Working Paper Series
Department of Economics
University of Verona

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WP Number: 17 October 2013

ISSN: 2036-2919 (paper), 2036-4679 (online)

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October 3, 2013

Abstract

This paper focuses on the evolution of child labour, fertility and human capital in an economy characterized by two types of workers, low- and high-skilled. This heterogeneity allows an endogenous analysis of inequality generated by child labour. More specifically, according to empirical evidence, we offer an explanation for the emergence of a vicious cycle between child labour and inequality. The basic intuition behind this result arises from the interdependence between child labour and fertility decisions. Furthermore, we investigate how child labour regulation policies can influence the welfare of the two groups in the short run, and the income distribution in the long run. We find that conflicts of interest may arise between the two groups.

JEL classification: J13; J24; J82; K31.
Keywords: Child Labour, Fertility, Human capital, Inequality.

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1 Introduction

Child labour is a persistent phenomenon in many developing countries despite being declared illegal at both the national and international levels. According to the International Labour Organization (2006a), in 2004, there were estimated to be more than 200 million child workers in the world (Edmonds, 2008).

A large body of literature has developed theoretical and empirical models to study the causes of child labour persistence.[1] The benchmark framework is based on two main axioms: the luxury axiom and the substitution axiom (Basu and Van, 1998 and Basu, 1999). Under the luxury axiom, parents send children to work if their income is below a certain threshold. According to the substitution axiom, adult labour and child labour are substitutes. These axioms lead to multiple equilibria in the labour market, with one equilibrium where the adult wage is low and children work, and another where the adult wage is high and children do not work.

This framework has been extended by Dessy (2000), Hazan and Berdugo (2002) and Doepke and Zilibotti (2005), who introduce endogenous fertility choices. They analyse the relationship between child labour, fertility and human capital showing the existence of multiple development paths. In early stages of development, the economy is in a development trap where child labour is abundant, fertility is high and output per capita is low. Technological progress allows a release from this trap because it gradually increases the wage differential between parental and child labour and hence the return of investment in education.[2]

However, these contributions do not consider the presence of inequality; the economy can follow different paths of development which are characterized – in equilibrium – by a single level of human capital and a low or high degree of child labour. We extend this framework to take into account two

[1] See, for example, Basu and Van, 1998; Basu, 1999, 2000; Baland and Robinson, 2000; Dessy, 2000; Dessy and Pallage, 2001, 2005; Ranjan, 1999, 2001

[2] For a review of the literature on the economics of child labour see Basu and Tzannatos (2003) and references therein.

groups of individuals with two different levels of human capital. The presence of this heterogeneity can induce the high-skilled dynasty to capture any increase in the return on human capital, by increasing the level of inequality during the process of development.

In this respect, our framework is closely related to the literature on inequality, differential fertility and economic growth. In particular, De la Croix and Doepke (2003, 2004), and Moav (2005) show that the differential fertility between the rich and the poor can offer an explanation for the persistence of poverty within and across countries. The basic idea is that the cost of child quantity increases with the parent's human capital since the opportunity cost of time is high. Consistent with empirical evidence, they obtain that high-income families choose low fertility rates and high investment in education. This implies that high income persists in the dynasty. On the other hand, poor households choose relatively high fertility rates with relatively low investment in their offspring's education. Therefore, their offspring are poor as well.[3]

We extend this framework in two directions: (i) we introduce a dynamic general equilibrium analysis of the ratio between unskilled and skilled wage. (ii) we take into account the role of child labour. The two extensions generate new features of the model and new policy implications if child labour regulation policies are implemented. The first result of our model is that, during the path of development, changes in educational choice of low- and high-skilled workers alter the equilibrium level of relative wages. Hence, unlike Moav (2005), in our general equilibrium setup the choice of a group of workers can change the decision of the other one. The main consequence is that, depending on the initial level of inequality, child labour regulation policies can bring about different results in the two groups. In this respect our work is related to Mookherjee et al. (2012). The negative relation between parental wage and fertility does not apply to unskilled workers. The possibility of child labour is crucial for this result, as Doepke (2004) pointed out,

[3] See also Dahan and Tsiddon (1998), Kremer and Chen (2002).

the value of children time is part of the opportunity cost of education (see p. 373). Thus we show that inequality persists even if all children have equal access to free, public education. The presence of child labour, by lowering the relative cost of raising children, leads to a higher fertility rate than that which would prevailed in the absence of child labour. In other words, child labour strengthens the vicious cycle between child labour and inequality.

