1} + \alpha_{2j} (\theta_t^{j+1})^2] = \upsilon_t^{S-1} \tag{26}$$

$$-p_t^r c_t^r + p_t^r \gamma^r + \beta^r (y_t - \sum_{r=1}^{R} p_t^r \gamma^r) = \upsilon_t^{S-1+r} \qquad \text{for } r = 1, ..., R-1. \tag{27}$$

Notice that one demand equation has been dropped because of the adding-up constraint.[3]

4.1 Estimation procedure

The structural system is estimated by FIML. The disturbance vector υ_t^i is assumed jointly normal, with variance-covariance matrix Σ, hence the log-likelihood function for the system of p equations is given by

$$\ln L(\Omega | data) = -\frac{pTN}{2} \ln(2\pi) - \frac{TN}{2} \ln |\Sigma| + \sum_{i=1}^{N} \sum_{t=1}^{T} \left(\ln |J_{i,t}| - \frac{1}{2} \upsilon_t^{i'} \Sigma^{-1} \upsilon_t^i \right) \tag{28}$$

[3]The equation for other non-food products has been dropped because of the adding-up restriction arising from the use of expenditures instead of quantities consumed in the demand system.

where $\Omega \equiv \{\alpha, \beta, \gamma, \psi, \Sigma\}$ is the vector of parameters to be estimated, N is the number of households in the sample, T the time periods and $J_{i,t}$ is the Jacobian transform matrix, since the equations (25)-(26) do not yield a closed form solution in the crop shares θ_t^j.

4.2 Data description

The data used in the empirical study come from the Ethiopian Rural Household Survey (ERHS) carried out by IFPRI, the Universities of Oxford and Addis Ababa. It is a longitudinal data set collected in six rounds from 1989 to 2004 covering 15 villages across the country and providing a sample of 1477 households. It is not representive of all rural Ethiopia but covers all different agro-climatic areas. Our selected sub-sample covers two villages of the SSNP region in southern Ethiopia, Adado and Aze Deboa, where coffee is produced.

Table 6: Descriptive statistics for Aze Deboa and Adado (1999)

Variable	**Aze Deboa**			**Adado**		
	n	**mean**	**sd**	**n**	**mean**	**sd**
Demographics and assets						
Household size	73	7.4	2.5	124	6.45	2.99
Dependency ratio	73	.33	.209	124	.448	.231
Livestock (tlu)	73	1.73	.956	124	.418	.704
Land size (ha)	74	.845	.446	134	.457	1.32
Land allocation						
Coffee share	68	.204	.139	134	.446	.127
Enset share	72	.167	.0981	134	.476	.14
Cereals share	74	.47	.201	7	.282	.266
Fruit/Trees share	19	.144	.128	20	.201	.108
Other crops share	44	.239	.157	23	.192	.119
Income sources - Consumption						
Livestock profits	73	50.2	154	115	20.2	85.1
Crops profits	74	923	631	132	1341	1133
Wage and other income	73	307	1217	124	518	1475
Food consumption	73	37.8	51.9	124	67.5	65.5
Expenditure shares						
Cereals share	73	.256	.148	120	.13	.108
Enset share	72	.167	.0981	134	.476	.14
Livestock products share	56	.157	.184	121	.225	.127
Other food share	73	.343	.173	124	.361	.139

Source: ERHS (round 5)

Table 7: Descriptive statistics for Aze Deboa and Adado (2004)

Variable	**Aze Deboa**			**Adado**		
	n	**mean**	**sd**	**n**	**mean**	**sd**
Demographics and assets						
Household size	74	8.07	2.19	126	6.29	2.38
Dependency ratio	74	.377	.193	126	.453	.23
Livestock (TLU)	74	2.26	1.5	124	.543	.872
Land size (ha)	74	4.2	20.6	126	1.49	4.19
Land allocation						
Coffee share	71	.119	.0603	125	.449	.148
Enset share	72	.124	.0742	126	.429	.168
Cereals share	72	.364	.189	24	.196	.189
Fruit/Trees share	66	.167	.147	31	.181	.149
Other crops share	59	.278	.171	33	.163	.136
Income sources - Consumption						
Livestock profits	74	90.1	278	123	21.9	238
Crops profits	74	1509	2648	118	1047	784
Wage and other income	74	141	566	126	28.4	84.5
Food consumption	73	111	134	126	41.1	45.6
Expenditure shares						
Cereals share	67	.37	.231	76	.243	.157
Enset share	72	.124	.0742	126	.429	.168
Livestock products share	58	.105	.138	78	.239	.145
Other food share	73	.479	.267	126	.536	.296

Source: ERHS (round 6)

5 Estimation results

We now turn to the estimation of the structural parameters of the system given by equations (26)-(27). The estimation is conducted using a balanced panel of $N = 166$ households (67 from Aze Deboa, 99 from Adado) over $T = 2$ years (1999 and 2004), for a total of 332 observations.

Table 8: Structural parameter estimates

Parameters	**Estimates**	**s.e.**	**Est./s.e.**	**Prob.**
Risk aversion determinants				
Intercept (ψ_0)	0.2323	0.0477	4.869	0.0000
Land size (ψ_1)	-0.0604	0.0206	-2.938	0.0033
Livestock owned (ψ_2)	0.1365	0.0076	17.915	0.0000
Education (ψ_3)	0.2191	0.0489	4.483	0.0000
Market participation (ψ_4)	-0.1296	0.0196	-6.625	0.0000
Property rights (ψ_5)	-0.1681	0.0515	-3.262	0.0011
Consumption preferences				
Cereals share (β_1)	0.1837	0.0172	10.66	0.0000
Enset share (β_2)	0.0775	0.0165	4.692	0.0000
Animal products share (β_3)	0.1529	0.0136	11.263	0.0000
Other food share (β_4)	0.4584	0.0227	20.214	0.0000
Non food share (β_5)	0.1275	0.0083	15.4537	0.0000
Cereals subsistence con. (γ_1)	0.2887	0.1468	1.966	0.0493
Enset subsistence con. (γ_2)	3.633	0.4759	7.633	0.0000
Animal prod. subsistence con. (γ_3)	-0.0154	0.0214	-0.723	0.4697
Other food subsistence con. (γ_4)	-0.1822	0.0945	-1.927	0.0539
Non food subsistence con. (γ_5)	0.1493	0.0048	30.867	0.0000
Agronomic constraints				
Intercept (α_0)	0.2361	0.0245	9.638	0.0000
Linear term enset (α_1)	1.0683	0.1112	9.603	0.0000
Quadratic term enset (α_2)	-1.5237	0.1211	-12.578	0.0000
Linear term cereals (α_3)	-0.0427	0.0193	-2.215	0.0268
Quadratic term cereals (α_4)	-0.3254	0.0558	-5.833	0.0000
Mean log-likelihood -22.253				

Since we are considering four (three free) crop production categories ($S = 4$) and five consumption categories ($R = 5$), the final system comprises two first order conditions, one agronomic constraint and four demand equations (because of the adding-up constraint) for a total of seven ($p = 7$) equations.

Table 8 reports the estimates of the structural parameters, together with the standard errors, calculated by inverting the computed Hessian, the associated t-ratio and probability values.
Parameters are estimated with high precision, as long as the assumptions and restrictions imposed by functional forms are acceptable.

As concerns consumption parameters, we observe that committed consumption is relatively high for enset, as expected being a staple crop, lower for cereals which are often used as a cash crop, while animal products and other food turn out to be inessential goods, as implied by the negative sign. Once covered subsistence consumption, households reveal high preferences for cereals, animal products and other richer food products in the residual food category.

Table 9: LES - Price elasticities

Cons./Price	**Cereals**	**Enset**	**Animal prod.**	**Other food**	**Non food**
Cereals	-0.845	-0.165	0.013	0.064	-0.207
	(0.072)	(0.031)	(0.017)	(0.027)	(0.028)
Enset	-0.012	-0.331	0.004	0.022	-0.07
	(0.006)	(0.07)	(0.006)	(0.011)	(0.016)
Animal prod.	-0.048	-0.228	-1.097	0.089	-0.285
	(0.025)	(0.047)	(0.143)	(0.046)	(0.039)
Other food	-0.052	-0.247	0.019	-1.113	-0.309
	(0.03)	(0.048)	(0.026)	(0.063)	(0.03)
Non food	-0.014	-0.068	0.005	0.026	-0.418
	(0.007)	(0.01)	(0.007)	(0.014)	(0.022)

Notes: Standard error in parentheses

Table 10: LES - Total expenditure elasticities

Item	**Estimate**	**Std. Err.**	**t-Ratio**
Cereals	1.1408	0.1407	8.1079
Enset	0.3872	0.0870	4.4488
Animal prod.	1.5693	0.2161	7.2607
Other food	1.7021	0.1551	10.9743
Non food	0.4686	0.0252	18.5965

Turning to price elasticities (Table 9), we see that own price elasticities are significant and have the expected signs; demand for animal products and other food is elastic, while that for staple crops, in particular enset, is quite inelastic, as expected. Nevertheless, the elasticity value close to one for cereals and the negative cross price elasticities seem to reveal that the staple crops are not real substitutes in consumption; rather, since we cannot rely on a simple price comovement explanation, cereals seem to play a double role as a food and cash crop, with consumption levels being reduced in favour of a higher marketed surplus to get advantage of periods characterized by high staple crops prices. In general, the importance of staple crops in households' budgets is reflected in the negative cross price elasticities with other consumption items and the same holds for non-food expenditure. Conversely, cereals, enset and other food may substitute for animal products, even though elasticities are not significant.

As shown in Table 10, income elasticity is quite high for cereals, an appreciated food crop, quite low for enset, an inferior staple crop, and high for other food and animal products. As to technology parameters, the high and significant value of the quadratic term for enset (α_2) indicates the importance of technical considerations of joint production with coffee, as expected because of intercropping, even though the negative sign is puzzling. The mild concave relationship between coffee and cereals suggests that economic factors are more important than agronomic constraints.

As regards the determinants of risk aversion, the coefficients are all significant and display plausible signs. Land size, used as a proxy of wealth has a positive sign, as suggested by theory, while the ownership of livestock seems to increase risk aversion. In general, the availability of liquidable assets, buffering against downside risks, is traditionally seen as reducing risk aversion, even though a tentative reverse causality explanation might be offered, whereas higher risk aversion leads households to accumulate precautionary saving in the form of liquidable assets. A higher education of the household's head tends to increase risk aversion, while an increase in the proportion of production sold on the market reduces it. The same result holds for the dummy reflecting the perceived property rights on own cultived land. This appears to be a strong result, confirming the arguments by Gautam et al. (2007) according to which tenure insecurity discourages long-term, potentially profitable but risky, investments, such as those in perennial crops, as concluded in their study carried out using Ethiopian data.

Table 11: Risk aversion coefficients by land size

Quantile	**Aze Deboa**	**Adado**
I	0.4557	0.1526
II	0.5098	0.1688
III	0.4279	0.1482
IV	0.4516	0.0574
Total	0.4664	0.1402

The predicted values for the relative risk aversion coefficient Ψ_t computed by land quartiles (Table 11) reflect the negative sign of the coefficient ψ_1 and thus, starting from the second quantile, tends to decrease with the amount of land owned. In fact, for very small landholdings, an effective portfolio diversification might be precluded. This pattern is evident for Adado, but the same is not true for Aze Deboa. The mean risk aversion coefficient for the whole distribution is low for Adado (0.14) and higher for Aze Deboa (0.46). This might be explained by the fact that Adado is an area highly specialized in coffee production, which remains a profitable but risky crop, while at Aze Deboa farming is more mixed. In general, the magnitude of the coefficient computed at the mean is lower than in Kurosaki and Fafchamps (2002), 1.83, and closer to that reported in Rosenzweig and Wolpin (1993), 0.96, for Indian farmers. The standard Arrow-Pratt coefficient of relative risk aversion R_t can obtained by applying equation (21).

6 Concluding remarks

In this work we have investigated crop portfolio choices by farmers as an instrument to reduce risk exposure in a rural environment characterized by imperfect or missing markets. The literature on the farm household (Roe and Graham-Tomasi (1986),Sadoulet and de Janvry (1995)) has demonstrated that in the absence of markets to copying with risk, separability between production and consumption decisions breaks down, so that the two economic decisions should be modelled jointly. Therefore, following Kurosaki and Fafchamps (2002), we set out a dynamic model of production and consumption under price and yield risk, from which structural estimates of technology, consumption and risk parameters are derived using longitudinal data from rural Ethiopia.

Empirical evidence shows that farmers portfolio is diversified and, to a more or lesser extent, both food and cash crops are included. Estimates of the determinants of risk aversion suggest that land size, used as a proxy of wealth, and institutional features related to the land tenure system are relevant factors in decreasing risk aversion, allowing households to engage in riskier activities.

This is reflected in the predicted risk aversion parameters computed by land quantiles, where at least in one village, risk aversion decreases with landholdings. Nonetheless, the overall low value of the coefficient compared to other studies may suggest two interpretations. First, the perennial nature of coffee and the related adjustment costs accompanying investment and disinvestment decisions, may prevent farmers from readily adjusting their portfolio to changed market conditions. Second, the agronomic system widespread in Southern Ethiopia and characterized by the joint cutivation of coffee, enset and to a lesser extent cereals, may prove effective in protecting farmers from adverse market and weather events. In particular, the role of enset, a poor but drought resistant staple crop, seems crucial in shielding farmers from the negative effects of highly volatile coffee and cereal prices.

As to further developments of the present work, it would seem interesting, as outlined in the theoretical model, to allow for choices concerning the investment in livestock also in the empirical specification, thus endogenizing the role of liquidable assets as instruments for consumption smoothing or for accumulating precautionary saving.

Another possibility would be to explicitely consider investments in production capacity in the form of replantings of existing stocks. Finally, the use of estimation methods such as maximum simulated likelihood might also allow to deal with corner solutions.

Appendices

A Derivation of estimable first order conditions

In order to obtain an estimable form for the approximated first order conditions (19), a parameterized version is required. Therefore, using (20), we first calculate $\bar{V}_y$, $\bar{V}_{yy}$ and $\bar{V}_{yp^r}$, evaluated at the expected values for prices, profits and incomes. Then, upon substituting such expressions into the first order conditions we obtain two second-order polynomial equations, FOC_{21} and FOC_{31}, in the endogenous variables θ^j. The first condition FOC_{21} is for instance given by

$$FOC_{2,1} \approx F_2 - F_1(\alpha_1 + 2\alpha_2\theta^2) + \sum_{j=1}^{3}[G_{j,2} - G_{j,1}(\alpha_1 + 2\alpha_2\theta^2)]\theta^j \tag{29}$$

where the terms F and G are functions of constructed variables, Z and W, and of the structural parameters β, γ and Ψ to be estimated. The subscripts $j, a = 1, 2, 3$ denote respectively coffee, enset and cereals, while $r = 1, 2, 3, 4, 5$ identify the consumption categories.[4] The variable F_a is constructed as

$$F_a = (1-\Psi)\sum_{j=1}^{S}\gamma^j Ep^j \sum_{r=1}^{R}\beta^r Z^1_{a,r} + \Psi \sum_{1=r}^{R}\gamma^r Z^2_{a,r} \tag{30}$$

$$- (1-\Psi)\sum_{r=1}^{R}\beta^r Z^3_{a,r} + \sum_{r=1}^{R}\gamma^r Z^4_{a,r} + \Psi Z^5_a + Z^6_a \tag{31}$$

while the variable $G_{j,a}$ is given by

$$G_{j,a} = (1-\Psi)\sum_{r=1}^{R}\beta^r W^1_{j,a,r} + \Psi W^2_{j,a} + W^3_{j,a}. \tag{32}$$

The variables Z and W are combinations of first and second moments of prices and profits and are defined as follows

$$Z^1_{a,r} = \frac{\mathrm{Cov}(\pi_a, p^r)}{LEp^r E\pi_1}, \quad Z^2_{a,r} = \frac{Ep^r}{E\pi_1} Z^1_{a,r}, \quad Z^3_{a,r} = (AE\pi_m + R)Z^1_{a,r}$$

$$Z^4_{a,r} = -\frac{E\pi_a Ep^r}{LE\pi_1}, \quad Z^5_a = \frac{A\mathrm{Cov}(\pi_a, \pi_m)}{LE\pi_1}, \quad Z^6_a = \frac{E\pi_a Z^3_{a,r}}{LE\pi_1 Z^1_{a,r}}$$

$$W^1_{j,a,r} = LE\pi_j Z^1_{a,r}, \quad W^2_{j,a} = \frac{A\mathrm{Cov}(\pi_j, \pi_a)}{E\pi_1}, \quad W^3_{j,a} = \frac{E\pi_j E\pi_a}{E\pi_1}.$$

[4] The subscripts for household, i, and time, t, have been dropped.

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Qualitative commodity market analysis under rational expectations: the case of the world cocoa market

1 Introduction

The analysis of primary commodity markets is relevant in many respects as: a) most agricultural commodities constitute basic staple food for populations worldwide; b) they still represent basic inputs in industrial production processes; c) primary commoditities are the major source of export revenues of many developing countries and on their production depends the livelihoods of millions of smallholders; d) they have become increasingly important as a financial asset as commodity prices display a countercyclical pattern that make them effective in a portfolio strategy.