As shown in Figure 1, we find empirical evidence of a positive relationship between inequality and child labour.[4] In this figure, we use the data on children not attending school (i.e.number of out-of-school children as a percentage of all primary school-age children) as a proxy of child labour given the shortage of data on child labour. Even if this measure presents the shortcoming that a child not attending school is not necessarily working, it is easier to monitor children not attending school than children who are working. In addition, the rate of children out of school should also give a measure of children working within the household or engaged in unofficial labour who are not included in the number of children economically active (see Cigno and Rosati, 2002). Note also that while still positive, the relationship between child labour and inequality has begun to flatten in recent years. A possible explanation for this result could be the increasing attention to child labour on the part of national and international organizations.

We develop an overlapping generations model with two types of workers – low- and high-skilled. According to the literature, we assume that child labour is a perfect substitute for unskilled adult labour but that children are relatively less productive. Adults allocate their time endowment between work and child rearing. They choose the number of children and their time allocation between schooling and work.

As a result, the model shows an intergenerational persistence in education levels: unskilled parents tend to have a high number of children and send them to work – a phenomenon called dynastic trap by Basu and Tzannatos

[4]Variability bands in Figure 1 show the level of variability present in the estimate. In particular, their width is determined by an estimate of the standard error (Bowman and Azzalini, 1997).

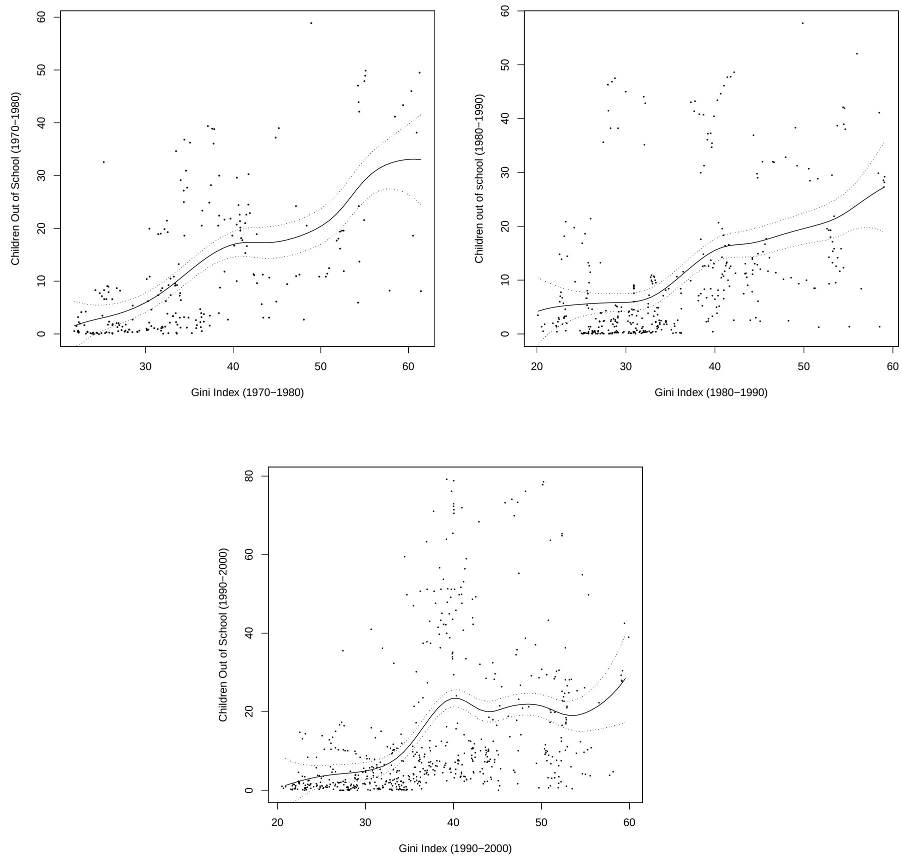


Figure 1: Children out of school and Gini Index (1970-1980, 1980-1990, 1990-2000). Nonparametric kernel smoother. Per capita GDP data are from Penn World Table 7.0. Gini Index data are from the World Income Inequality Database. Children out of school data are from World Development Indicators (2010)

(2003) – whereas skilled parents have low fertility rates and tend to invest more in education. This effect together with the differential fertility between low- and high-skilled parents can produce a continuous increase in inequality and child labour during the transition and an average impoverishment within the country in the long run.

Furthermore, we provide a new perspective on the effects of child labour regulation policies on the relationship between inequality and development. Indeed, policies aiming to regulate child labour, by affecting fertility and educational choices of skilled and unskilled parents, have large effects on income distribution in the long run.