One feature displayed by commodity markets is the lagged response by economic agents to market signals even in time series with annual periodicity, as observed by Lord (1991). This is most true for perennnial crops, such as cocoa, coffee, tea or rubber, cultivated in developing countries where, in addition to the biological gestation lag, market imperfections and domestic policies generate adjustment costs which cause suppliers to fully react to price changes only after several years. The delayed adjustments may generate temporary, even though not necessarily short-lived, market disequilibria from long term equilibrium relationships, resulting in quite long price cycles.[1]

In fact, because of the low price elasticity of world demand, the dynamics of commodity prices is mainly driven by supply shocks, with stocks fluctuating widely with a cyclical pattern, the cycles coinciding with those of production. Such a close correspondence between stocks and price movements can be observed in Figure 2 of next chapter, which shows the pattern of world cocoa price and the stock-to-use ratio, an indicator of cocoa availability monitored by industry analysts. Stocks of course may play a role of price stabilizers, absorbing excess supply, and that was the goal of the buffer stocks operating under several International Cocoa Agreements, even though they are not effective in case of price spikes due to stockouts.

Modelling of commodity markets has a long history. Traditional world commodity models (Akiyama and Duncan, 1984; Ghosh et al., 1987; Trivedi, 1990) typically comprise: a) blocks of domestic supply and demand equations, geographically disaggregated; b) price transmission equations linking local prices to a world reference price; c) a price equation relating price to world stocks or a proxy for world supply market-balance. Such models are typically estimated equation by equation. The tea model developed by Trivedi (1990) is a relevant variant as the world price is determined within a separate rational expectations (RE) model including a stockholding equation. The RE solution is used in an estimable price equation obtained by the inversion of the stock equation.

Some models focused on speculative stockholding and set price as a function of a single state variable, total availability, equal to production plus carry-

[1] Labys (2006) identifies price cycles, their amplitude and duration dependence, for some 21 primary commodities using monthly data over the period 1960-1995. For cocoa and coffee he finds 15 and 16 price cycles, whith a mean overall duration of about 25 and 27 months, respectively. In the case of cocoa, the maximum duration in contraction phases is 67 months, 134 for coffee, the longest among all investigated commodities.

overs, trying to address through dynamic programming the non linearities arising from the non-negativity constraints related to possible stockouts (Deaton and Laroque, 1992; Wright and Williams, 1989; Peterson and Tomek, 2005).

Other authors stressed the importance of market imbalances and the role of quantity variables for modelling expectations (Hwa, 1985; Ghosh et al., 1987; Gilbert and Palaskas, 1990). In particular, Gilbert and Palaskas (1990), unlike traditional RE models, regress price changes on expected quantity variables. The rationale is that, as argued by Ghosh et al. (1987), expected future price changes are in general uninformative, since arbitraged to equal storage cost less convenience yield; rather, expected future supply-demand balances may provide an indication of the direction in which the price must adjust if the market clears over time. Though, as adjustments will eliminate such imbalances, the relevant expected future imbalances are those calculated at a reference price.

Gilbert (1995), along this stream of literature, reduces the information set used in a model for the aluminium market to a limited number of state variables, called market fundamentals, capturing excess supply (or demand) in the short and long-term. The basic intuition is that agents will be willing to hold stocks if excess supply is positive in the short term but not in the long term too. The model allows to obtain predicted values for stocks and prices and to test the rational expectations hypothesis in stockolding behaviour, conditional on the validity of the model.

In this chapter we build on this latter contribution, specifying an aggregate global model for a perennial crop, namely cocoa, accounting for speculative stockholding behaviour, given the historical importance of physical stocks in the cocoa market. In the model specification we proceed in two steps: we first set out a rather general ARDL model, which allows to properly deal with the lagged response and is meant to represent an encompassing specification, containing several features useful to modelling perennial crop markets in different frameworks.[2] In fact, depending on the specific modelling purposes and data availability, sensible economic restrictions can then imposed. In the second step, we derive a stripped-down version of the model and adopt a solution method which, in a rational expectations framework, makes use of the concept of market imbalance (Gilbert, 1995). In deriving the solved or reduced form in price and stocks, we generalize the definition of market fundamental accounting for the lags in supply response characterizing perennial crops. A possible research question is in fact whether market participants anticipate future market imbalances, by including past (typically supply) shocks into their information set, as we would expect in a rational expectations framework.

The chapter is organized as follows. Section 2 presents the general specification of the model, where a particular emphashis is placed on the supply side given the peculiar features characterizing perennial crop production. Section 3 illustrates the analytical derivation of the solved form, through the construction of the market fundamentals, and of the set of restrictions stemming from the

[2]For instance, the ARDL specification proves convenient as it can be readily written in ECM form, particularly suitable to describe long-term equilibrium relationships and the related adjustment processes.

rational expectations hypothesis. Section 4 provides in a deterministic setting and using a short run model, on overview of the conditions determining the dynamic stability of the system and a simulation of price and stock response to shocks hitting the system, using the price rational expectations solution and different combinations of values for the structural parameters. The concluding remarks are given in Section 5.

2 Modelling perennial crop markets

2.1 Production

Perennial crops, such as cocoa, display typical features which make the traditional nerlovian approach not suitable to modeling supply response, namely: a) the existence of a biological gestation lag between planting and obtaining yield during which supply conditions may change; b) the bearing of adjustment costs related to the removal and planting of trees; c) the fact that the productivity of trees varies systematically with age; d) the heterogeneous nature of the tree stock, since the age-yield profile and productive life depend on technical change and hence are not invariant with respect to the date (*vintage*) of the investment.

The heterogenous nature of capital stock and the time-varying productivity have been traditionally dealt with by making use of the concept of *potential* output q_t^p, given by the sum, over all mature vintages v, of the tree stock $K(t,v)$ still in production multiplied by yields $\delta(t,v)$ associated to each specific vintage[3]

$$q_t^p = \sum_v \delta(t,v)K(t,v), \ \forall v. \tag{1}$$

Potential output represents a vintage production function under the assumption of a constant coefficients technology, where variable inputs are combined in fixed proportions to capital stock of different vintages.

Data on stock composition and age-yield profiles are often lacking and therefore simplifying assumptions are usually adopted. For instance, if yields are assumed to depend only on age and are positive from age k through age m, the second equality in equation (2) holds

$$q_t^p = \sum_v \delta_{t-v}K_{t-v} \cong \sum_{i=k}^{m} \delta_i K_{t-i} \cong \bar{\delta} \sum_{i=k}^{m} K_{t-i} \tag{2}$$

and if we further assume that yields are uniform over time, we get the third equality. These are of course strong assumptions as we are in fact ruling out technical change and the possibility of accounting for capital heterogeneity.

In order to improve production capacity the farmer has the option of either planting trees on land previously uncultivated or allocated to different crops, or replacing existing aging stands with new trees. As argued by (Ruf et al., 2004), new planting and replanting decisions are qualitatively different, as the

[3]we assume a constant density of planting d so that area planted $A(t,v) = dK(t,v)$ represents a good proxy for the tree stock.

slash-and-burn system does not imply the additional opportunity cost of investment of foregoing a present, though declining, output. These adjustment costs, associated to the partial irreversibility of investment, liquidity constraints and uncertainty about future economic conditions may result in inaction bands, where farmers do not respond to price changes, making the standard neoclassical theory of investment not suitable for explaining investment decisions typical of a mature industry (Hill, 1996).

These considerations motivated the attempts of modelling the two investment decisions separately (French and Matthews, 1971; Hartley et al., 1987; Akiyama and Trivedi, 1987), even though data availability has severely limited the widespread use of such modeling strategy.[4]

Therefore, if we are forced to neglect the age composition of the tree stock, the evolution of total productive land A_t can be written as

$$A_t = A_{t-1} + M_t - U_t + \xi_t \tag{3}$$

where M_t denotes new areas entered into production, U_t areas temporarily out of production because of uprootings of aging trees aimed at replantings or permanently loss to that specific crop production, while ξ_t represents any stochastic shock negatively affecting the tree stock such as frosts, fires and deseases.

Since data on uprootings, plantings or replantings are hardly available at agggregate level, coming usually from ad hoc household surveys, we follow Mehta and Chavas (2008) specifying a simplified law of motion

$$A_t = \vartheta A_{t-1} + N_t \tag{4}$$

where N_t denote net additions to productive areas and $(1 - \vartheta)$ is the annual depreciation rate.

In deriving the supply equation of the global model we proceed by specifying a structural model of supply along the lines of the seminal work by Wickens and Greenfield (1973) on the coffee market in Brazil. Their supply model comprises a vintage production function, given by potential output as defined above, an investment equation for new plantings and a harvesting equation relating actual to potential output, aimed at capturing short-run supply response.

The optimal level of investment I_t^* in their model is derived by maximizing the discounted flow of expected net revenues from the investment subject to the production function (1)

$$V_t = \sum_{t=0}^{\infty} \beta^t [(p_t^e - s_t^e) q_t^p - F_t - g(I_t)] \tag{5}$$

where s_t^e is the expected unit cost of harvesting, F_t are fixed costs, $g(I_t)$ is a nonlinear function representing planting costs and β is the discount factor. By assuming a quadratic function for $g(I_t)$, the time path solution for I_t is a linear

[4]The use of the Kalman filter in models cast in state-space form tries to overcome the shortcomings related to lack of statistical data (Kalaitzandonakes and Shonkwiler, 1992).

function of discounted expected net revenues R_t^e

$$R_t^e = \sum_{i=0}^{\infty} \beta^i \delta_i (p_{t+i}^e - s_{t+i}^e) \tag{6}$$

where δ_i are age-specific yields and the unobservable variables are proxied by distributed lags of prices and variable input costs, if any are available. Since cocoa production is labour intensive, a suitable proxy for unit costs would be the wage rate in the agricultural sector.[5] More in general, the availability of labour force has been historically an important factor in the early development of the cocoa sector in most producing countries, characterized by the slash-and-burn system.

The substitution of expected price with distributed lags of course introduces a measurement error and makes such naïve expectations not coherent with the rational expectations framework. Nevertheless, in major cocoa producing countries were operating, until the liberalization process took place in the 1990s, marketing boards and caisse systems where a guaranteed farm gate price were announced at the beginning of each crop year, eliminating uncertainty at least for the nearby harvest.[6]

At this regard, we remark that the specification of an aggregated global model unfortunately prevent us from incorporating institutional features specific to single countries. This can be problematic in pre-liberalization periods where domestic policies drive a substantial wedge betweeen producer and border prices, insulating producers from international price variability. While the price transmission issue could be better adressed in a disaggregated model via price linkage equation relating domestic producer prices to the world reference price, we may try to loosly control for domestic policies through the use of time dummies referring to the steps in the liberalization processes occuring in the main producing countries, such as Côte d'Ivoire. Furthermore, dummy variables may also allow us to control in a simple way for asymmetric supply response and inaction bands arising from the above mentioned adjustment costs and uncertain future market conditions in a liberalized framework.

The relevance of such effects is an empirical matter, and the use of an aggregated supply function might blur and hide locally relevant effects. The use of annual data and the related necessity of preserving degrees of freedom suggest parsimony in the use of such dummies, and the same rationale should apply to the choice of the appropriate lag length.[7]

[5] Other outlays are related to the purchase of seedlings, fertilizers and pesticides.

[6] This price was calculated by subtracting to a cif price all marketing costs along the supply chain, including levies, margins and stabilization funds used to guarantee minimum producer prices in period of declining world prices. The cif price was calculated as a weighted average of the prevailing spot price and the price obtained by selling forward part of the future harvest. For this reason the futures price for harvest time delivery could be considered a good proxy of the reference price faced by producers at planting time and, in general, of the extent of the investment by the state in the cocoa sector for the incoming crop year, in terms of subsidized inputs and infrastructures.

[7] While information criteria provide guidance, a relevant lag length for planting decisions could be 3 to 5 years, as this is the gestation lag before the plant starts bearing fruits, even

Therefore, a rather general distributed-lag linear specification for the investment equation, describing areas entered into production in period t, can be written as

$$N_t = \mu_t^N + \alpha_c(L)p_t + \alpha_f(L)f_t + \alpha_{cf}(L)p_t^{cf} + \boldsymbol{\alpha}_{\mathbf{w}}(L)\mathbf{w}_t \tag{7}$$

where f_t denotes the futures price of cocoa, p_t the spot cocoa price, p_t^{cf} the spot price of substitute crops, such as coffee, and $\mathbf{w}_t$ is a vector including the unit cost of inputs or in general any other monetary factor affecting investment decisions. The formulation of the intercept, $\mu_t^N = \mu_0^N + \mu_1^N D_t^N$, is general enough to include time dummies or deterministic terms. The coefficients $\alpha_i(L)$ are polynomials in the lag operator L that in a very general formulation are given by

$$\alpha_i(L) = \sum_{j=0}^{s}(\alpha_{ij} + \alpha_{ij}^{+}D_i^{+} + \alpha_{ij}^{B}B_i + \alpha_{ij}^{B+}B_iD_i^{+})L^j \tag{8}$$

where D_i^+ and B_i are dummies denoting positive price changes and policy changes, such as liberalizations, respectively .

The harvesting equation aims to explain discrepancies between potential output q_t^p, as a result of past investments, and actual production due to short run supply response to current and lagged cocoa prices, resulting in more intense applications of variable inputs such as pesticides and fertilizers, and possibly to other exogenous factors $\mathbf{w}_t$, and is given by

$$q_t = \mu_t^h + \gamma_p q_t^p + \gamma_c(L)p_t + \boldsymbol{\gamma}_{\mathbf{w}}(L)\mathbf{w}_t + \gamma_q q_{t-1} + \xi_t^h \tag{9}$$

where one period lagged production q_{t-1}, while balancing the equation, may capture possible autocorrelation arising from shocks affecting the tree stock due to pest and deseases. The idea is that the permanent component of the composite shock is captured by lagged production so that the disturbance term ξ_t^h can be assumed to be an *iid* process, reflecting weather events.

Apart from using Kalman-filtering techniques, structural estimation of the supply system, given by equations (9), (7) and (2), usually turns out to be problematic because of data constraints. For these reasons, we prefer to derive the reduced form of the system, as in Wickens and Greenfield (1973), which represents the model supply equation. Thus, assuming that $q_t^p = \bar{\delta}A_t$, with constant average yields, writing (4) as

$$A_t = (1 - \vartheta L)^{-1}N_t \tag{10}$$

and substituting q_t^p into (9) we get

$$q_t = \mu_t^q + \beta_q(L)q_t + \beta_c(L)p_t + \beta_{cf}(L)p_t^{cf} + \beta_f(L)f_t + \boldsymbol{\beta}_{\mathbf{w}}(L)\mathbf{w}_t + u_t^q \tag{11}$$

where

$$\beta_{cf}(L) = \alpha_{cf}(L)\gamma^p\bar{\delta}\kappa, \quad \beta_f(L) = \alpha_f(L)\gamma_p\bar{\delta}\kappa, \quad \kappa = (1 - \vartheta L)^{-1}$$
$$\beta_c(L) = (\alpha_c(L)\gamma_p\bar{\delta} + (1 - \vartheta L)\gamma_c(L))\kappa, \quad \boldsymbol{\beta}_{\mathbf{w}}(L) = (\boldsymbol{\alpha}_{\mathbf{w}}(L)\gamma_p\bar{\delta} + (1 - \vartheta L)\boldsymbol{\gamma}_{\mathbf{w}}(L))\kappa$$
$$\beta_q(L) = \gamma_q L, \quad \mu_t^q = (\mu_t^N\gamma_p\bar{\delta} + (1 - \vartheta L)\mu_t^h)\kappa, \quad u_t^q = \xi_t^h.$$

though 8 to 11 years pass before the plant reaches full production.

In what follows, because of data constraints, the vector $\mathbf{w}_t$ will be substituted by the scalar x_{2t} denoting the exchange rate of major producing countries.

One problem with this approach is the difficulty of identifying the structural parameters from the reduced form. Furthermore, the inclusion of a large number of explanatory variables in the investment equation implies the addition of a corresponding number of lag structures, reducing drastically the degrees of freedom.[8]

2.2 Consumption

Consumption demand c_t, measured by total world grindings, is specified as a function of a distributed lag of cocoa price p_t, the price of cocoa substitues p_t^v, such as vegetable fats and oils, and world income x_{1t}, as measured by the weighted GDP of major consuming countries. In the general specification, lags in consumption have also been introduced to possibly account for long-term equilibrium relationships between consumption and GDP

$$c_t = \mu_t^c + \varphi_c(L)c_t + \varphi_p(L)p_t + \varphi_v(L)p_t^v + \varphi_x(L)x_{1t} + u_t^c. \tag{12}$$

2.3 Stockholding

Stock demand may include transaction, precautionary and speculative components. In Muth's famous speculative inventory model the incentive to carry over an additional unit of stock is given by

$$P_{t+1|t}^e - (1 + R_t + \delta)P_t + c(S_t) \tag{13}$$

where P_t is the spot commodity price, R_t the interest rate, δ a constant depreciation rate and $c(S_t)$ the marginal convenience yield, a positive return related to the availabilty of the commodity in periods of supply shortages.

Neglecting depreciation and convenience yield, the equilibrium conditions can be summarized by the Kuhn-Tucker inequalities

$$P_t \geq \frac{P_{t+1|t}^e}{1 + R_t} : \quad S_t \geq 0 \tag{14}$$

where arbitrage leads to a situation where either stocks are zero and the incentive to carry additional stocks is non-positive, or stocks are positive and the incentive is zero.

The non negativity restriction and the possibility of stockout may result in non-linear price response as an increase in production can be partially absorbed by increased stockholding, moderating the price fall, but the reverse is not true, unless sufficient carryovers from previous period are available. The

[8] The identification issue, along with the inadequate representation of the vintage technology, have led some authors to reject the econometric approach altogether and adopt a dynamic programming approach, gauged more appropriate to the complexity of the problem (Bellman and Hartley 1985, Trivedi 1986, Weaver 1989).

resulting non-linearities can be dealt with only by numerical approximation or simulation techniques even in fairly simple models (Lowry et al., 1987; Deaton and Laroque, 1996).

Note that we ignore the non negativity constraint, assuming a positive convenience yield high enough to ensure that positive stocks are always held. Hence, our log-linear stockholding equation is

$$s_t = \mu_t^s + \eta_s(L)s_t + \eta_e(p_{t+1}^e - p_t - r_t) + u_t^s \tag{15}$$

where η_e is taken as a constant parameter and lower-case letters denote as before logged variables.

2.4 Expectations

Agents form their expectations about future market developments following the rational expectations hypothesis, so that

$$p_{t+1} = E[p_{t+1}|\Omega_t] + \epsilon_{t+1} \tag{16}$$

where ϵ_{t+1} is un unforecastable innovation such that $E_t\epsilon_{t+1} = 0$.

2.5 Market clearing

The model is closed by the market clearing condition[9] where total availability, given by production plus lagged carryovers s_{t-1}, equals demand for final consumption and for stocks

$$q_t + s_{t-1} = c_t + s_t. \tag{17}$$

2.6 Exogenous variables dynamics

In a rational expectations framework, it is important to find a suitable representation of the stochastic process governing the exogenous variables dynamics as they represent the forcing variables of the system. In this section we simply present the chosen specifications, deferring to the next chapter further comments based on results from the unit root tests.

As concerns income x_{1t}, we adopt a random walk specification with a drift term

$$x_{1t} = \gamma_{10} + x_{1,t-1} + w_{1t}. \tag{18}$$

The same specification is used for the exchange rate x_{2t}, thus

$$x_{2t} = \gamma_{20} + x_{2,t-1} + w_{2t}. \tag{19}$$

[9] A few studies develop and estimate global commodity models where interestingly a futures market is introduced. Kawai (1983) derives the rational expectations equilibrium solution of both spot and futures market in a model with stockholding very close to the one developed in this work. Palm and Vogelvang (1986) estimate a short-run model for the world coffee market in which, together with futures, other prices along the supply chain are modelled. In all these cases, a clearing condition for the futures market is added to the model.

As regards the interest rate r_t, an ARMA(1,1) specification with drift term is chosen

$$r_t = \gamma_{30} + \rho_3 r_{t-1} + w_{3t} \tag{20}$$

where the disturbance $w_{3t} = \upsilon\epsilon_{t-1} + \epsilon_t$ includes an autoregressive component.

3 The estimable model specification

We now present the model specification used for estimation in the next chapter, obtained from the general formulation by reducing the order of the lag polynomials of the quantity variables, eliminating prices other than spot cocoa prices and the dummies related to asymmetric price response. The simplification, albeit at the cost of losing generality, eases the illustration of the solution method, originally proposed by Gilbert (1995) to analyze the world aluminium market and successively used by Perali and Pieroni (2004) for the US corn market.

The world cocoa model used to illustrate the solution method is given by the following set of equations:

Consumption

$$c_t = \mu_t^c + \varphi_p p_t + \varphi_x x_{1t} + u_t^c \tag{21}$$

Production

$$q_t = \mu_t^q + \beta_q q_{t-1} + \beta_p(L) p_t + \beta_x x_{2,t-1} + u_t^q \tag{22}$$

Stock demand

$$s_t = \mu_t^s + \eta_s s_{t-1} + \eta_e (p_{t+1|t}^e - p_t - r_t) + u_t^s \tag{23}$$

Expectations

$$p_{t+1|t}^e = E[p_{t+1}|\Omega_t] \tag{24}$$

Market clearing

$$q_t + s_{t-1} = c_t + s_t. \tag{25}$$

The equations for the stochastic processes of the exogenous variables (18)-(20) complete the model.

3.1 Market fundamentals

We now solve the above structural system into a solved or reduced form in price and stocks depending on a reduced set of state variables. To this end, following Gilbert (1995), we make use of two constructed variables measuring market imbalance in the short and long term. As argued before, the rationale is that agents will be willing to hold stocks if the short-term fundamental is weak, as excess supply prevails in the market, but not if the long-term fundamental is weak too.

As competitive markets always clear, we need to define the market imbalances at a reference price $\bar{p}$. This can be thought of as the price which on average clears the market or the steady state equilibrium price.

Proposition 1. Short term market fundamental
Given the structural model (21)-(25), the short-term market fundamental z_{1t} is given by the excess supply calculated at the reference price $\bar{p}$ as

$$z_{1t} = q_t - \beta_p(L)(p_t - \bar{p}) - c_t + \varphi_p(p_t - \bar{p}) + (s_{t-1} - \bar{s}). \tag{26}$$

We can interpret z_{1t} as the net addition to stocks were the market clearing price equal to the reference price. It is obtained from the market clearing condition (25) by adjusting supply and demand with the response to the price differential $(p_t - \bar{p})$. Notice how, through the use of the lag polynomial $\beta_p(L)$, we have allowed for possible long-term supply effects on the current market imbalance. Letting $y_{1t} = p_t - \bar{p}$, $y_{2t} = s_t - \bar{s}$ and using the definition of z_{1t}, we can write the market clearing condition as

$$y_{1t} = -\lambda(z_{1t} - y_{2t}) \tag{27}$$

where $\lambda = (\beta_p(L) - \varphi_p)^{-1}$. Such expression will prove useful in the following derivations.

Proposition 2. Long term market fundamental
We define the long-term market fundamental as the gap between production at the reference price $\bar{p}$ and the trend in consumption $\hat{c}_t$[10]

$$z_{2t} = q_t - \beta_p(L)(p_t - \bar{p}) - \hat{c}_t. \tag{28}$$

The possibility of defining a steady-state equilibrium price relies eventually on the stationarity of the price stochastic process, so that shocks are ultimately transitory. On the other hand, permanent shifts in supply or demand due to technical change or sustained investment may well determine structural breaks in the series, thus changing the long-run equilibrium price. Therefore, the price to which reference is made in calculating market imbalances has to be updated according to the structural evolution of the system.[11]

Proposition 3. Dynamics of short term fundamental
Substituting from equations (22)-(25) into (26), set at time $t+1$, the evolution of the short-term market fundamental is derived as

$$z_{1,t+1} = \xi_{1t} + \theta_{12}z_{2t} + \theta_{13}x_{1t} + \theta_{14}x_{2,t-1} + \phi_{12}y_{2t} + \varphi_{11}y_{1,t-1} + \epsilon_{1,t+1} \tag{29}$$

where

$$\begin{aligned} \xi_{1t} &= \beta_p(L)\bar{p} - \varphi_p\bar{p} + \mu^q_{t+1} - \mu^c_{t+1} - \varphi_x\gamma_{10} + \beta_x\gamma_{20} + \beta_q\hat{c}_t \\ \epsilon_{1,t+1} &= u_{2,t+1} - u_{1,t+1} + \beta_x w_{2,t+1} - \varphi_x w_{1,t+1} \\ \theta_{12} &= \beta_q, \quad \theta_{13} = -\varphi_x, \quad \theta_{14} = \beta_x, \quad \varphi_{11} = \beta_q\beta_p(L), \quad \phi_{12} = 1. \end{aligned}$$

[10]In the empirical application of next chapter we shall use the fitted consumption values as a proxy for consumption trend.
[11]Such considerations suggest using a moving average of appropriate window size in computing the reference price, as it has been done in the empirical application of next chapter.

Proposition 4. Dynamics of long term fundamental
The evolution of the long-term market fundamental is obtained in turn by substituting from equations (22)-(25) into (28) at time $t+1$

$$z_{2,t+1} = \xi_{2t} + \theta_{22} z_{2t} + \theta_{24} x_{2,t-1} + \varphi_{21} y_{1,t-1} + \epsilon_{2,t+1} \tag{30}$$

where

$$\begin{aligned} \xi_{2t} &= \mu^{q}_{t+1} + \beta_p(L)\bar{p} + \beta_x \gamma_{20} + \beta_q \hat{c}_t - \hat{c}_{t+1} \\ \epsilon_{2,t+1} &= u_{2,t+1} + \beta_x w_{2,t+1} \\ \theta_{22} &= \beta_q, \quad \theta_{24} = \beta_x, \quad \varphi_{21} = \beta_q \beta_p(L). \end{aligned}$$

Stacking $z_{1,t+1}$ and $z_{2,t+1}$ along with the stochastic processes of the exogenous variables we get the dynamic system

$$\begin{pmatrix} z_{1,t+1} \\ z_{2,t+1} \\ x_{1,t+1} \\ x_{2t} \\ r_{t+1} \end{pmatrix} = \begin{pmatrix} \xi_{1t} \\ \xi_{2t} \\ \gamma_{10} \\ \gamma_{20} \\ \gamma_{30} \end{pmatrix} + \begin{pmatrix} 0 & \phi_{12} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} . \begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} + \begin{pmatrix} \varphi_{11} & 0 \\ \varphi_{21} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} . \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \end{pmatrix} +$$
$$\begin{pmatrix} 0 & \theta_{12} & \theta_{13} & \theta_{14} & 0 \\ 0 & \theta_{22} & 0 & \theta_{24} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \rho_3 \end{pmatrix} . \begin{pmatrix} z_{1t} \\ z_{2t} \\ x_{1t} \\ x_{2,t-1} \\ r_t \end{pmatrix} + \begin{pmatrix} \epsilon_{1,t+1} \\ \epsilon_{2,t+1} \\ w_{1,t+1} \\ w_{2t} \\ w_{3,t+1} \end{pmatrix} \tag{31}$$

which can be written in matrix form as

$$\mathbf{z}_{t+1} = \boldsymbol{\xi}_t + \Phi_0 \mathbf{y}_t + \Phi_1 \mathbf{y}_{t-1} + \Theta \mathbf{z}_t + \boldsymbol{\epsilon}_{t+1} \tag{32}$$

where $\mathbf{z}_t = (z_{1t}, z_{2t}, x_{1t}, x_{2,t-1}, r_t)'$ is the (5×1) vector of the new state variables, $\mathbf{y}_t = (y_{1t}, y_{2t})'$ is the (2×1) vector of price and stocks expressed in terms of deviations from the respective reference values, $\mathbf{y}_{t-1} = (y_{1,t-1}, y_{2,t-1})'$ is the (2×1) vector of the lagged endogenous, $\boldsymbol{\xi}_t$ is a (5×1) vector of intercepts, $\boldsymbol{\epsilon}_{t+1}$ a (5×1) vector of disturbances, while Θ (5×5), Φ_0 (5×2) and Φ_1 (5×2) are matrices of intermediate parameters.

The explicit inclusion in the above system of the one-lagged vector of the endogenous variables is functional to the following solution of a short-run model, where $\beta_p(L)$ is restricted to β_{1p}. This choice is motivated by the dynamic properties of the statistical series, as investigated in the next chapter.

3.2 The model solution

The linearity of the system allows to solve it with respect to the vector of the endogenous variables $\mathbf{y}_t$ to obtain

$$\mathbf{y}_t = \mathbf{a}_0 + A_0 \mathbf{z}_t + A_1 \mathbf{y}_{t-1} + \mathbf{v}_t \tag{33}$$

that is the solved price and stock equations

$$p_t = a_{10} + a_{11}z_{1t} + a_{12}z_{2t} + a_{13}x_{1t} + a_{14}x_{2,t-1} + a_{15}r_t + a_{16}p_{t-1} + v_{1t} \tag{34}$$

$$s_t = a_{20} + a_{21}z_{1t} + a_{22}z_{2t} + a_{23}x_{1t} + a_{24}x_{2,t-1} + a_{25}r_t + a_{26}s_{t-1} + v_{2t}. \tag{35}$$

In order to derive an explicit representation for the matrices A_0 and A_1, first substitute (32) into $\mathbf{y}_{t+1}$ and take the expectation to get

$$E\mathbf{y}_{t+1} = \mathbf{a}_0 + A_0\boldsymbol{\xi}_t + A_0\Theta\mathbf{z}_t + A_0\Phi_0\mathbf{y}_t + (A_0\Phi_1 + A_1)\mathbf{y}_{t-1}. \tag{36}$$

Then, write equations (27) and (23) in system form as follows

$$\begin{pmatrix} 1 & -\lambda \\ \eta_e & 1-\eta_s L \end{pmatrix}(\mathbf{y}_t - \mathbf{a}_0) = \begin{pmatrix} 0 & 0 \\ \eta_e & 0 \end{pmatrix} E\mathbf{y}_{t+1} + \begin{pmatrix} -\lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\eta_e \end{pmatrix}(\mathbf{z}_t - \boldsymbol{\zeta}) \tag{37}$$

which in matrix form is

$$H\mathbf{y}_t = JE\mathbf{y}_{t+1} + K\mathbf{z}_t + k \tag{38}$$

where $k = H\mathbf{a}_0 - K\boldsymbol{\zeta}$. Substituting (36) into the above modified structural system and factoring similar terms we get

$$(H - JA_0\Phi_0)\mathbf{y}_t = (K + JA_0\Theta)\mathbf{z}_t + J(A_0\Phi_1 + A_1)\mathbf{y}_{t-1} + \tau_0 \tag{39}$$

where $\tau_0 = k + J(\mathbf{a}_0 + A_0\boldsymbol{\xi}_t)$. It follows that an implicit expression for matrices A_0 and A_1 can be obtained from

$$\mathbf{y}_t = \underbrace{W(K + JA_0\Theta)}_{A_0}\mathbf{z}_t + \underbrace{WJ(A_0\Phi_1 + A_1)}_{A_1}\mathbf{y}_{t-1} + W\tau_0. \tag{40}$$

where $W = (H - JA_0\Phi_0)^{-1}$. The coefficients of the response matrix A_0 thus obtained are a combination of structural and reduced form parameters, and by manipulating it a set of restrictions stemming from the rational expectations hypothesis can be derived, as it will be shown in the next section.

Before proceding further, the explicit representation of the A_0 matrix is provided by following the steps just illustrated. Therefore, from equation (34) we compute the rational expectations of Ep_{t+1} as

$$\begin{aligned} Ep_{t+1} = {} & a_{10} + a_{16}p_t + a_{13}\gamma_{10} + a_{14}\gamma_{20} + a_{15}\gamma_{30} + z_{2t}\left(a_{11}\theta_{12} + a_{12}\theta_{22}\right) + \\ & x_{1t}\left(a_{13} + a_{11}\theta_{13}\right) + x_{2,t-1}\left(a_{14} + a_{11}\theta_{14} + a_{12}\theta_{24}\right) + a_{11}\xi_{1t} + a_{12}\xi_{2t} + \\ & a_{15}\rho_3 r_t + \left(s_t - \bar{s}\right)a_{11}\phi_{12} + \left(p_{t-1} - \bar{p}\right)\left(a_{11}\varphi_{11} + a_{12}\varphi_{21}\right) \end{aligned} \tag{41}$$

and substitute it into the modified system (37) to derive the reduced form in price and stocks (40), whose slope coefficients form the A_0 and A_1 matrices. The elements of the second column of A_0 are for instance given by

$$a_{12} = \frac{\lambda\left(-a_{11}\eta_e\theta_{12} - a_{12}\eta_e\theta_{22}\right)}{\Delta}, \quad a_{22} = -\frac{a_{11}\eta_e\theta_{12} + a_{12}\eta_e\theta_{22}}{\Delta}$$

where $\Delta = \eta_e\left(a_{16}\lambda - \lambda + a_{11}\phi_{12}\right) - 1$.

3.3 The REH restrictions

One peculiar feature of rational expectations models is that they yield a set of non-linear cross equations restrictions linking structural and reduced form parameters.

As discussed in Gilbert (1995), a first set of restrictions verify the logical consistency of the estimated price and stock equations with the first stage production and consumption estimates. They allow to infer the coefficients of the price equation from those of the stock equations and viceversa.

Proposition 5. Logical restrictions
Given the matrix A_0 of the reduced form slope coefficients and the parameter λ, which is a function of the price response coefficients in the structural supply and demand equations, the logical restrictions are given by

$$a_{21} = 1 + \frac{1}{\lambda}a_{11}, \quad a_{2j} = \frac{1}{\lambda}a_{1j}, \quad j = 2, 3, 4, 5. \tag{42}$$

Proof 5.
Dividing the second row elements of A_0 by the corresponding elements of the first row it is easy to verify that

$$\frac{a_{22}}{a_{12}} = \frac{-\eta_e(a_{11}\theta_{12} + a_{12}\theta_{22})}{-\lambda\eta_e(a_{11}\theta_{12} + a_{12}\theta_{22})} = \frac{1}{\lambda}$$
$$\frac{a_{23}}{a_{13}} = \frac{-\eta_e(a_{13} + a_{11}\theta_{13})}{-\lambda\eta_e(a_{13} + a_{11}\theta_{13})} = \frac{1}{\lambda}$$
$$\frac{a_{24}}{a_{14}} = \frac{-\eta_e(a_{14} + a_{11}\theta_{14} + a_{12}\theta_{24})}{-\lambda\eta_e(a_{14} + a_{11}\theta_{14} + a_{12}\theta_{24})} = \frac{1}{\lambda}$$
$$\frac{a_{25}}{a_{15}} = \frac{\eta_e(1 - a_{15}\rho_3)}{\lambda\eta_e(1 - a_{15}\rho_3)} = \frac{1}{\lambda}.$$

The first logical restriction is instead derived as follows

$$\frac{a_{21} - 1}{a_{11}} = \frac{\lambda\eta_e(a_{16} - 1) - 1}{\lambda(1 - a_{11}\eta_e\phi_{12})} = \frac{1}{\lambda}. \qquad \square$$

A second set of restrictions provide a check of the consistency of the estimated price equation (34) with the rational expectation of price in the stock demand equation (23).