In particular, we show that child labour regulation (CLR) policies, if enforced, significantly shape the quantity/quality trade-off by inducing an increase in education and therefore lower the level of inequality in the long run. However, CLR policies are likely to run into enforcement problems since they can asymmetrically reduce the attainable level of utility of the two groups (see, for instance, Ranjan, 2001). In particular, the effect of a CLR policy strictly depends on the initial level of inequality. In an economy with low inequality – i.e. the wage differential between skilled and unskilled parents is low – a CLR policy induces a decline in the welfare of high-skilled workers since restrictions on the child labour market reduce both their wage and the income of child labour. Unskilled workers face two opposite effects: a negative effect due to the loss of child labour and a positive effect due to the rise of the unskilled wage. If the productivity of child labour is sufficiently low, a CLR policy induces an increase in the welfare of unskilled parents. On the other hand, if inequality is high – i.e. skilled agents send their children only to school – a CLR policy does not affect the utility of skilled agents and decreases the utility of unskilled agents.

Finally, if the economy is in the long-run equilibrium, we show that in order to reduce child labour significantly through CLR policies, the human capital of high-skilled workers and their income may well have to decrease. This happens when the marginal return of human capital accumulation is low.

The rest of the paper is structured as follows. Section 2 describes the basic structure of the model. Section 3 presents the properties of the short-run general equilibrium. Section 4 shows the long-run dynamics of the economy. Section 5 derives the implications of CLR policies and Section 6 concludes.

2 The Model

We analyse an overlapping-generations economy which is populated by N_t individuals. Each of them is endowed with a level of human capital, h_t^i. This level is endogenously determined by parents' choice concerning their children's time allocation between labour and schooling. Adults can supply skilled or unskilled labour, while children can only supply unskilled labour. This setup is consistent with much of the literature on income distribution and development (see, for instance, Galor and Zeira, 1993), and was also recently introduced in the issue of child labour and economic growth (i.e. Hazan and Berdugo, 2002).

2.1 Production

We assume that labour is the only production factor. According to Doepke and Zilibotti (2005), production occurs according to a constant-returns-to-scale technology using unskilled and skilled labour as inputs.[5] Since the aim of the paper is to investigate the relations between child labour, unskilled and skilled labour, for the sake of argument, we abstract from capital in the production function. Thus the output produced at time t is

$$Y_t = \psi(H_t)^\mu (L_t)^{1-\mu} = \psi(s_t)^\mu L_t, \tag{1}$$

where $s_t \equiv H_t/L_t$ is the ratio of skilled H_t to unskilled labour L_t employed in production in period t, and $\psi > 0$ and $0 < \mu < 1$ are technological

[5] See also, Dahan and Tsiddon (1998), Galor and Mountford (2008).

parameters.[6] In each period t, firms choose the level of unskilled labour, L_t, and the efficiency units of labour, H_t, so as to maximise profits. Thus the wage of unskilled workers, i.e. w_t^u, and the wage rate per efficiency unit, w_t^s, are

$$w_t^u = \psi(1-\mu)\,(s_t)^{\mu}\,, \tag{2}$$

and

$$w_t^s = \psi\mu\,(s_t)^{\mu-1}\,. \tag{3}$$

Since each adult i is endowed with a certain level of human capital h_t^i, he/she chooses to work as unskilled if, and only if, $w_t^s h_t^i < w_t^u$, while he/she works as skilled if, and only if, $w_t^s h_t^i > w_t^u$. If, for instance, the level of human capital were uniformly distributed in the population, given the level of wages w^s and w^u, there would be an adult i^* such that $w_t^s h_t^{i^*} = w_t^u$. All the adults with a level of human capital $h^i < h^{i^*}$ would choose to work as unskilled, while the adults with a level $h^i > h^{i^*}$ would choose to work as skilled. In terms of income, agents with $h^i > h^{i^*}$ obtain a wage proportional to their level of human capital, while agents with $h^i < h^{i^*}$ obtain the same wage, w^u, irrespective of their level of human capital.[7]

2.2 Preferences

Members of generation t live for two periods: childhood and adulthood. In childhood, individuals may either work, go to school or both. In adulthood, agents supply unskilled or skilled labour. Individuals' preferences are defined over consumption, i.e. c_t^i, the number of children n_t^i, and the human capital

[6] As an alternative interpretation, we are assuming that labour contributes to production through two distinct services, physical effort ("brawn") and mental effort ("brain"). Unskilled labour and children provide physical effort, while skilled labour provides mental effort. For instance, this argument can be found in Stokey (1996).