Proposition 6. REH restrictions
Given the matrix A_0 of the reduced form slope coefficients, the parameter λ, the logical restrictions (42) and the matrices of intermediate parameters Φ_0 and Θ, the rational expectations restrictions are given by

$$\frac{a_{22}}{a_{21}} = \frac{(a_{16}a_{12} + \theta_{22}a_{12} - a_{12} + a_{11}\theta_{12})}{a_{11}\,(a_{16} - 1)} \tag{43}$$

$$\frac{a_{23}}{a_{21}} = \frac{(a_{13}a_{16} + a_{11}\theta_{13})}{a_{11}(a_{16} - 1)} \tag{44}$$

$$\frac{a_{24}}{a_{21}} = \frac{(a_{14}a_{16} + a_{11}\theta_{14} + a_{12}\theta_{24})}{a_{11}(a_{16} - 1)} \tag{45}$$

$$\frac{a_{25}}{a_{21}} = \frac{(a_{16}a_{15} + \rho_3 a_{15} - a_{15} - 1)}{a_{11}(a_{16} - 1)} \tag{46}$$

Proof 6.
In what follows we illustrate the analytical derivation of the REH restriction (43). The other restrictions are analogously obtained. Substituting the rational expectation of price (41) into the stock demand equation (23), and equating the coefficient of z_{2t} with the corresponding coefficient in the reduced form stock equation (35), we get

$$a_{22} = -\frac{a_{11}\eta_e\theta_{12} + a_{12}\eta_e\theta_{22}}{\eta_e(a_{16}\lambda - \lambda + a_{11}\phi_{12}) - 1} \tag{47}$$

which still depends on the parameters λ and η_e.

An intermediate step of the proof thus consists in deriving an analytical expression for the free parameter η_e. This is achieved by repeating the first step above for the z_{1t} coefficients and combining the resulting expression with the first logical restriction as given below

$$a_{21} = \frac{\lambda(-a_{11}\eta_e\theta_{12} - a_{12}\eta_e\theta_{22})}{\eta_e(a_{16}\lambda - \lambda + a_{11}\phi_{12}) - 1}, \quad a_{21} = 1 + \frac{1}{\lambda}a_{11}.$$

After cancelling out λ and solving for η_e we get

$$\eta_e = \frac{a_{21}}{a_{11}(a_{16} + a_{21}\phi_{12} - 1)}. \tag{48}$$

We can now substitute for η_e into (47), use the logical restriction $a_{22} = \lambda^{-1}a_{12}$ for cancelling out λ, and solve for a_{22} to get upon simplifying

$$\frac{a_{22}}{a_{21}} = \frac{(a_{16}a_{12} + \theta_{22}a_{12} - a_{12} + a_{11}\theta_{12})}{a_{11}(a_{16} - 1)}. \qquad \square$$

It turns out that the structural parameter η_s can be recovered following the same procedure used to deriving η_e. Thus, equating coefficients of the variable s_{t-1} in the reduced form equations and using $a_{26} = \lambda^{-1}a_{16}$, it follows that

$$\eta_s = \frac{(a_{16} - 1)(a_{11}a_{26} - a_{16}a_{21})}{a_{11}(a_{16} + a_{21}\phi_{12} - 1)}. \tag{49}$$

By using estimates of the reduced form parameters, such restrictions can be tested through Wald tests. Rejection of the restrictions implies the rejection of the rational expectations provided the model and the expectations are well specified. In fact, the rejection may well be regarded as a rejection of the model conditional on the validity of the REH.

Furthermore, as shown in Appendix A, the logical and REH restrictions allow to recover the structure of the model from the estimated solved form and to investigate the qualitative effects of changes in the state variables on price and stocks, as illustrated in Appendix B.

4 The dynamic analysis of the system

In this section we provide a brief description of the conditions determining the dynamic stability of the system, using a short run model where further restrictions are imposed. In particular, the price lags in the supply equation are reduced to one, $\beta_p(L) = \beta_{1p}$, and $\eta_s = 0$. The equilibrium analysis of the model is indeed important as it may permit to verify whether the theoretical model is able to generate a dynamic path consistent with the observed realization of the stochastic process of the main endogenous variable of the system.

Solving equations (21)-(25) with respect to p_t, we get the stochastic price difference equation

$$p_t = -\frac{\eta_e}{\eta_e - \varphi_p} p_t^e + \frac{\eta_e}{\eta_e - \varphi_p} p_{t+1}^e + \frac{(\eta_e - \beta_{1p})}{\eta_e - \varphi_p} p_{t-1} + X_t \tag{50}$$

where X_t includes the exogenous variables and all disturbances. For illustration purposes, we confine our analysis to a deterministic setting, which in a rational expectations framework is equivalent to perfect foresight, i.e. $p_{t+1}^e = p_{t+1}$. Thus, substituting the expectational variables with observed realizations we get the second order non-homogeneous difference equation

$$p_{t+1} - \phi_1 p_t - \phi_2 p_{t-1} = Z_t \tag{51}$$

where

$$\phi_1 = \frac{2\eta_e - \varphi_p}{\eta_e}, \quad \phi_2 = \frac{\beta_{1p} - \eta_e}{\eta_e}, \quad Z_t = \frac{\mu_t + \omega_t}{\eta_e}$$

and

$$\begin{aligned} \mu_t &= -\mu_t^c + \mu_t^q - \Delta\mu_t^s \\ \omega_t &= -u_t^c + u_t^q - \Delta u_t^s + \beta_x x_{2,t-1} + \eta_e \Delta r_t - \varphi_x x_{1t}. \end{aligned}$$

By the superposition principle the general solution p_t of a non-homogeneous difference equation can be written as the sum of the general solution to the homogeneous equation $p_t^{(g)}$ and any particular solution to the non-homogeneous equation $p_t^{(p)}$.

A particular solution of interest, when is well defined ($1 - \phi_1 - \phi_2 \neq 0$), is the steady state equilibrium, where all stochastic variables have been set to their long-term values $\bar{Z}$

$$\bar{p} = -\frac{\eta_e \bar{Z}}{\beta_{1p} - \varphi_p}. \tag{52}$$

It can be shown (Neusser, 2009) that this equilibrium is asymptotically stable (i.e. all solutions converge to $\bar{p}$) if and only if the following conditions are satisfied

$$\begin{array}{rlcr} (i) & 1 - \phi_1 - \phi_2 > 0 & & \varphi_p - \beta_{1p} > 0 \\ (ii) & 1 + \phi_1 - \phi_2 > 0 & \text{or} & -\beta_{1\text{p}} + 4\eta_e - \varphi_p > 0 \\ (iii) & 1 + \phi_2 > 0 & & \beta_{1p} > 0. \end{array} \tag{53}$$

In general, the dynamic stability of the system depends on the roots of the characteristic equation

$$\lambda^2 - \phi_1 \lambda - \phi_2 = 0 \tag{54}$$

which in terms of the original parameters are given by

$$\lambda_{1,2} = \frac{2\eta_e - \varphi_p \pm \sqrt{\varphi_p^2 - 4\eta_e\varphi_p + 4\beta_{1p}\eta_e}}{2\eta_e} \tag{55}$$

whose nature is determined by the sign of the discriminant $\Delta = \varphi_p^2 - 4\eta_e\varphi_p + 4\beta_{1p}\eta_e$.

It turns out that the signs of the structural parameters predicted by the theory ($\beta_{1p} > 0$, $\varphi_p < 0$, $\eta_e > 0$) are such that $\Delta > 0$, therefore we are given two real and distinct roots. In this case, the general solution to the second order non-homogeneous difference equation is given by

$$p_t^{(g)} = c_1\lambda_1^t + c_2\lambda_2^t + p_t^{(p)} \tag{56}$$

which can be written as

$$p_t^{(g)} = \lambda_2^t \left[c_2 + c_1 \left(\frac{\lambda_1}{\lambda_2} \right)^t \right] + p_t^{(p)}. \tag{57}$$

If without loss of generality we assume that $|\lambda_2| > |\lambda_1|$, then the asymptotic behaviour of the solution depends on the value of the larger root λ_2, since $(\lambda_1/\lambda_2)^t \to 0$ as $t \to \infty$. Then, six possible cases emerge depending on the value of λ_2:

1. $\lambda_2 > 1$, $c_2\lambda_2^t$ diverges (the system is unstable);
2. $\lambda_2 = 1$, $c_2\lambda_2^t$ remains constant at c_2;
3. $0 < \lambda_2 < 1$, $c_2\lambda_2^t$ decreases monotonically to zero (the system is stable);
4. $-1 < \lambda_2 < 0$, $c_2\lambda_2^t$ oscillates around zero, alternating in sign, but converges;
5. $\lambda_2 = -1$, $c_2\lambda_2^t$ alternates between the values c_2 and $-c_2$;
6. $\lambda_2 < -1$, $c_2\lambda_2^t$ alternates in sign but diverges.

At this regard, it is immediate to verify that the stability condition (i) is not satisfied and therefore the system is unstable as can be seen also from the values of λ_2 reported in Table 1, computed for plausible ranges of values of the structural parameters, namely $\varphi_p \in [.05, .4]$, $\beta_{1p} \in [.05, .4]$, and $\eta_e \in [.3, 2.5]$.

Since we have postulated the rational expectations hypothesis, agents are assumed to be forward looking and incorporate future developments of shocks ω_t into their decisions. Therefore, following Neusser (2009), we conjecture that a sensible particular solution can be given by

$$p_t^{(p)} = \sum_{j=-\infty}^{\infty} \psi_j \omega_{t-j}. \tag{58}$$

We proceed using the method of undetermined coefficients[12]and thus substituting (58) into (51)

$$\sum_{j=-\infty}^{\infty} \psi_j \omega_{t+1-j} = \phi_1 \sum_{j=-\infty}^{\infty} \psi_j \omega_{t-j} + \phi_2 \sum_{j=-\infty}^{\infty} \psi_j \omega_{t-1-j} + Z_t. \tag{59}$$

and equating terms we get

$$\begin{aligned} &\dots \\ \psi_0 &= \phi_1 \psi_{-1} + \phi_2 \psi_{-2} \\ \psi_1 &= \phi_1 \psi_0 + \phi_2 \psi_{-1} + \frac{1}{\eta_e} \\ \psi_2 &= \phi_1 \psi_1 + \phi_2 \psi_0 \\ &\dots \end{aligned}$$

The coefficients ψ_j's follow second order homogeneous difference equations

$$\begin{aligned} \psi_j &= \phi_1 \psi_{j-1} + \phi_2 \psi_{j-2} \quad j \geq 1 \\ \psi_{-j} &= \phi_1 \psi_{-j-1} + \phi_2 \psi_{-j-2} \quad j \geq 1 \end{aligned}$$

whose solutions are given by

$$\begin{aligned} \psi_j &= d_1 \lambda_1^j + d_2 \lambda_2^j \\ \psi_{-j} &= e_1 \lambda_1^j + e_2 \lambda_2^j. \end{aligned}$$

Table 1 shows that, for plausible values of the structural parameters, the roots are on either side of the unit circle. Therefore, if we look for a stationary solution, we have to rule out the explosive part by setting $d_2 = 0$ and $e_2 = 0$. Then, noting that the two solutions coincide for $j = 0$, which implies $d_1 = e_1$, we calculate the common constant d by exploiting the fact that both solutions

[12]An extensive review of different solution methods to linear rational expectations models is given in Pesaran (1987).

must satisfy the initial value condition $\psi_1 = \phi_1\psi_0 + \phi_2\psi_{-1} + 1/\eta_e$. Substituting for ψ_1, ψ_0 and ψ_{-1}, leads to

$$d\lambda = \phi_1 d + \phi_2 d\lambda + \frac{1}{\eta_e} \quad \Rightarrow \quad d = \frac{\eta_e^{-1}}{\lambda(1-\phi_2) - \phi_1} \tag{60}$$

where $\lambda \equiv \lambda_1$ and, as a consequence, $\lambda_2 = -\phi_2\lambda^{-1}$. The general solution to non-homogeneous difference equation (51) is thus given by

$$p_t = c_1\lambda^t - c_2\phi_2^t\lambda^{-t} + d\sum_{j=-\infty}^{\infty}\psi_j\omega_{t-j} \tag{61}$$

whereas in order to impose a boundedness condition we must set $c_2 = 0$ and assume that the infinite sum converges as $t \to \infty$. In what follows, we simulate the response of price and stocks to shocks using some combinations of the structural parameters reported in Table 1.

Table 1: Simulations

	φ_p	η_e	β_{1p}	Δ	λ_1	λ_2
S1	-0.05	0.3	0.05	0.1225	1.6667	0.5000
S2	-0.05	0.3	0.4	0.5425	2.3109	-0.1442
S3	-0.05	2.5	0.05	1.0025	1.2102	0.8098
S4	-0.05	2.5	0.4	4.5025	1.4344	0.5856
S5	-0.4	0.3	0.05	0.7000	3.0611	0.2722
S6	-0.4	0.3	0.4	1.1200	3.4305	-0.0972
S7	-0.4	2.5	0.05	4.6600	1.5117	0.6483
S8	-0.4	2.5	0.4	8.1600	1.6513	0.5087
S9	-0.21	2.35	0.06	2.5821	1.3866	0.7028

For simplicity, we drop the interest rate in the stockholding equation and express price exclusively as a function of past and expected future shocks by setting also $c_1 = 0$.[13] The stochastic term ω_{t-j} in equation (61) is a composite disturbance as it includes shocks arising from various parts in the system, including exchange rate, interest rate and demand shocks, which can have offsetting effects. Though, given the relevance of supply shocks in generating price cycles in agricultural markets we may focus on u_{t-j}^q.

Assume that a unitary unexpected transitory supply shock hits the market at $t = 0$. As can be seen in Figure 1 (bottom part), in all scenarios the price immediately falls, while inventories (upper part) rise as prices are expected to increase in the future due to the transitory nature of the shock. Then price smoothly goes up to the previous level as stocks are gradually run down. The different simulations use values of the structural parameters typical of perennial

[13]The constant c_1 could have been otherwise initialized at p_0 by solving an initial value condition.

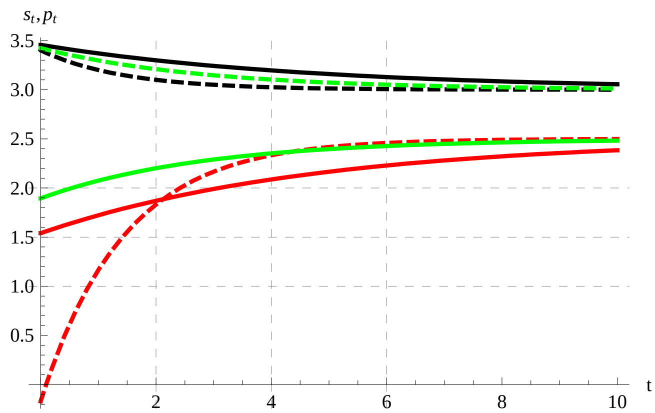

Figure 1: Unanticipated supply shock at t=0

crop markets, characterized by low short-term supply and demand elasticities. Though, we observe how in scenario S3 (solid red/black lines), characterized by a high value of the parameter η_e, the price fall is less pronounced than in scenario S1 (dashed red/black line), where the stock elasticity is low. A similar smooth pattern is observed also in scenario S9 (solid/dashed green line), which differs from S3 practically only for a higher demand elasticity. We may argue that a greater responsiveness of stocks to expected price gains seems to have in this model a stabilizing effect on price, a relatively larger value of the small root increasing the persistency of both price and stocks series.

Suppose now that a positive supply shock is expected to hit the market in period 0. As shown in Figure 2, at time $t-5$, when the shock is announced, market participants expect the price to fall in the future and therefore want to get rid of their stocks trying to selling them already at this time. As a result, the price starts to fall before the shock actually occurs. When finally the positive shock takes place in period 0, the excess supply sharply increases stocks, depressing price at its lowest level. Inventories are then decumulated to their initial levels, but only gradually, as agents expect the price to move up again. Beginning from period 0, the market adjusts like in the previous case as shocks are again assumed to be transitory. Similar considerations apply here concerning the different simulation scenarios. The price decrease at time 0 in scenario S1 is much more pronounced than in the other cases and the low root implies a relatively higher discounting of future events, so that adjustments occur more rapidly.

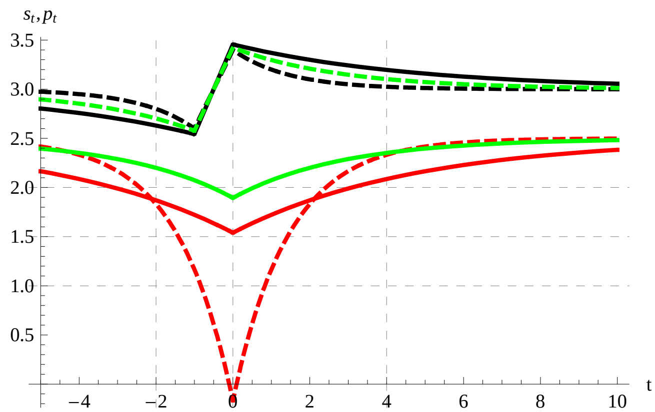

Figure 2: Anticipated supply shock at t=0

5 Concluding remarks

In this chapter we specified a model for the global market of a perennial crop, namely cocoa, under linear rational expectations. One feature common to primary commodity markets is the lagged supply response by producers to market signals. In fact, domestic policies, imperfect price transmission along the supply chain and substantial adjustment costs under uncertain market conditions may render supply quite unresponsive in the short run to world market prices. This is most true for perennial crops where the biological gestation lag further accentuates such characteristics.

As a result, given the low demand elasticity, stocks fluctuate widely in response to supply shocks and a fairly stable relationship between stock levels and world price can be observed in the market. Nevertheless, stocks may fail to absorbe market imbalances and the use of commodities as a financial asset in a balanced portfolio may also determine temporary departures of price from its equilibrium time path, exacerbating price spikes and producing bubble effects with possibly destabilizing long-run effects on the market.

Such stylized facts led us on the one hand to pay attention to the derivation of the supply equation and on the other hand to focus on the role of stocks, by including a speculative stockholding equation in the structural model and deriving a dual solved or reduced form in price and stocks. The solution method follows Gilbert (1995) in reducing the state variables to a reduced set including, along with world income, interest rate and exchange rate, two constructed variables representing short and long run market imbalances, whose inclusion appears theoretically appealing for the reasons explained above.

The consideration, anticipated in Ghosh et al. (1987) and stressed in Gilbert (1995), that the implications of REH may be limited for long term market developments because agents do not possess relevant information, motivated us to extend the definition of market fundamental accounting for past supply shocks, given that the forecasting horizon for perennial crops should be longer than for other commodities.

The rational expectations hypothesis generates as usual a set of nonlinear cross equation restrictions linking structural and reduced form parameters, which provide indirect tests of the REH and in general of model specification. Such restrictions will be tested in the next chapter using estimates from a stripped-down version of the model.

Furthermore, we investigated, in a deterministic setting, the stability conditions of a short-run version of the model, deriving the price rational expectations solution. We noticed how the model generates one explosive and one stable root, as usual in a rational expectations framework. Therefore, in order to obtain a bounded time path for the price solution, the explosive root must be ruled out and stationarity conditions imposed. We further simulated the impact of a positive supply shock on price and stocks under different scenarios given by alternative combinations of values for the structural parameters. Results suggest that low values of supply and stock demand elasticities, generating in turn relatively higher values of the stable root, tend to increase the persistency of the series. A greater sensitivity of stock demand to expected capital gains seems to have a stabilizing effect on price, deflating the explosive root and making adjustments to long-term equilibria more gradual. Conversely, a high demand elasticity would tend to amplify market imbalances.

Appendices

A Recovery of structural parameters

The structure of the model can be recovered using the logical and REH restrictions which link the structural and reduced form parameters. From the definitions of the intermediate parameters in equations (29) and (30) we note that $\theta_{12} = \theta_{22} = \beta_q$. Thus, solving (43) with respect to β_q we get

$$\beta_q = -\frac{(a_{16} - 1)(a_{12}a_{21} - a_{11}a_{22})}{(a_{11} + a_{12})\, a_{21}}. \tag{A.1}$$

Similarly, noting that $\theta_{13} = -\varphi_x$ and $\theta_{14} = \theta_{24} = \beta_x$, we can solve (44) and (45) for β_x and φ_x, respectively, to obtain

$$\beta_x = \frac{-a_{13}a_{16}a_{21} - a_{11}a_{23} + a_{11}a_{16}a_{23}}{a_{11}a_{21}} \tag{A.2}$$

$$\varphi_x = \frac{-a_{14}a_{16}a_{21} - a_{11}a_{24} + a_{11}a_{16}a_{24}}{(a_{11} + a_{12})\, a_{21}}. \tag{A.3}$$

In order to recover the price supply elasticity β_p, since $\varphi_{11} = \beta_q\beta_p$, we first derive the intermediate parameter φ_{11}. To this end, we equate the reduced form coefficients of p_{t-1} in (34) and (40)

$$a_{16} = \frac{\lambda(-a_{11}\eta_e\varphi_{11} - a_{12}\eta_e\varphi_{11})}{(a_{16}\lambda - \lambda + a_{11})\,\eta_e - 1}$$

and substituting $a_{12} = \lambda a_{22}$ and (48) into the above expression to eliminate λ and η_e we get

$$\varphi_{11} = -\frac{a_{16}(-a_{12}a_{21} + a_{12}a_{16}a_{21} + a_{11}a_{22} - a_{11}a_{16}a_{22})}{(a_{11} + a_{12})\, a_{12}a_{21}}. \tag{A.4}$$

We can now substitute (A.4) and (A.1) into $\varphi_{11} = \beta_q\beta_p$ to get, upon simplifying,

$$\beta_p = \frac{a_{16}}{a_{12}}. \tag{A.5}$$

Finally, in order to derive the price demand elasticity φ_p, we subtitute for λ and β_p into $\lambda^{-1} = \beta_p - \varphi_p$ to obtain

$$\varphi_p = \frac{a_{16} + a_{22}}{a_{12}}. \tag{A.6}$$

B Qualitative analysis

This appendix investigates the qualitative effects of changes in the state variables on the dependent variables of the model, by inferring the signs of the A_0 matrix coefficients of the solved form in price and stocks from previous knowledge of the signs of the structural parameters based on economic theory.

The adopted solution method, based on market fundamentals, prevent us from carrying out a traditional comparative statics analysis by differentiating the solved form (40), as the resulting terms are a combination of reduced form and structural parameters. Nevertheless, from plausible conjectures about the values of the structural parameters and the reduced form lagged endogenous coefficients, based on statistical grounds, we can tentatively predict the solved form signs using the logical and REH restrictions and the expressions derived in Appendix A. In what follows, we illustrate through propositions the procedure used with reference to market fundamentals as the effects of changes in the other state variables, reported in Table 2, are analogously obtained.

The first step consists in determining the signs of the structural parameters from the causal effects suggested by economic theory in the structural relationships.

Assumption 1. Structural parameters signs
The expected signs of the structural parameters in system (21)-(25) are the following: $\varphi_p < 0$, $\varphi_x > 0$, $\beta_p > 0$, $\beta_x > 0$, $\beta_q > 0$, $\eta_e > 0$.

These signs are justified by observing that the effect on demand of own price φ_p and income φ_x are respectively negative and positive. As concerns supply, the own price elasticity β_p is expected to be positive, as such is the effect on production of a competitive devaluation of the exchange rate β_x. The positive sign of β_q is related to the observed autocorrelation in production, as a result of the slow changes in the tree stock due to high adjustment costs, a typical feature of perennial crops. Finally, the positive stock demand elasticity to expected capital gains, η_e, is postulated by the theory of speculative storage.

Since $\lambda^{-1} = \beta_p - \varphi_p$, from ASS.1 it follows that $\lambda > 0$. This in turn allows to make the following statement

Proposition B.1. Symmetry condition
Given ASS.1 and the logical restrictions (42), the signs of corresponding coefficients in the price and stock equations coincide

$$\text{sign}[a_{1j}] = \text{sign}[a_{2j}] \quad \text{for} \quad j = 2, 3, 4, 5.$$

Assumption 2. Stationarity condition
The effect of previous period price on current price in the solved form is such that $0 < a_{16} < 1$.

This assumption is based on the statistical analysis developed in the following chapter and on the desirable stationary property of the price series, which lead us to rule out unit and explosive roots.

Proposition B.2. Long term fundamental effect
Given ASS.1, ASS.2, PROP.B.1 and equation (A.5), the effect of the long-term market fundamental z_{2t} on price and stocks is negative.

Proof B.2
Using ASS.1 and ASS.2 we can sign the parameters in equation (A.5) as follows

$$\overbrace{\beta_p}^{-} = \frac{\overbrace{a_{16}}^{+}}{a_{12}} \tag{62}$$

from which it follows that $a_{12} < 0$ and, by PROP.B.1, $a_{22} < 0$.[14] □

Proposition B.3. Short term fundamental effect
Given ASS.1, ASS.2, PROP.B.1, PROP.B.2, equation (A.1) and the first logical restriction, the predicted effect of the short-term market fundamental z_{1t} is negative on price and positive on stocks.

Proof B.3
These results hinge on further mild assumptions about the values of the structural parameters. In fact, the latter result follows from the first logical restriction, $a_{21} = 1+\lambda^{-1}a_{11}$, by observing that, for plausible values of λ and a_{11}, $a_{21} > 0$ regardless of the sign of a_{11}. In order to sign a_{11}, consider equation (A.1)

$$\overbrace{\beta_q}^{+} = -\frac{\overbrace{(a_{16}-1)}^{-}(\overbrace{a_{12}a_{21}}^{-} - a_{11}\overbrace{a_{22}}^{-})}{(a_{11} + \underbrace{a_{12}}_{-})\underbrace{a_{21}}_{+}}$$

where from PROP.B.1 we know that $a_{12} < 0$ and $a_{22} < 0$. We can note that the signs of the left and right-hand side members are coherent, without imposing any further assumptions on the values of the reduced form coefficients, only if $a_{11} < 0$. The same result holds also if we check the consistency of the signs in equation (44), assuming $a_{23} > 0$ and $a_{13} > 0$. □

Given these signs, it is easy to verify, using the REH restrictions (44) and (45), the positive effect of income x_{1t} and exchange rate $x_{2,t-1}$ on both price and stocks. The predicted negative effect of r_t in both equations, through (46), is instead a likely outcome for plausible values of ρ_3 and a_{15}. Finally, we observe how the consistency of the signs in (48) implies the further restriction on the reduced form parameters $a_{16} + a_{12} < 1$.

[14] This result in turn implies that, for (A.6) to be valid, $|a_{16}| > |a_{22}|$.

Table 2: Comparative statics

	z_{1t}	z_{2t}	x_{1t}	$x_{2,t-1}$	r_t
price	↓	↓	↑	↑	↓
stocks	↑	↓	↑	↑	↓

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On the estimation of the world cocoa market model

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1 Introduction

In this chapter we estimate a parsimonious short-run specification of the world cocoa market model developed in the previous chapter. The choice is motivated by the possible shortage of degrees of freedom implied by the use of annual data and by the opportunity of staying close, for the sake of comparability, to the specification adopted in the equilibrium analysis. In what follows we shall make reference to the results of the previous chapter, reporting here only the structural model, the reduced form and the set of restrictions derived before.

The goals of this chapter are the following: first, we aim to provide a consistent estimate of the solved form in price and stocks developed previously, making use of the short and long term market fundamentals, and test the restrictions stemming from the rational expectations hypothesis; second, we compare these estimates with those obtained from time series models, namely vector autoregressions (VAR), using the same dependent variables.

As to the first objective, the reduced form is estimated using the generalized method of moments (GMM) because of the endogeneity of the market fundamentals. Since we are estimating a short-run model, the REH restrictions provide a test of whether market participants (specifically stockholders) form their expectations rationally, or consistently with the postulated model, using an information set which is in fact limited in time, apparently neglecting events far in the past.[1] We then estimate another version of the model where a time dummy is included to account for a structural break detected investigating the statistical properties of the series.

As regards the second goal, one of the most important critics to the traditional approach to econometric modelling, as exemplified by the Cowles Commission works, which motivated the introduction and widespread adoption of VAR modelling within the London School of Economics methodology, concerned the insufficient attention paid to the statistical model, resulting in an inaccurate specification of the system dynamics and unjustified exclusion restrictions (Sims et al., 1990).

Conversely, the inability in traditional VAR models of accounting for contemporaneous correlations between variables, as suggested by economic theory, instead a typical feature of simultaneous equation models, was indeeed a limitation which led to the development of structural VAR modelling (Amisano and Giannini, 1997). In this respect, the traditional VAR approach can be seen as backward looking, as opposed to the use of all information available up to the present period in forming expectations about future variables, in turn affecting the present values, typical of rational expectations models.

Therefore, we present different VAR models, moving from simple a-theoretical to more structured specifications where exogenous variables from the solved form are included. A comparison between the GMM estimates of the reduced form and those from a restricted VAR model with a matching specification is then

[1] An alternative interpretation is that, for instance, the current or lagged price already incorporates past events having future impact on market developments, as we would expect in a rational expectations framework.

offered.

This exercise must be considered an attempt towards a comparison of full information versus reduced form models which make use of only price data or a very limited number of state variables (Wright and Williams, 1989; Deaton and Laroque, 1992).

As to the organization of this chapter, in Section 2 we provide a brief description of the world cocoa market. The dynamic characteristics of the series used in the model for a proper dynamic specification in Section 3. Section 4 illustrates the specification of the structural model, while Section 5 presents the GMM estimates of the reduced form, the results from different VAR models and a comparison between the GMM estimates and those from the restricted VAR with a matching specification. Section 6 reports the concluding remarks.

2 The world cocoa market

This section gives a brief description of the cocoa market, illustrating the major past events and some recent developments occurring along the supply chain.

Cocoa production is concentrated in countries located in the equatorial belt (within $10°N$ and 10°S of the equator) where the climate conditions are favourable for growing the cocoa tree (*Theobroma cacao*). The natural habitat is the lower storey of the evergreen rainforest which provides the necessary humidity, temperature (it should be not less than 18-21 degrees C. on average) and shade. Rainfall is the main climatic factor affecting yields and must be abundant and well distributed throughout the year (preferably between 1500mm and 2000mm of annual rainfall level).
The cocoa tree flowers in two cycles of six months, yielding two harvests per year. In most African countries the main harvest lasts from October to March, while the mid harvest (typically much less abundant) from May to August. It takes from three to five years of gestation before the plant starts bearing fruits, after that yields rapidly increase to reach a peak after 8 or 11 years, depending on the varieties. Then, yields remain constant until 20-25 years before steadily declining, though the tree is productive for about 40 years.

Western and Central Africa are the most important producing regions with four countries (Côte d'Ivoire, Ghana, Nigeria, Cameroon) providing about 64% of world production (crop year 2009/2010). Côte d'Ivoire alone accounts for 34% of total supply, while other major producers include Indonesia (17%), Brazil (4%), Ecuador (4%) and Malaysia which is though recently withdrawing from cocoa production.

Despite a remarkable increase of grindings at the origin (from 33.6% in 2001/02 to about 40% in 2009/10), mostly located in Côte d'Ivoire and Malaysia (together 47% of total origin grindings), cocoa processing continues to be mainly undertaken in importing countries, notably Europe (41%), the Netherlands (13%) being the world largest cocoa-processing country, and the United States (10%).

As concerns demand, measured in terms of apparent consumption, given by the sum of grindings plus net imports of cocoa products, either final or semi-processed, converted in beans equivalent, it is concentrated in developed countries, mainly Western Europe and North America. United States, Germany, France and United Kingdom are, in the order, the single largest consumer countries (average 1997/98-2005/06).

Figure 1 shows the pattern of world production, grindings and stocks over the last fifty years. After averaging at about 1.5 million tonnes until the mid-eighties, production went on a constant growth path, despite the prolonged descending trend in world prices which followed the price spikes in the late seventies, caused by the frosts in Brazil (see Figure 2). The sustained growth was partly due to the investments occurred worldwide following the earlier price increases, the arrival of Indonesia as a major producer and the continued support to domestic producer prices in major producing countries in spite of the falling world prices.[2]

Though, such pricing policies became soon no longer sustainable, as the attempts of raising world prices through the purchases of buffer stocks under several International Cocoa Agreements turned out to be ineffective, so that most state-owned marketing agencies went bankrupt. Under the pressure of international donors, a liberalization process started in most African countries leading to the progressive dismantlement of existing marketing boards or caisse systems[3].

Despite the increasing trend in consumption, the excess supply determined a structural break in stocks and the stocks-to-use ratio jumped to levels permanently above 0.4, as can be seen from Figure 2. The stocks-to-use ratio (SUR) is an overall indicator of world cocoa availability widely used in the industry, as there is a fairly steady relationship between world market prices and this ratio, as appears also in Figure 3. Apart from the shift in the intercept occuring in the late seventies, a negative relationship between the world price and the SUR indicator clearly emerges. This evidence motivates investigating further the empirical relationship between stocks and prices as we try to pursue in the next sections.

As to the industry structure, cocoa is a typical smallholder crop as almost 90% per cent of cocoa production worldwide comes from smallholdings below 5 hectares. At the end of the 1990s, the global number of cocoa producers was estimated at about 14 millions, about 75% of whom in Africa. The international trade of cocoa is instead dominated by a limited number of large multinationals. The most remarkable events in recent years have been the processes of vertical integration along the supply chain and horizontal concentration, with mergers of large multinationals. Major international traders have in fact started to ver-

[2]In most African countries, the whole cocoa domestic chain was heavily controlled by the government through state-owned marketing agencies. They either directly handled the physical delivery of the produce from the farm gate to the ports (marketing board system), or set all prices and margins along the supply chain, releasing export licences (caisse system).

[3]The first African country to liberalize its cocoa sector was Nigeria in 1986-1987, followed by Cameroon in stages, during 1989-1991 and 1995. In Côte dIvoire the disengagement of the State began in 1994 and was not complete until 1999, while Ghana is still not fully liberalized.

Figure 1: World cocoa production, grindings and stocks

tically integrate upstream, taking over local exporters, in order to secure supply and downstream, engaging in the first stages of cocoa processing. At present, some two-thirds of total grindings is done by the top ten firms, with three large multinationals (ADM, Cargill and Barry Callebaut) dominating the market. If the market power of such multinationals is balanced downstream by the strength of large chocolate manufacturers, the upstream integration has raised concerns of a possible abuse of buying power against a multitude of unorganized local producers.

3 The dynamic characteristics of the series

This section analyzes the statistical properties of the variables used in the model in order to identify the order of integration of the series for a correct dynamic specification. In a rational expectations framework, it is furthermore important to find a proper representation of the stochastic process of the exogenous variables, given the role they play as predictors of future dated variables.

As a preliminary operation, all price series are deflated using the US CPI, a convenient deflator used in Deaton and Laroque (2003). The same deflator is used to obtain the real income, computed as the consumption weighted GDP of the main consuming countries. Further details on definitions and data sources are given in the Appendix.