[7] We believe that this feature of the economy can partly take into account the education-occupation mismatches. In our model such mismatches are in our model endogenously determined by the level of wages in the two labour markets and do not result in income penalties.

of children h^i_{t+1}.[8] The utility function of an agent i of generation t is given by

$$U^i_t = \alpha \ln c^i_t + (1-\alpha)\ln(n^i_t h^i_{t+1}), \tag{4}$$

where $\alpha \in (0,1)$ is the altruism factor.

We suppose that children are born with some basic human capital, which can be increased by attending school. In particular, human capital of children in period $t+1$ is an increasing, strictly concave function of the time devoted to school, that is

$$h^i_{t+1} = a(b + e^i_t)^{\beta}, \tag{5}$$

where $a, b > 0$ and $\beta \in (0,1)$.[9]

Parents allocate their income between consumption and child rearing. In particular, raising each born child takes a fraction $z \in (0,1)$ of an adult's income. In addition, parents allocate the time endowment of children between schooling, $e^i_t \in [0,1]$, and labour force participation $(1-e^i_t) \in [0,1]$. We assume that, each child can offer only $\theta \in [0,z)$ units of unskilled labour. The parameter $\theta < 1$ implies that children are relatively less productive than unskilled adult workers. The assumption $\theta < z$ implies that the cost of having children is positive even if parents choose to send them always to work. In other words, we assume that the cost of raising each child for unskilled parents – i.e. zw^u_t the forgone income – is higher than children's wage i.e. θw^u_t. In other words, it is not possible to increase income by simply "producing" more children.

[8] As is clear from equation (4), we assume that parents are aware of the human capital of their children rather than their income. As pointed out by De la Croix and Doepke (2003) and Galor (2005), we believe that this is a more realistic assumption. Moreover, the results of the model are not crucially affected by this choice. In particular, the inclusion of the future income of children, instead of their human capital does not change the results. See for instance Hazan and Berdugo (2002).

[9] See e.g. Galor and Tsiddon (1997), Galor and Weil (2000) and De la Croix and Doepke (2004).

As we pointed out above, while children can work only as unskilled workers, parents will choose to work in the sector that guarantees them the highest income. Thus, each household has two potential sources of income: i) parent income, $I_t^i = \max\{w_t^s h_t^i, w_t^u\}$ and, ii) child income, $(1-e_t^i)\theta w_t^u$.

The budget constraint is therefore[10]

$$c_t^i \leq (1 - z n_t^i) I_t^i + (1 - e_t^i)\theta w_t^u n_t^i. \tag{6}$$

2.3 Individual choices

Each household chooses c_t^i, n_t^i and e_t^i so as to maximize the utility function (4) subject to the budget constraint (6). Given the wage ratio, the optimal consumption, the optimal schooling and the optimal number of children chosen by member i are

$$c_t = \alpha I_t^i; \tag{7}$$

$$e_t^i = \begin{cases} 0 & \text{if } r_t^i \leq \frac{\theta(\beta+b)}{\beta z}, \\ \frac{r_t^i \beta z - \theta(\beta+b)}{\theta(1-\beta)} & \text{if } \frac{\theta(\beta+b)}{\beta z} \leq r_t^i \leq \frac{\theta(1+b)}{\beta z}, \\ 1 & \text{if } r_t^i \geq \frac{\theta(1+b)}{\beta z}; \end{cases} \tag{8}$$

and

$$n_t^i = \begin{cases} \frac{(1-\alpha) r_t^i}{z r_t^i - \theta} & \text{if } r_t^i \leq \frac{\theta(\beta+b)}{\beta z}, \\ \frac{(1-\alpha)(1-\beta) r_t^i}{z r_t^i - \theta(1+b)} & \text{if } \frac{\theta(\beta+b)}{\beta z} \leq r_t^i \leq \frac{\theta(1+b)}{\beta z}, \\ \frac{1-\alpha}{z} & \text{if } r_t^i \geq \frac{\theta(1+b)}{\beta z}; \end{cases} \tag{9}$$

where $r_t^i \equiv I_t^i / w_t^u$ is the wage differential between parental and child labour which is

[10] We do not consider in this model the issue of inter-generational transfers. We leave this extension for future research.

$$r_t^i = \begin{cases} 1 & \text{if } w_t^s h_t^i \leq w_t^u, \\ \frac{w_t^s h_t^i}{w_t^u} & \text{if } w_t^s h_t^i > w_t^u. \end{cases} \quad (10)$$

In the absence of child labour, with public education, parents, irrespective of their level of income, would have chosen the maximum level of education and the minimum level of fertility i.e. $(1-\alpha)/z$. The presence of child labour, by lowering the cost of children, leads to a higher fertility rate and different fertility and educational choices between skilled and unskilled parents.