Table 1 reports the results of different unit root tests on the series transformed in logarithms over the sample period 1970-2010. The ADF test (Dickey and Fuller, 1979) and the PP test (Phillips and Perron, 1988) share the same null hypothesis of the presence of a unit root, but adopt different methods to

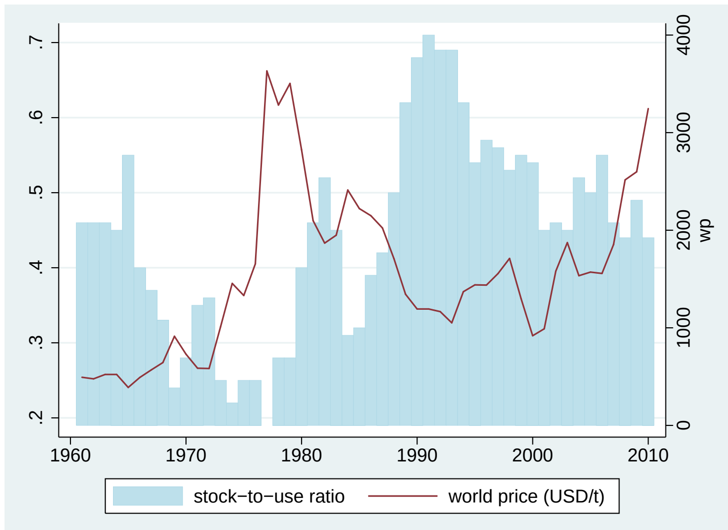

Figure 2: World cocoa price and stock-to-use ratio (1)

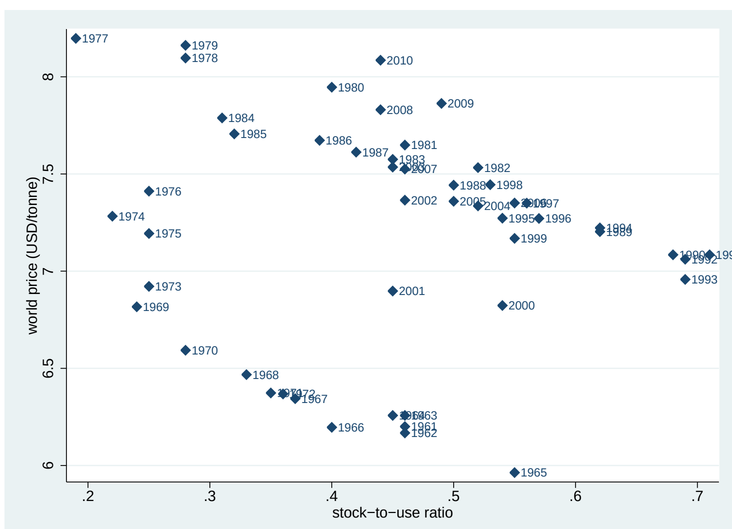

Figure 3: World cocoa price and stock-to-use ratio (2)

account for serial correlation. In the ADF test a sufficient number of lagged first differences is included to ensure that residuals are innovations[4], while in the PP test Newey-West consistent estimate correction for autocorrelation is implemented. Conversely, in the KPSS test (Kwiatkowski et al., 1992) the null is the trend (or level) stationarity of the series. Inference using this test is therefore complementary to that obtainable from the former tests.

Whereas the series exhibit clear upward trends, as in the case of production, consumption and real income, a linear time trend has been included in the regressions. The test statistics and the corresponding MacKinnon approximate p-values from both tests indicate that, at 5% significance level, the null of unit root is not rejected for all series except for income (ADF) and production (PP), for which the alternative of trend stationarity is accepted[5]. Coherently the null of trend (level) stationarity is rejected by the KPSS test for all series, included income and production actually. All series become stationary after first differencing (results are not reported), so they appear to be integrated of order one $I(1)$.

The theoretically appealing stationarity of the price series does not emerge from the tests, as suggested also by visual inspection of the real price series of cocoa, coffee, rubber and palm oil (Figure 4). Along with the well documented comovement of commodity prices, the picture shows the long descending trend in prices started in the 1980s and partially offset by the increases of the last decade[6]. In general, the combination of long price cycles, arising from the complex dynamics of perennial crops, with potentially multiple structural breaks in long annual time series, make it difficult to draw conclusive answers from unit root tests.

A problem of these tests is the lack of power in the presence of structural breaks in the series, so that the null hypothesis of unit root is overly accepted. Several tests have been deviced to address this problem, where the unit root hypothesis is tested allowing for a change in the mean or trend (or both) of the series (Perron, 1989; Perron and Vogelsang, 1992; Zivot and Andrews, 1992) and where the optimal breakpoint is endogenously determined. Table 1 presents results from two of such tests, the Zivot and Andrews (ZA) test, where the unit root hypothesis is tested allowing for a single endogenously determined structural break in the series, either in the intercept or trend (or both), and the Clemente et al. (1998) test (CMR), which extends the test of Perron and Vogelsang (1992) to the case of multiple structural breaks, more precisely a double shift in the mean in the additional outlier version (AO2). In Table 1 are reported the minimum t-statistics, the 5% critical value of the left-tailed test and the endogenous optimal break(s). The ZA test does not reject the null of unit root for all but the production series, and the same outcome hold for the CMR test, with the exclusion of the interest rate series. Moreover, both tests seem to

[4]The Pormanteau (Q) test has been applied to the ADF residuals; for almost all series a single lagged difference is sufficient to generate approximately white noise residuals.

[5]Though, at 10% significance level, the ADF test does reject the unit root for the exchange rate and interest rate series.

[6]Stationarity can be achieved by reducing the sample to the period 1986-2010.

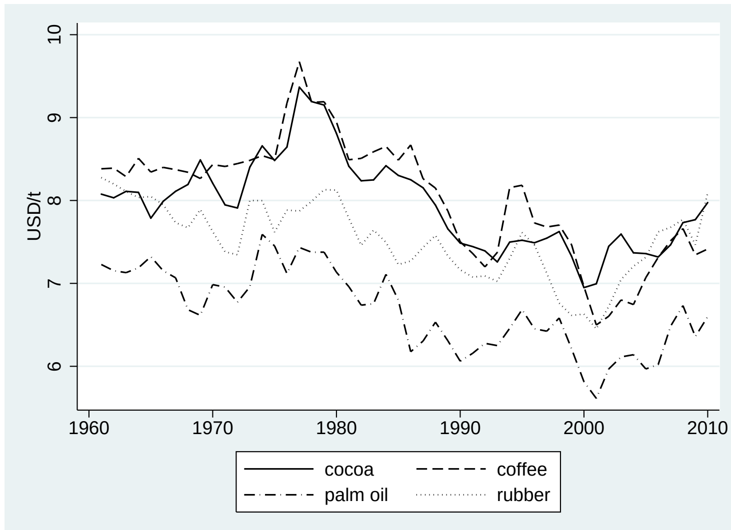

Figure 4: Real prices of selected perennial crops

suggest the existence of a structural break in both quantity and price series in the mid-eighties, possibly in 1986. We will try to exploit this fact using a time dummy in the following estimation.

4 The model specification

In this section we briefly review the structural model of the world cocoa market in the basic specification already presented in the previous chapter. As discussed, such specification proves convenient as the model can be quite easily solved in terms of the market fundamentals and, since the aim is to provide an acceptable representation of price and stocks behaviour at global level, it allows to focus on the main driving variables. Conversely, we are admittedly overlooking cross price effects and institutional features, likely relevant in explaining supply response, included asymmetric price transmission along the supply chain due to domestic policies or market power issues. That would probably require the specification of a more disaggregated model.

The model consists of three behavioural equations for supply, demand and stockholding and a market clearing identity. Consumption demand c_t, measured by total world grindings, is postulated as a function of real current cocoa price p_t and real income x_{1t}, given by the weighted GDP of major consuming countries

$$c_t = \mu_t^c + \varphi_p p_t + \varphi_x x_{1t} + u_t^c. \quad (1)$$

In the short run model supply is specified as a function of previous period production, the lagged cocoa price p_{t-1} (we rule out contemporaneous price

Table 1: Unit root tests

	q	c	s	x_1	x_2	r	p	p^{cf}	p^{oi}	p^{rb}
ADF	-3.131^b	-2.304^b	-1.593	-3.834^b	-1.413^a	-2.594	-1.82	-1.494	-1.913	-2.010
p-value	0.0991	0.4318	0.4871	0.0149	0.0829	0.0943	0.3704	0.5366	0.3261	0.2824
KPSS	0.178^b	0.198^b	1.11	0.211^b	1.03	0.495	0.705	0.888	0.957	0.712
5% c.v.	0.146	0.146	0.463	0.146	0.463	0.463	0.463	0.463	0.463	0.463
PP	-3.848^b	-2.332^b	-1.497	-2.635^b	-1.389	-1.462	-1.573	-1.316	-1.653	-1.919
p-value	0.0143	0.4163	0.5351	0.2639	0.5874	0.552	0.4974	0.622	0.4553	0.3234
ZA	-6.714	-4.2	-3.015^c	-4.76	-3.869^c	-3.804^c	-3.055	-3.586	-2.193	-4.183
5% c.v.	-5.08	-5.08	-4.8	-5.08	-4.8	-4.8	-5.08	-5.08	-5.08	-5.08
optimal break	1985	1974	1986	1979	1994	2001	1988	1987	2000	1998
CMR (AO2)	-1.722	-2.709	-3.523	-3.283	-5.046	-5.68	-4.776	-3.853	-2.469	-3.584
5% c.v.	-5.49	-5.49	-5.49	-5.49	-5.49	-5.49	-5.49	-5.49	-5.49	-5.49
optimal break	1986, 2001	1986, 1997	1983, 1989	1974, 1989	1982, 1995	1979, 1998	1975, 1986	1986, 2002	1975, 1982	1986, 2005

Notes: Sample 1970-2010. (a) a drift term is included; (b) a linear trend is included; (c) only intercept included; default is both intercept and trend.

response) and the lagged exchange rate of major producing countries $x_{2,t-1}$

$$q_t = \mu_t^q + \beta_q q_{t-1} + \beta_{1p} p_{t-1} + \beta_x x_{2,t-1} + u_t^q. \tag{2}$$

Stock demand linearly depends on the expected gain from carrying stocks into the next period, given by the difference between expected $p^e_{t+1|t}$ and current price, net of storage costs proxied by the interest rate r_t

$$s_t = \mu_t^s + \eta_s s_{t-1} + \eta_e (p^e_{t+1|t} - p_t - r_t) + u_t^s \tag{3}$$

where η_e is taken as a constant parameter.
Expectations are assumed to be rational, or model consistent, exploiting all information available until the present period Ω_t

$$p^e_{t+1|t} = E[p_{t+1}|\Omega_t]. \tag{4}$$

The model is closed by the market clearing condition.

$$q_t + s_{t-1} = c_t + s_t. \tag{5}$$

In a rational expectations framework, it is important to find a suitable representation of the stochastic process governing the exogenous variables dynamics as they represent the forcing variables of the system. As regards income x_{1t}, the ADF test suggests a trend stationary process, while the other tests possibly indicate a random walk specification with a drift term[7]. We decided for the latter option, so that we have

$$x_{1t} = \gamma_{10} + x_{1,t-1} + w_{1t}. \tag{6}$$

As to the exchange rate x_{2t}, the unit root hypothesis is accepted by all tests, though only weakly, even accounting for the structural break in 1994 corresponding to the devaluation of the Franc CFA. Therefore, we chose a random walk with drift specification given by

$$x_{2t} = \gamma_{20} + x_{2,t-1} + w_{2t}. \tag{7}$$

As concerns the interest rate r_t, the CMR test rejects the unit root while the ADF and KPSS only marginally fail to do it. Thus, we specify an ARMA(1,1) process with drift term

$$r_t = \gamma_{30} + \rho_3 r_{t-1} + w_{3t} \tag{8}$$

where the disturbance $w_{3t} = \upsilon\epsilon_{t-1} + \epsilon_t$ includes an autoregressive component. As we can see, a certain degreee of arbitrariness is present and hence room is left for further experimentation.

[7]A debate is still alive in macroeconomics as to whether GDP is better represented by a random walk or a trend stationary process.

5 Estimation methods and results

In the previous chapter we illustrated how the structural model (1)-(5) can be solved, under the rational expectations hypothesis, through the use of the short and long term market fundamentals, z_{1t} and z_{2t} respectively. The solved or reduced form in price and stocks (9)-(10) is what we shall estimate in this section, whereas the original set of explanatory variables is substituted by a new vector of state variables $(z_{1t}, z_{2t}, x_{1t}, x_{2,t-1}, r_t)$ plus the lagged endogenous

$$p_t = a_{10} + a_{11}z_{1t} + a_{12}z_{2t} + a_{13}x_{1t} + a_{14}x_{2,t-1} + a_{15}r_t + a_{16}p_{t-1} + v_{1t} \quad (9)$$

$$s_t = a_{20} + a_{21}z_{1t} + a_{22}z_{2t} + a_{23}x_{1t} + a_{24}x_{2,t-1} + a_{25}r_t + a_{26}s_{t-1} + v_{2t}. \quad (10)$$

The first step in the estimation process is the computation of the market fundamentals. We report the definition of the short run excess supply at the reference price which is given by

$$z_{1t} = q_t - \widehat{\beta}_{1p}L(p_t - \bar{p}_t) - c_t + \widehat{\varphi}_p(p_t - \bar{p}_t) + (s_{t-1} - \bar{s}_t). \quad (11)$$

One first empirical question regards the choice of the reference price $\bar{p}_t$ at which the market imbalance is computed. As discussed in the previous chapter, a moving average of appropriate length would seem appropriate. We experimented with different window sizes and eventually decided for a five years moving average, so that the relevant price is compared with the five preceding years when supply response is evaluated. The same rationale applies to the calculation of the reference stocks level $\bar{s}_t$.[8]

In order to proceed with the computation, we also need consistent estimates of the price coefficients in the structural supply and demand equations. As concerns consumption (1), we postulated that world grindings respond instantaneously to current price and income, and we need therefore to instrument price because of the endogeneity with the quantity demanded. The additional instruments we selected are the lagged weighted GDP of major producing countries[9] and the lagged fCFA/US$ exchange rate, all in logarithms. The overall fit of the two-step efficient GMM estimation of the consumption equation is good ($R^2 = 0.96$), the coefficients are significant at 5% level, display the expected signs and economically meaningful magnitudes. The price elasticity ($\widehat{\varphi}_p = -0.21$) is slightly higher than in previous works, while the income elasticity is not far from unity ($\widehat{\varphi}_x = 0.84$). Hansen J-statistic $\chi^2(1) = 0.269$ (p-val 0.6042) confirms the validity of the instruments as the orthogonality conditions are met.

Turning to the supply equation (2), in our short-run model a single (one period lagged) price coefficient $\widehat{\beta}_{1p}$ is estimated. The OLS estimates are quite satisfactory ($R^2 = 0.96$) as all coefficients show the expected signs, even though most are significant only at 10% level. The short-run price elasticity is low

[8]Notice how the use of a time varying reference value requires the addition of a time subscript in the notation.

[9]For the list of countries see the Appendix.

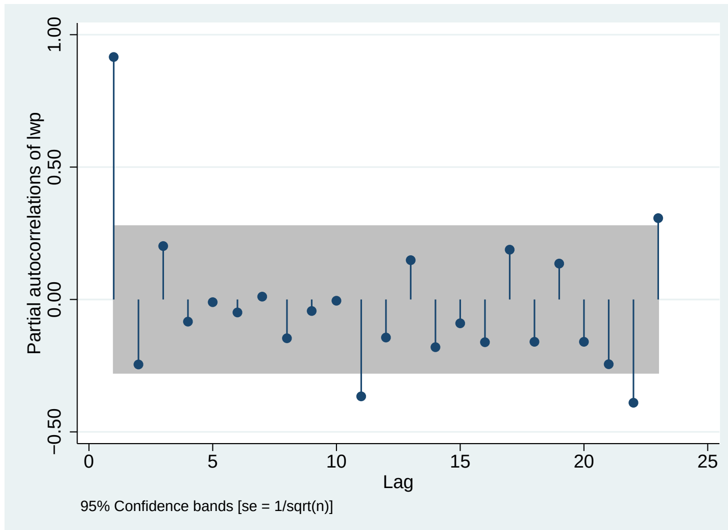

Figure 5: Partial autocorrelations of world cocoa price

($\widehat{\beta}_{1p} = 0.06$) and the effect of lagged production ($\widehat{\beta}_q = 0.82$) is positive and relevant, both expected results for a perennial crop, while the positive sign of the exchange rate coefficient ($\widehat{\beta}_x = 0.10$) is coherent with the effect on production of a depreciation of the local currency. Finally, the step-type time dummy variable D_{86} included in the regression to account for the structural break identified in the previous section is positive and significant ($\widehat{\mu}_t^q = 0.30$).

If we were to estimate a long-run version, in order to decide on the lag length, we know from the age yield profile of the cocoa tree that typically four to five years pass before the plant starts bearing fruits, then yields rapidly increase to reach a peak at about eight or eleven years for hybrid and traditional varieties, respectively. We have confirmation of this pattern by looking at the price partial autocorrelations plotted in Figure 5.[10]

The long-run market fundamental has been defined as the difference between production at the reference price and the consumption trend. We use as a proxy for consumption trend the fitted values from the estimated consumption equation $\widehat{c}_t$[11], so that we have

$$z_{2t} = q_t - \widehat{\beta}_{1p} L(p_t - \bar{p}_t) - \widehat{c}_t. \tag{12}$$

Figure 6 shows the pattern of the short and long term market fundamentals over the sample period 1070-2010.

[10] Moreover, from the estimation of a full distributed lag supply equation we got positive and quantitatively relevant price coefficients associated to the lags 5, 8 and 11. We eventually chose lag 8 which, while saving on degrees of freedom, turns out to be significant in the VAR estimation (as lag 11, actually).