Note that, if the parents find it convenient to perform unskilled work, since $r_t^i = 1$, their choices on fertility and education only depend on the relative cost of child-raising, i.e. z/θ – see equations (8) and (9). This feature results from the substitutability between child and unskilled labour.

On the other hand, when parents choose skilled work, educational and fertility choices strictly depend on the wage differential between parental and child labour.[11] In particular, when it increases, the optimum number of children declines and the time allocated to children's schooling increases because the relative importance of children's earnings declines.[12]

3 General Equilibrium: Short Run

The last result highlights the emergence of a marked asymmetry between agents who offer skilled and unskilled work. For the sake of argument, we assume that in the initial period, t, the population is divided into two groups which are endowed with two different levels of human capital, a low level of human capital h_t^u – the low-skilled workers – and a high level h_t^s – the high-skilled workers. We show that this difference may persist across generations.

[11] If we introduce a direct cost of schooling, the threshold level of parental income below which parents send their children to work will rise, and hence the incidence of child labour will rise. Although such an assumption may be more realistic, its introduction would mean explicitly introducing a sector for education since we want to endogenously determine the level of wages in a general equilibrium framework.

[12] According to the existing literature, the model shows a trade-off between quantity and quality of children. See, for instance, Hazan and Berdugo (2002) for an analysis of a similar model under constant wages.

As we pointed out above, an important feature of this framework is that, if $w_t^s h_t^u < w_t^u$, low-skilled workers choose to work as unskilled, while if $w_t^s h_t^u > w_t^u$ they would prefer to work as skilled. Thus, given perfect mobility of labour, at equilibrium the wage ratio must satisfy $w_t^s h_t^u \leq w_t^u$; otherwise all the labour force would offer skilled labour, which is not possible given equation (2). A similar argument applies to high-skilled workers; thus $w_t^s h_t^s \geq w_t^u$. Therefore, for any $h_t^u \leq h_t^s$, at equilibrium

$$h_t^u \leq \frac{w_t^u}{w_t^s} \leq h_t^s. \tag{11}$$

From equations (10) and (11), it holds that

$$r_t^u = 1, \tag{12}$$

and

$$1 \leq r_t^s \leq \frac{h_t^s}{h_t^u}, \tag{13}$$

for all $t \in \mathbb{N}_0$. Thus choices of education and fertility of the two groups can be obtained substituting $r_t^u = 1$ and r_t^s in equations (8) and (9) respectively.

Inequality (11) points out that three distinct regimes arise in this framework. Two regimes are corner solutions. If at equilibrium $w_t^s h_t^s = w_t^u$, high-skilled workers are indifferent to doing skilled or unskilled work. On the other hand, if at equilibrium $w_t^s h_t^u = w_t^u$, a fraction of low-skilled workers work as skilled. In the other case, when $h_t^u < \frac{w_t^u}{w_t^s} < h_t^s$, the low-skilled only work as unskilled and high-skilled only as skilled.

It is worth pointing out that if in a certain period t, market equilibrium implies $w_t^s h_t^s = w_t^u$, in period $t+1$ there will be no difference between low- and high-skilled workers, since all the population gets the same adult income and makes the same schooling and fertility decisions. This argument does not apply when $w_t^s h_t^u = w_t^u$, since in that case high-skilled workers get a higher income equal to $w_t^s h_t^s$ which is greater than w_t^u if $h_t^s > h_t^u$.

In what follows we provide the equilibrium characterization in the short run in the two economic regimes in question, – i.e. $w_t^s h_t^u < w_t^u$ and $w_t^s h_t^u = w_t^u$. Then, in Section 4, we investigate the long-run dynamics of the system. The main result of this analysis is that, starting from $w_t^s h_t^u < w_t^u$, the inequality of the economy will increase and the system will move towards the regime $w_t^s h_t^u = w_t^u$.[13]

3.1 Internal equilibrium

Let us assume that in period t, $1 < r_t^s < \frac{h_t^s}{h_t^u}$. As we pointed out above, under this condition low-skilled workers find it convenient to work as unskilled and high-skilled as skilled. Thus, at equilibrium – if it exists – the economy is characterized by two classes of income ($w_t^s h_t^s > w_t^u$), which make different fertility and schooling decisions – see equations (8) and (9).

Thus the aggregate demand is

$$D_t = c_t^u N_t^u + c_t^s N_t^s, \tag{14}$$

where N_t^u and N_t^s are, respectively, the number of low- and high-skilled agents, and from equations (2), (3) and (7),

$$c_t^u = \alpha\psi(1-\mu)s_t^{\mu}, \tag{15}$$

$$c_t^s = \alpha h_t^s \psi \mu s_t^{\mu-1}. \tag{16}$$

At time t, the supply of unskilled labour is given by the labour supplied by low-skilled adults, i.e. $(1 - zn_t^u)N_t^u$, plus the labour supplied by the children of low- and high-skilled parents, i.e. $(1-e_t^u)n_t^u N_t^u$ and $(1-e_t^s)n_t^s N_t^s$. At equilibrium this supply must be equal to the total demand of unskilled labour, L_t, that is,

$$L_t = (1 - zn_t^u)N_t^u + \theta[(1-e_t^u)n_t^u N_t^u + (1-e_t^s)n_t^s N_t^s]. \tag{17}$$

Moreover, the supply of skilled labour must be equal to the demand for skilled labour H_t, that is

$$H_t = (1 - zn_t^s)h_t^s N_t^s. \tag{18}$$

[13] As will be clearer later, the presence of a fertility differential drives this result. See, for instance, De la Croix and Doepke (2003, 2004).

From equations (1), (14), (15) and (16), the equilibrium in the goods market yields

$$L_t = \alpha \frac{(1-\mu)s_t N^u + \mu h^s N^s}{s_t}. \tag{19}$$

The ratio between equations (18) and (19) defines the equilibrium level of s_t

$$s_t^* = \frac{h_t^s}{1-\mu}\left[\frac{1-zn_t^s}{\alpha} - \mu\right]\frac{1}{x_t}, \tag{20}$$

where $x_t \equiv N_t^u/N_t^s$. Note that in period t, s_t^* depends only on the choice of n_t^s.[14]

The other variables N_t^s, N_t^u and h_t^s depend on choices made in period $t-1$. In order to understand the relation between s_t^* and n_t^s it is convenient to rewrite r_t^s. From equations (2), (3) and (20), we obtain:

$$r_t^s = \frac{w_t^s h_t^s}{w_t^u} = \frac{\mu\alpha x_t}{1 - zn_t^s - \mu\alpha}, \tag{21}$$

which depends only on n_t^s. This function takes different values according to the value of r_t^s. Thus, at equilibrium

$$r_t^{s*} = \begin{cases} \frac{2\theta\alpha\mu x_t}{\theta(1-\alpha\mu)+z\alpha\mu x_t - \sqrt{\Delta_1(x_t)}} & \text{if} \quad x_t \leq x_2 \\ \frac{2\theta(1+b)\alpha\mu x_t}{\theta(1+b)(1-\alpha\mu)+z\alpha\mu x_t - \sqrt{\Delta_2(x_t)}} & \text{if} \quad x_2 \leq x_t \leq x_3 \\ \frac{\mu}{1-\mu}x_t & \text{if} \quad x_3 \leq x_t \end{cases} \tag{22}$$

where $\Delta_1(x_t) = [z\alpha\mu x_t - \theta(1-\alpha\mu)]^2 + 4\theta(1-\alpha)z\alpha\mu x_t$ and $\Delta_2(x_t) = [z\alpha\mu x_t - \theta(1+b)(1-\alpha\mu)]^2 + 4\theta(1+b)(1-\beta)(1-\alpha)z\alpha\mu x_t$; $x_2 = \frac{\theta(b+\beta)[b\alpha(1-\mu)-\beta(1-\alpha)]}{z\alpha\mu\beta b}$, $x_3 = \frac{\theta(1-\mu)(1+b)}{\mu\beta z}$.

[14]The fact that the ratio s^* does not depend on education and fertility choices of unskilled is an implication of the trade-off between quantity and quality of offspring in the utility of the parents and of the perfect substitutability between children and unskilled labour (see Dessy, 2000). A higher (lower) labour supply on the part of low-skilled parents is induced by a decline (increase) in fertility which exactly offsets the lost of labour supplied by their children.

Note that the equilibrium value of r_t^{s*} depends only on the ratio between the number of low- and high-skilled workers. Moreover, in an internal equilibrium it must hold that $1 < r_t^{s*} < \frac{h_t^s}{h_t^u}$. Thus for some values of x_t there may be no internal solution. Figure 2 clarifies this result. The function r_t^{s*} is a piecewise function defined in the interval $\underline{x} \leq x_t \leq \bar{x}$ – that implies $1 < r_t^{s*} < \frac{h_t^s}{h_t^u}$ – where an internal equilibrium always exists.[15] In the case presented in Figure 2, as long as x_t increases r_t^{s*} becomes equal to $\frac{h_t^s}{h_t^u}$ before reaching the level $\frac{\theta(1+b)}{\beta z}$, that is the level which ensures $e_t^s = 1$. In Appendix A we show that the derivative of r_t^{s*} with respect of x_t is always positive.