[11] Other trend extraction techniques are available, from polynomial and ARMA models to other filters used in macroeconometrics (Hodrick and Prescott, 1997; Baxter and King, 1999).

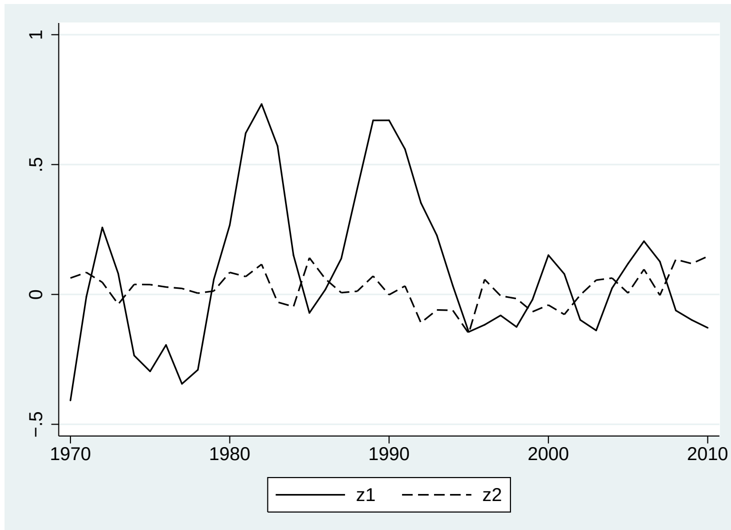

Figure 6: Market fundamentals

Furthermore, from the supply and demand equations we get all the necessary estimates to compute the intermediate parameters ϕ_{ij} and θ_{ij} (the elements in the first two rows of the matrices Φ_0 and Θ of the previous chapter) appearing in the evolution of the market fundamentals, $z_{1,t+1}$ and $z_{2,t+1}$, respectively, and in the REH restrictions.[12]

5.1 The reduced form estimates

5.1.1 GMM estimation

Once computed the market fundamentals, we can now turn to the estimation of the solved price and stock equations. Since z_{1t} and z_{2t} are, by construction, endogenous regressors, we have to use instrumental variable methods. This fact is, as expected, confirmed by Wu-Hausman endogeneity tests applied to both equations.

As it has been shown before (see Figure 3), a contemporaneous (negative) relation exists between price and stocks. Therefore, it seems appropriate to estimate the solved form as a system as the estimates would benefit in terms of efficiency. As to the estimation method, Perali and Pieroni (2004) used three stage least squares (3SLS).[13] Peculiar features of 3SLS estimation are that the full set of insruments is used for all equations and the error terms are assumed

[12] An alternative procedure is to re-estimate the supply and demand equations jointly with the solved price and stock equations, thus getting new estimates of the parameters to be used in testing the restrictions, with likely efficiency gains (Gilbert, 1995).

[13] Gilbert (1995) first ran single equation regressions and then employed an iterated NL3SLS algorithm for the full system imposing different set of restrictions.

to be *iid*. Therefore we decided to pursue GMM estimation as it allows the use of equation specific instruments and produces efficient estimates that are robust to heteroskedasticity (and possibly autocorrelation if HAC correction is employed). Moreover, if the number of moment conditions available are in excess with respect to the number of parameters to be estimated, tests of overidentifying restrictions allow to check the validity of instruments and in general the specification of the model.

One major drawback from using instruments which are not relevant is that GMM estimates can be inconsistent. In order to check on instruments relevance, after controlling pairwise correlations between excluded instruments and endogenous regressors, we have run auxiliary first-stage regressions of market fundamentals on the full set (included and excluded) instruments. Low values of the partial R^2 and of the F-test on the excluded instruments may all signal possible problems of weak identification. Moreover, non-negligible descrepancies between the partial R^2 and the Shea partial R^2 may suggest possible collinearity between the instruments. In all these cases parsimony in the use of instruments is a good strategy.

We then performed single equation GMM estimates running C (or difference in Sargan) tests to decide on the best set of instruments to use for each equation. At this stage, we also checked the Kleibergen-Paap F statistic to further control for possible weak identification issues; the not particularly high values suggest that there is probably scope for further improvements in this direction. After these checks, the following sets of additional instruments have been selected: for the price equation, the one-period lagged values of stocks level s_{t-1}, palm oil price p_{t-1}^{oi} and a production weighted food production index for major cocoa producers FPI_{t-1}; for the stock equation, the one-period lagged values of stock and production levels, coffee price p_{t-1}^{cf} and cocoa price p_{t-1}, and the production weighted population in major producing countries POP_{t-1}. As we shall see, the tests conducted on the system GMM estimates confirm the validy of such instruments.

Table 2 reports the results of the iterated GMM estimates of the solved form, for the base model and one including a time dummy for the structural break in 1986, using the sample 1970-2010. Restricted estimates from a bivariate VAR(C1r) model including the same set of exogenous variables are also reported, where cross lagged endogenous terms have been dropped.

The overall fit of the GMM system estimation of the price and stock equation in both base and break models is quite good. The Hansen-J statistics, distributed as a $\chi^2(4)$ (four being the number of overidentifying restrictions), confirm the validity of the instruments (i.e. uncorrelated with the error terms and correctly excluded instruments). As a general remark, we observe that the autoregressive terms tend to absorbe much of the explanatory power in the regression, as they are large and highly significant (this is true in particular for the lagged price) and display positive signs, thus showing a positive short-run persistence in the series. Using standard errors robust to heteroskedasticity, few other coefficients are actually significant at 5% significance level. The signs are the same in both models, while the inclusion of the time dummy improves the

Table 2: GMM vs VAR(C1r) estimates of reduced form

	GMM			**VAR(C1r)**	
	base	**break**		**base**	**break**
		price equation			
cons	-0.529	-8.723	*cons*	-0.759	-11.325
	(-0.1)	(-1.74)		(-0.14)	(-1.54)
z_{1t}	-0.414	-0.124	$\hat{z}_{1t}$	-0.452	-0.284
	(-1.74)	(-0.6)		(-2.12)	(-1.27)
z_{2t}	-0.855	-1.730	$\hat{z}_{2t}$	-0.971	-2.324
	(-0.87)	(-1.91)		(-0.96)	(-1.93)
x_{1t}	0.129	0.458	x_{1t}	0.113	0.587
	(0.56)	(1.97)		(0.46)	(1.76)
$x_{2,t-1}$	-0.242	-0.140	$x_{2,t-1}$	-0.211	-0.193
	(-1.55)	(-1)		(-1.25)	(-1.18)
r_t	-0.055	-0.157	r_t	-0.078	-0.166
	(-0.52)	(-1.38)		(-0.72)	(-1.45)
p_{t-1}	0.891	0.941	p_{t-1}	0.949	0.954
	(7.72)	(7.58)		(9.3)	(9.31)
s_{t-1}			s_{t-1}		
D_{86}		-0.317	D_{86}		-0.418
		(-2.51)			(-2.02)
		stock equation			
cons	-15.599	-3.522	*cons*	-5.997	3.760
	(-1.45)	(-0.54)		(-1.35)	(0.68)
z_{1t}	1.164	0.679	$\hat{z}_{1t}$	0.386	0.225
	(2.23)	(2.11)		(2.03)	(1.19)
z_{2t}	-1.974	-0.544	$\hat{z}_{2t}$	0.256	1.488
	(-0.95)	(-0.4)		(0.38)	(1.89)
x_{1t}	0.905	0.325	x_{1t}	0.332	-0.109
	(1.55)	(0.95)		(1.49)	(-0.41)
$x_{2,t-1}$	0.187	0.081	$x_{2,t-1}$	0.038	0.021
	(0.96)	(0.64)		(0.32)	(0.2)
r_t	-0.285	-0.130	r_t	-0.040	0.040
	(-1.7)	(-1.31)		(-0.56)	(0.54)
p_{t-1}			p_{t-1}		
s_{t-1}	0.157	0.357	s_{t-1}	0.743	0.752
	(0.36)	(1.44)		(6.61)	(6.95)
D_{86}		0.356	D_{86}		0.380
		(3.1)			(2.7)
η_e	-2.662	-8.790		-2.552	-4.438
	(-1.76)	(-0.61)		(-1.96)	(-1.09)
η_s	-0.274	-0.518		-0.239	-0.391
	(-0.89)	(-0.41)		(-0.48)	(-0.4)
Hansen's J $\chi^2(4)$	1.180	2.577			
p-value:	(0.7578)	(0.4616)			
Log likelihood				50.446	54.573
AIC				-11.102	-10.977

Notes: t-statistics in parenthesis. Sample 1970-2010.

significance of coefficients in the price equation (the reverse is true for the stock equation).

In the price equation, short and the long-term fundamentals display the expected negative sign[14], as excess supply tends to depress current prices, an high z_{2t} reducing in particular the incentive to carry stocks forward, even tough it is not significant in the base model (at 10% in the break one). The absolute value of z_{2t} is rather high in the base model, approximately twice as large as z_{1t} (significant at 10%). The positive sign of z_{1t} (significant at 5%) in the stock equation correctly reflects the immediate impact of excess supply on stocks level, while the negative sign of z_{2t} is in our opinion consistent with the reduced incentive to accumulating stocks in face of a weak long-term fundamental.

Income elasticity is correctly signed in both equations, but only significant in the break model. The negative sign of interest rate in the stock equation is coherent with a speculative stockholding explanation as postulated in our model, as an increased cost of storage would reduce the incentive to carry over stocks, in turn depressing current price. Conversely, an interpretation which exploits the countercyclical nature of commodity prices could be offered to a possible positive sign of the interest rate in the price equation. In fact, interest rates tend to be high in late expansion and early recession phases, exactly when commodity prices are typically high. The opposite signs of the exchange rate in the two equations look consistent with an increased supply by major producers following a competitive devaluation, in turn possibly depressing world prices and increasing stock levels. Though, both interest rate and exchange rate variables are not significant in both models. Finally, the time dummy included for accounting for the structural break in 1986 is significant in both equations and display the expected signs, increasing the mean stock level and depressing the mean price.

As concerns the free structural parameters η_e and η_s, they can be recovered from the estimated solved form by manipulating the response matrix A_0, as discussed in the previous chapter. In particular, it has been shown that the elasticity of stock demand with respect to expected capital gains is given by

$$\eta_e = \frac{a_{21}}{a_{11}\left(a_{16} + a_{21}\phi_{12} - 1\right)}. \tag{13}$$

The estimates of η_e from both models are not satisfactory, as display negative signs and not much plausible large values, even though in general we would expect the speculative component of stock demand to be very responsive to even small expected capital gains. The sign of η_s is negative as well and not significant. Such results deserve further investigation.

We now turn to compare the GMM estimates of the reduced form with those from a restricted VAR(C1r) model including the same exogenous variables, where the cross lagged endogenous variables have been dropped, as reported in Table 2.[15] As a general remark, we observe again that also in the VAR estimates

[14] A useful comparison can be made with the signs derived from the theoretical model in Appendix B of previous chapter through a comparative statics analysis.

[15] For details on the VAR model we defer to the next section.

the autoregressive components, positive and rather close to unity (also for stocks in this case), seem to absorbe much of the explanatory power as the other exogenous regressors, apart from z_{1t}, z_{2t} (in the price equation) and marginally x_{1t}, result not significant in both equations. The signs of the GMM and VAR estimates to a very large extent coincide in both models, except for the positive sign of z_{2t} in the VAR stock equation. Also the magnitudes of coefficients are largely comparable, in particular in the price equation, while discrepancies are larger in the stock equation, notably for short and long-run fundamentals. The same considerations made before concerning the time dummy and the derived parameters η_e and η_s hold here.

5.1.2 VAR models

This section presents the estimation of different VAR models in the attempt to more fully account for the dynamics of the system. As discussed so far, the role of stocks is crucial in explaining the, at least, short term dynamics of world prices. Therefore, we shall continue to analyze the relationship between price and stocks in an substantially bivariate framework, keeping the specification close to the derived solved form. In our exercise we shall move from an atheoretical specification, where only lagged values of price and stocks are used as explanatory variables, to a more structured one where additional exogenous regressors from the solved form are included. Other alternative specifications were of course available, increasing for instance the VAR dimension or modelling explicitely long-run relations in VECM form. Nevertheless, that would have implied specifying ultimately a different model, possibly in a non rational expectations framework (Deaton and Laroque, 2003) and we decided to leave it for future work.

A first choice to make concerns whether to estimate a model in levels or first-differences. Results from the preliminary analysis on the stationarity of the series do not provide definitive answers. Even though the unit root hypothesis cannot be rejected for both series over the whole sample, if we account for the structural break in 1986 and restrict accordingly the sample period, the price and stocks variables can be considered $I(0)$ processes. In the end, we decided to keep the variables in (log) levels, using the sample 1970-2010 and comparing benchmark estimates from various models with those including a time dummy to control for the shift in the mean.[16]

In order to decide on the VAR length, lag selection criteria (FPE, AIC, SBIC, HQIC) have been used. As can be seen from Table 3, they unambigously seem to suggest that a single lag is sufficient to control for the persistency of the processes. Hence, a VAR(1) will provide our benchmark model, even though from the univariate analysis on the price series and a priori knowledge on the age-yield profile of the cocoa tree we know that other lags can be relevant. We thus experimented with additional lags and found significant results from a VAR

[16]However, the point estimates from an integrated VAR are consistent, as long as the dynamics is correctly specified, and can be used for forecasting purposes (Sims et al., 1990). Moreover, the lag selection criteria are still valid.

including also a lag 8 or 11. In what follows only results obtained using a lag 8 are presented. The general formulation of the VAR can be written as

$$\mathbf{y}_t = \boldsymbol{\mu} + B(L)\mathbf{y}_t + B_0\mathbf{x}_t + \mathbf{v}_t \tag{14}$$

where $\mathbf{y}_t$ is a $K \times 1$ vector of endogenous variables, $B(L)$ is a matrix polynomial of order p, $\mathbf{x}_t$ is an $L \times 1$ vector of exogenous regressors, B_0 a $K \times L$ matrix of coefficients, $\boldsymbol{\mu}$ a $K \times 1$ vector of constants and $\mathbf{v}_t$ a $K \times 1$ vector of disturbances. We assume that a sufficient number of lags have been included so that the v_t can be considered as white noises. In our basic specification $\mathbf{y}_t = (p_t, s_t)'$ is a 2×1 vector while B_0 is restricted to be $\mathbf{0}$. As to the lag polynomial, after checking the lag selection criteria and exploiting a priori information, we can restrict it to B_1L in the short-run specification (model A1) and to $B(L) = B_1L + B_8L^8$ in the long-run version (model A2).

In the extended specifications we add exogenous variables from the solved form, whereas the fitted values of the market fundamentals, $\widehat{z}_{1t}$ and $\widehat{z}_{2t}$, substitute the original variables because of the known endogeneity problems. The full vector of the exogenous variables thus becomes $\mathrm{x}_t = (\widehat{z}_{1t}, \widehat{z}_{2t}, x_{1t}, x_{2,t-1}, r_t, D_{86})'$, as the time dummy has been included. We estimate a model including the entire set of exogenous variables (model C1) and a restricted version which excludes the fundamentals (model B1), in both cases with or without the time dummy.

Table 4 reports the estimates of the VAR(1) models presented above. The overall fit of the models, as measured by the log-likelihood, in general improves as we move from simple to more structured specifications and a further improvement is achieved by including the time dummy, whose effect is negative in the price equation, as it captures the declining trend in prices, and positive in the stocks equation reflecting the increased stocks level.

Turning to the single coefficients, the autoregressive terms in model A1(d) are positive and significant in both equations, and the magnitude and significance is largely preserved in models B1 and C1. Conversely, cross lagged endogenous variables, s_{t-1} negatively impacting on current price and p_{t-1} positively affecting stocks, loose relevance and significance in models with other exogenous regressors. In particular, the inclusion of $\widehat{z}_{1t}$ in model C1, which is negative and significant, seems to make the effect of s_{t-1} negligible, not surprisingly as by construction it includes lagged stocks. The same considerations made above concerning the expected signs of the coefficients should hold here. In the stocks equation, the signs of x_{1t}, $x_{2,t-1}$ and r_t remain the same in models B1 and C1, while the same is not true in the price equation, where the coefficients are however poorly estimated.

We said before that, from a priori information, we know that a lag 8 may be relevant. Therefore, in Table 5 we present estimates from the preceding VAR models where an additional lag 8 is included. The signs of the one lagged endogenous are confirmed. The eight lagged price negatively impacts on current price, as s_{t-8} does, an expected outcome since past price dynamics may have triggered investment in new production capacity. In the stock equation both p_{t-8} and s_{t-8} have a positive effect on current stocks. Remarkably, in the benchmark (without dummy) specification the eight lagged endogenous variables remain significant also in model B2 and C2. As to the other exogenous variables, we simply observe that they are in general poorly estimated, included the time dummy, apart from income in the price equation and, marginally, the long-term fundamental.

The analysis of residuals (not reported) from the preceding estimated models suggest that disturbances can be considered as innovations. In particular, the Breush-Godfrey tests exclude the presence of residual autocorrelation, while the Jarque-Bera tests do not reject the hypothesis of normality of residuals. Furthermore, the VAR models satisfy the stability conditions as all eigenvalues of the companion matrix lie inside the unit circle.

Table 3: Lag order selection criteria

lag	**LL**	**LR**	**FPE**	**AIC**	**HQIC**	**SBIC**
0	-45.7986		0.03529	2.33164	2.36208	2.41523
1	40.67	172.94*	0.00063*	-1.69122*	-1.59991*	-1.44045*
2	43.6973	6.05460	0.00066	-1.64377	-1.49158	-1.22583
3	45.5666	3.73860	0.00074	-1.53983	-1.32677	-0.95471
4	47.9156	4.69810	0.00081	-1.45930	-1.18535	-0.70700

Notes: * denotes lag chosen by the criterion. LL (Loglikelihood), LR (Likelihood Ratio), FPE (Final Prediction Errror), AIC (Akaike), HQIC (Hannan-Queen), SBIC (Schwartz).