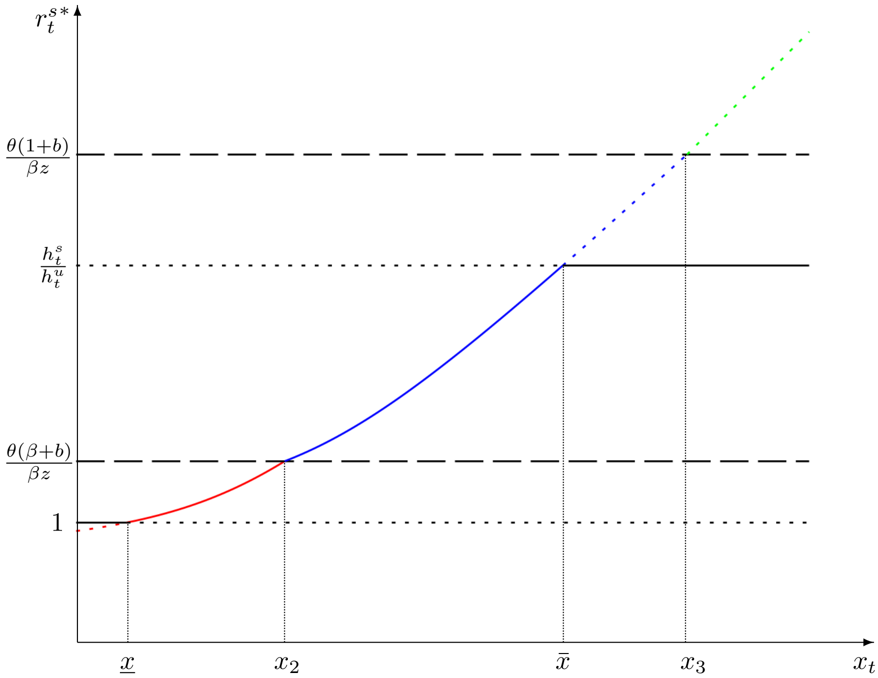


Figure 2: A numerical illustration of r_t^{s*} as a function of x_t. An internal equilibrium exists if, and only if, $\underline{x} \leq x_t \leq \bar{x}$. Value of parameters: $\alpha = 0.9$; $\mu = 0.3$; $z := 0.3$; $\theta = 0.25$; $\beta = 0.4$; $b = 0.2$; $a = 1$.

Given r_t^{s*}, it is easy to get the equilibrium values for all the other variables of the model. Note, that for $x_t \leq x_2$ high- (and low-) skilled workers do not invest in education ($e_t^s = 0$), for $x_2 < x_t < x_3$ high-skilled workers send their

[15] The boundaries $\underline{x}$ and $\bar{x}$ can cross the function r_t^{s*} in each of the three intervals depending on the values of parameters. Thus many different cases may arise, but such an analysis does not give much insight.

children to work and to school ($0 < e_t^s < 1$), while for $x_3 \leq x_t$ they send their children only to school ($e_t^s = 1$).

3.2 Corner solutions

The previous analysis shows that the equilibrium only depends on the level of x_t, that is the ratio between low- and high-skilled workers. If this ratio is smaller than $\underline{x}$, the number of high-skilled is so large, given the available technology, that the wage of the high-skilled equals the wage of the unskilled ($w_t^s h_t^s = w_t^u$). This means that for any $0 \leq x_t \leq \underline{x}$, $r_t^u = r_t^{s*} = 1$ (see Figure 2).

On the other hand, if $x_t \geq \bar{x}$, there are so few high-skilled workers with respect to low-skilled, given the available technology, that the efficiency wage (w_t^s) is high enough to allow low-skilled to work as skilled, getting the same wage as unskilled workers, i.e. $w_t^s h_t^u = w_t^u$. Thus $r_t^{s*} = \frac{h_t^s}{h_t^u}$.