5.2 Testing the REH restrictions

A peculiar feature of rational expectations models is that they yield a set of non-linear cross equations restrictions linking structural and reduced form parameters. Tests of such restrictions provide a means of indirectly verifying the rational expectations hypothesis conditional on the validity of the model or equivalently, and perhaps preferably, of testing the validity of model specification under the REH. In the previous chapter we have illustrated the analytical derivation of the restrictions which we now test using the estimates of the intermediate and reduced form parameters from both base and break models.

Since we are using estimates from a short-run model, and in particular short-run supply and demand elasticities have been employed in costructing the market fundamentals, what we are testing is in fact whether stockholders form their expectations rationally, or consistently with the postulated model, but using a short-term information set, neglecting events far in the past affecting supply conditions and potentially determining future market imbalances. Although the restrictions stem from and are theoretically consistent with the solved form GMM estimates, the specification used in the restricted VAR(C1r) model matches the derived reduced form, allowing us to run the tests using the VAR estimates of the response matrix A_0, as from Table 2.

As discussed in Gilbert (1995), a first set of restrictions verifying the logi-

Table 4: Comparison of VAR models (1)

	A1	**B1**	**C1**	**A1d**	**B1d**	**C1d**
			price equation			
cons	3.9079	-1.762	3.655	3.377	-7.616	-6.92
	(2.22)	(-0.26)	(0.49)	(1.84)	(-1.04)	(-0.76)
p_{t-1}	0.7014	0.6761	0.815	0.691	0.634	0.834
	(5.59)	(3.9)	(4.1)	(5.49)	(3.76)	(4.35)
s_{t-1}	-0.226	-0.345	0.1125	-0.12	-0.25	0.091
	(-1.9)	(-1.64)	(0.36)	(-0.78)	(-1.19)	(0.3)
x_{1t}		-0.171	-0.313		-0.185	-0.28
		(-1.02)	(-1.76)		(-1.14)	(-1.64)
$x_{2,t-1}$		0.3416	-0.048		0.596	0.424
		(0.87)	(-0.11)		(1.48)	(0.87)
r_t		-0.073	0.0242		-0.073	-0.07
		(-0.78)	(0.2)		(-0.8)	(-0.56)
$\hat{z}_{1t}$			-0.667			-0.48
			(-1.99)			(-1.41)
$\hat{z}_{2t}$			0.0194			-1.38
			(0.02)			(-1.04)
D_{86}				-0.16	-0.325	-0.39
				(-1.01)	(-1.87)	(-1.89)
			stock equation			
cons	-2.29	-3.077	-7.598	-1.34	2.814	2.15
	(-1.92)	(-0.64)	(-1.44)	(-1.18)	(0.58)	(0.35)
p_{t-1}	0.2007	0.2293	0.1743	0.219	0.272	0.157
	(2.36)	(1.88)	(1.25)	(2.8)	(2.42)	(1.21)
s_{t-1}	1.1073	1.0991	0.7791	0.922	1.003	0.799
	(13.71)	(7.41)	(3.59)	(9.48)	(7.19)	(3.99)
x_{1t}		0.0179	0.1194		0.032	0.091
		(0.15)	(0.96)		(0.3)	(0.79)
$x_{2,t-1}$		0.0244	0.3162		-0.231	-0.12
		(0.09)	(1.03)		(-0.86)	(-0.36)
r_t		-0.031	-0.124		-0.031	-0.04
		(-0.46)	(-1.46)		(-0.51)	(-0.42)
$\hat{z}_{1t}$			0.4632			0.287
			(1.97)			(1.26)
$\hat{z}_{2t}$			-0.543			0.747
			(-0.67)			(0.83)
D_{86}				0.291	0.328	0.36
				(2.92)	(2.84)	(2.59)
Log likelihood	40.670	49.089	52.340	46.538	53.695	56.272
AIC	-7.465	-7.875	-8.034	-7.75	-8.1	-8.23

Notes: t-statistics in parenthesis. Sample: 1970-2010.

Table 5: Comparison of VAR models (2)

	A2	**B2**	**C2**	**A2d**	**B2d**	**C2d**
			price equation			
cons	9.1066	-7.648	-4.158	10.793	-13.47	-9.845
	(3.03)	(-0.99)	(-0.51)	(2.49)	(-1.26)	(-0.89)
p_{t-1}	0.6027	0.323	0.544	0.5816	0.3397	0.557
	(4.3)	(1.59)	(2.33)	(3.96)	(1.65)	(2.36)
p_{t-8}	-0.295	-0.304	-0.429	-0.364	-0.134	-0.273
	(-2.00)	(-1.91)	(-1.97)	(-1.86)	(-0.50)	(-0.91)
s_{t-1}	-0.119	-0.544	-0.247	-0.169	-0.511	-0.228
	(-0.95)	(-2.41)	(-0.76)	(-1.08)	(-2.22)	(-0.69)
s_{t-8}	-0.417	-0.697	-0.797	-0.522	-0.48	-0.598
	(-2.08)	(-2.82)	(-2.63)	(-1.86)	(-1.3)	(-1.49)
x_{1t}		1.048	0.8121		1.1921	0.962
		(2.26)	(1.54)		(2.38)	(1.7)
$x_{2,t-1}$		-0.031	-0.049		-0.107	-0.116
		(-0.19)	(-0.25)		(-0.56)	(-0.53)
r_t		0.024	0.0064		-0.007	-0.027
		(0.26)	(0.06)		(-0.07)	(-0.21)
$\hat{z}_{1t}$			-0.333			-0.317
			(-0.99)			(-0.94)
$\hat{z}_{2t}$			-1.658			-1.696
			(-1.13)			(-1.15)
D_{86}				0.1187	-0.22	-0.209
				(0.54)	(-0.79)	(-0.77)
			stock equation			
cons	-6.446	-4.158	-6.404	-3.444	0.2914	-1.983
	(-3.24)	(-0.77)	(-1.13)	(-1.23)	(0.04)	(-0.26)
p_{t-1}	0.2847	0.388	0.224	0.2472	0.375	0.214
	(3.07)	(2.72)	(1.38)	(2.62)	(2.61)	(1.31)
p_{t-8}	0.229	0.324	0.4363	0.1066	0.1939	0.315
	(2.35)	(2.91)	(2.89)	(0.85)	(1.04)	(1.52)
s_{t-1}	1.0228	1.106	0.9099	0.9348	1.0814	0.895
	(12.26)	(7.01)	(4.05)	(9.28)	(6.72)	(3.95)
s_{t-8}	0.3349	0.497	0.5961	0.1475	0.3305	0.441
	(2.52)	(2.88)	(2.83)	(0.82)	(1.28)	(1.59)
x_{1t}		-0.205	-0.062		-0.315	-0.179
		(-0.63)	(-0.17)		(-0.9)	(-0.46)
$x_{2,t-1}$		-0.114	-0.118		-0.056	-0.066
		(-0.99)	(-0.84)		(-0.42)	(-0.43)
r_t		-0.102	-0.079		-0.078	-0.053
		(-1.56)	(-0.97)		(-1.1)	(-0.61)
$\hat{z}_{1t}$			0.2083			0.195
			(0.9)			(0.83)
$\hat{z}_{2t}$			1.3995			1.429
			(1.38)			(1.4)
D_{86}				0.2113	0.1683	0.163
				(1.50)	(0.87)	(0.86)
Log likelihood	44.143	58.766	61.342	49.045	59.259	61.857
AIC	-7.439	-8.152	-8.278	-7.678	-8.176	-8.303

Notes: t-statistics in parenthesis. Sample: 1970-2010.

cal consistency of the estimated price and stock equations with the first stage production and consumption estimates is given by

$$a_{21} = 1 + \frac{1}{\lambda} a_{11}, \quad a_{2j} = \frac{1}{\lambda} a_{1j}, \quad j = 2, 3, 4, 5. \tag{15}$$

Table 6 reports the Wald statistics of the logical restrictions from the four estimated models over the period 1970-2010. Considering the GMM estimates, the single restrictions easily pass the tests in all cases and the overall set is accepted in both models, even though more weakly in the break model. The single restrictions are not rejected also using the VAR(C1r) estimates, except for z_{2t} and the overall set in the break model. This result does not come as a surprise since the GMM and VAR estimates are quite close.

Similar considerations can be made by looking at the results from the tests of

Table 6: Wald tests of logical restrictions

		GMM		**VAR(C1r)**	
j	**variable**	**base**	**break**	**base**	**break**
1	z_{1t}	0.28	0.47	1.23	0.87
		(0.5992)	(0.492)	(0.2665)	(0.3519)
2	z_{2t}	0.33	0.16	1.01	14.18
		(0.565)	(0.6892)	(0.3137)	(0.0002)
3	x_{1t}	0.20	0.74	0.12	1.35
		(0.6556)	(0.3904)	(0.7336)	(0.2462)
4	$x_{2,t-1}$	1.68	0.00	0.97	0.58
		(0.1955)	(0.9567)	(0.3235)	(0.4473)
5	r_t	0.23	0.70	0.03	2.21
		(0.6318)	(0.4031)	(0.8716)	(0.1376)
Overall $\chi^2(5)$		2.23	9.36	8.42	17.89
		(0.8162)	(0.0955)	(0.1346)	(0.0031)

Notes: t-statistics reported are $\chi^2(1)$. P-values are given in parentheses.

the REH restrictions, which provide a check of the consistency of the estimated price equation with the price expectation in the stock demand equation (3). They have been derived in the previous chapter as

$$\frac{a_{22}}{a_{21}} = \frac{(a_{16}a_{12} + \theta_{22}a_{12} - a_{12} + a_{11}\theta_{12})}{a_{11}(a_{16} - 1)} \tag{16}$$

$$\frac{a_{23}}{a_{21}} = \frac{(a_{13}a_{16} + a_{11}\theta_{13})}{a_{11}(a_{16} - 1)} \tag{17}$$

$$\frac{a_{24}}{a_{21}} = \frac{(a_{14}a_{16} + a_{11}\theta_{14} + a_{12}\theta_{24})}{a_{11}(a_{16} - 1)} \tag{18}$$

$$\frac{a_{25}}{a_{21}} = \frac{(a_{16}a_{15} + \rho_3 a_{15} - a_{15} - 1)}{a_{11}(a_{16} - 1)} \tag{19}$$

Table 7 presents the Wald statistics of the four REH restrictions given above. Using either the GMM or the VAR(C1r) estimates, the single restrictions and the overall sets are not rejected for both models, the break one performing slightly better. Such results are encouriging as suggest a substantially correct basic specification of the model and are supportive of the view that the relevant information set of stockholders is quite limited in time.

Table 7: Wald tests of REH restrictions

	GMM		**VAR(C1r)**	
equation	**base**	**break**	**base**	**break**
16	0.28	0.11	0.15	0.13
	(0.5984)	(0.7407)	(0.6986)	(0.7201)
17	0.49	0.16	0.20	0.15
	(0.4851)	(0.6906)	(0.6523)	(0.6984)
18	0.51	0.14	0.19	0.11
	(0.4731)	(0.7066)	(0.6618)	(0.7376)
19	0.53	0.17	0.2	0.14
	(0.4676)	(0.6792)	(0.6552)	(0.7047)
Overall $\chi^2(4)$	1.12	1.37	0.68	0.46
	(0.8908)	(0.8499)	(0.9532)	(0.9777)

Notes: t-statistics reported are $\chi^2(1)$. P-values in brackets.

6 Concluding remarks

In this chapter we estimated the solved or reduced form in price and stocks of a short-run rational expectations model of the world cocoa market, including a speculative stockholding equation. The choice of focusing on these two variables is motivated by the apparent stable relationship over time existing between world price, as measured by the ICCO indicator price, and the stocks-to-use ratio, an indicator of cocoa availabity at global level monitored by industry analysts. In this respect, the adoption of a solution method using two constructed variables measuring excess or shortage of supply in the market, as proposed by Gilbert (1995), has been deemed appropriate.

Hence, a reduced form in the market fundamentals and exogenous variables such as world income, exchange rate and interest rate has been estimated using the generalized method of moments (GMM) to account for the endogeneity in the fundamentals. The GMM estimates, robust to possible heteroskedasticity, are quite satisfactory. The estimated coefficients present theoretically consistent signs and meaningful magnitudes, apparently confirming the hypothesized effect of market fundamentals on current price and stocks. The lagged endogenous variables are mostly significant, confirming the validity of their inclusion in the statistical model. A model including a time dummy to account for a structural break has also been estimated yielding similar results and a slightly superior fit. Conversely, the derived estimates of the stock elasticities are not satisfactory suggesting further investigation in this direction.

The restrictions stemming from the hypothesized rational expectations have been tested using estimates from both models. The single restrictions and the overall set are not rejected, suggesting an acceptable model specification and the validity of the model consistent expectations.

At this regard, we remark that the failure at rejecting the REH restrictions using estimates from short-run models seems to imply that the information set used by market participants in forming their expectations about future market developments is essentially limited in time. Our initial guess that past events, mostly supply related, might be incorporated in expectations would not seem to be supported by empirical evidence, at least as far as stockholding decisions are concerned. Nevertheless, based on preliminary results from long lag structure VAR models, testing using also estimates from a long-run version model could possibly corroborate these findings.

An alternative approach, aiming at possibly providing a better representation of the underlying statistical model, in particular the short-run dynamics, has also been pursued by specifying different vector autoregressive models in the price and stocks dimension. Specification tests pointed to a VAR(1) as a suitable representation of price and stocks dynamics which we augmented in alternative versions with an additional exogenous variables from the solved form, supposed to have a contemporaneous impact on the endogenous variables and thus paralleling the GMM estimation of the reduced form. Using a priori information, we also included a lag eight which turned out to be significant, signalling that efforts towards a better modelling of supply dynamics, as tentatively attempted in the previous chapter work, are worth pursuing.

The GMM estimates of the solved form and those from a restricted VAR model, with a matching specification, have been then compared. The results are to a large extent similar, also as concerns the acceptance of the REH restrictions, confirming the validity of the short-run statistical model and the relevance of the autoregressive components which tend though to crowd out other state or forcing variables in the model.

Finally, we observe that the ongoing process of vertical integration, both upstream and downstream, and the increased interest in supply conditions by international traders, may justify a modelling exercise focused on market fundamentals or tentatively exploiting the information set used by global players, despite speculative bubbles on financial markets might turn away prices from market fundamentals for prolonged periods and even generate permanent effects.

7 Data appendix

Data used in the estimation have been collected from various sources. Figures on world production, grindings, stocks, apparent consumption and the world indicator price come from the International Cocoa Organization (ICCO). Data on area harvested, production, trade and producer prices for individual countries are from FAOSTAT database (FAO). Macroeconomic data such as GDP, exchange rates, interest rates, consumer price indexes and other international commodity prices come from the International Monetary Fund (IMF) and the World Bank (WB). We now provide definitions and further details on the single series.

Data on world production, grindings and stocks of cocoa beans are measured in thousand tonnes. The stock series is computed from the annual supply/demand balance and from two base year stocks estimates. The ICCO Secretariat uses an estimate of 325,000 tonnes in 1973/74 and of 1,682,000 tonnes in 2003/04 as the base year figures. World end-of-season stocks of cocoa beans are calculated as current production adjusted for loss in weight (net production) minus seasonal grindings plus previous stocks. *Source: Quarterly Bulletin of Cocoa Statistics (ICCO).*

World income is calculated as a weighted average of the GDP (in current US dollars) of major consuming countries. [17] The weights are given by average apparent domestic consumption computed over the period 1997/1998 through 2005/2006. *Source: World Development Indicators (WB)).*

Apparent domestic consumption is given by the sum of grindings plus net imports of cocoa products, either finished or semiprocessed, expressed in beans equivalent. For the purpose of determining the beans equivalent of cocoa products, the following conversion factors are used: cocoa butter 1.33, cocoa cake and powder 1.18, cocoa paste/liquor and nibs 1.25. *Source: Quarterly Bulletin of Cocoa Statistics (ICCO).*

The world cocoa price is the crop year average of the ICCO daily price for cocoa beans, unit US$ per tonne. The latter price is calculated as the average of the quotations of the nearest three active futures trading months on NYSE Liffe Futures and Options and ICE Futures US at the time of London close. *Source: Quarterly Bulletin of Cocoa Statistics (ICCO).*

The international commodity prices used have been converted in US$ per tonne. Rubber and palm oil prices are for production originating from Malaysia, the coffee price is for the Robusta variety (Uganda origin), the one cultivated mostly in cocoa producing countries. *Source: International Financial Statistics (IMF).*

The exchange rate is given by the annual average of the franc CFA per US$ used in fourteen African countries, included Côte d'Ivoire and Cameroon. *Source: International Financial Statistics (IMF).*

The interest rate used is the annual average return on US three-month Trea-

[17] The most important countries in terms of domestic apparent consumption are the United States, Germany, France, Belgium-Luxemburg, UK, Italy, Russian Federation, Japan, Brazil, Spain, Canada, Poland.

sury Bills. *Source: International Financial Statistics (IMF).*

The price deflator is the US consumer price index, all items (100=2004). *Source: International Financial Statistics (IMF).*

The food production index and the population figures are weighted averages for major producing countries.[18] The weights used are the time series of the production shares for those countries. *Source: World Development Indicators (WB).*

[18]The list of countries includes Côte d'Ivoire, Ghana, Nigeria, Cameroon, Indonesia, Malaysia, Brazil, Ecuador, Dominican Republic and Papua New Guinea.

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