From equations (2) and (3), the condition $w_t^s h_t^u = w_t^u$ implies that

$$s_t = \frac{\mu h_t^u}{1-\mu}. \tag{23}$$

Furthermore, given that $r_t^{s*} = \frac{h_t^s}{h_t^u}$, the choices of fertility and education are given in period t. Let ϕ be the fraction of low-skilled adults that work as unskilled and $1 - \phi$ the fraction of low-skilled adults that work as skilled. Thus, at any period, the equilibrium conditions in the two labour markets require

$$L_t = (1 - zn_t^u)\phi_t N_t^u + \theta[(1 - e_t^u)n_t^u N_t^u + (1 - e_t^s)n_t^s N_t^s], \tag{24}$$

$$H_t = (1 - zn_t^s)h_t^s N_t^s + (1 - zn_t^u)(1 - \phi_t)h_t^u N_t^u. \tag{25}$$

From equations (24) and (25), there will be only one value of ϕ which satisfies equation (23), that is:

$$\phi_t^* = \frac{(1-\mu)[(1 - zn_t^u)x_t h_t^u + (1 - zn_t^s)h_t^s] - \mu h_t^u \theta[(1 - e_t^u)n_t^u x_t + (1 - e_t^s)n_t^s]}{(1 - zn_t^u)x_t h_t^u} \tag{26}$$

Hence, the equilibrium values for all the variables of the model are determined.

4 Long-Run Dynamics

Fertility choices of the two groups affect the relative size of high and low-skilled workers. In the long run, the fertility differential is crucial in determining the dynamics of the wage ratio, and hence the dynamics of human capital.

Since $N^u_{t+1} = n^u_t N^u_t$ and $N^s_{t+1} = n^s_t N^s_t$, the population dynamics is given by

$$N_{t+1} = n^u_t N^u_t + n^s_t N^s_t. \tag{27}$$

Thus the dynamics of $x_t \equiv N^u_t / N^s_t$ is given by

$$x_{t+1} = \frac{n^u_t}{n^{s*}_t} x_t, \tag{28}$$

Note that since both n^u_t and n^{s*}_t are decreasing functions of income, $n^u_t \geq n^{s*}_t$.

The long-run equilibrium may be defined as a trajectory in which individual choices do not change over time. Since choices at any period t are affected by income, in a long-run equilibrium the wage ratio must be constant, which means that there must be a constant proportion of skilled and unskilled labour. However, as long as there is inequality in the economy (the income of high-skilled is higher than the income of low-skilled), the fertility choices between the two groups are different and the ratio x_t changes over time. This implies that any short-run internal equilibrium cannot be a long-run equilibrium.

There are three limiting cases, where inequality disappears after the first period. First, if, $\frac{z}{\theta} \geq \frac{(1+b)}{\beta}$, the relative cost of child-raising is so high that low-skilled workers choose the minimum family size and send their children only to school. The fertility choices of high and low-skilled workers are the same and all the population will be characterized by the maximum level of human capital. Second, if, $\frac{z}{\theta} \leq \frac{(\beta+b)}{\beta}$ and in period $t = 0$, $x_0 \leq x_2$, high-skilled workers send their children to work, since the relative cost of child-raising is low with respect to the return ofnhuman capital accumulation. Thus the next generation will have the minimum level of human capital: the differences between the two classes disappear. Third, if in period $t = 0$, $x_0 \leq \underline{x}$, high-skilled workers get a level of wage equal to w_0^u. Also in this case, their choice of fertility and education are the same as that of low-skilled workers.

Beside the above cases, the economy follows a transition with increasing inequality. For any $\max\{\underline{x}, x_2\} < x_t < \bar{x}$, the fertility of high-skilled workers is permanently lower than the fertility of low-skilled workers. Thus, from equation (28), during the transition x_t increases over time. This change directly affects the wage ratio since the supply of unskilled labour increases more than the supply of skilled labour. However, the increase in x_t induces an increase in r_t^{s*} (see Figure 2), which in turns brings about an increase in the human capital of children of high-skilled parents, i.e. h_{t+1}^s. Both those consequences favour the group of high-skilled workers.

This process generates a continuous increase in the inequality and an increase in child labour, since generation by generation the fraction of high-skilled workers decreases and becomes richer whereas the fraction of low-skilled workers increases and becomes poorer.

Since the descendants of the high-skilled group cannot obtain an income lower than their parents, thus, the choice of education cannot diminish. In other words, the human capital of high-skilled workers tends to increase over time. The increase in h_t^s leads to an increase in $\bar{x}$, which may allow the dynamics of human capital to reach its maximum level. However, since the accumulation of human capital is bounded, the continuous increase in x_t implies that in a certain time period, for instance $t = \tilde{t}$, the population ratio x_t reaches the threshold level $\bar{x}$ at which the wage ratio becomes constant, i.e. $w_t^u / w_t^s = h_t^u$ (see Figure 2). This implies that in the time interval $t > \tilde{t}$ low-skilled workers are indifferent between working as skilled and unskilled and the proportion of skilled and unskilled workers will be constant to maintain a constant wage ratio.[16] We summarize the above results in the following proposition.

Proposition 4.1. *The economy admits only one equilibrium with inequality. Thus equilibrium will be reached if, and only if, at the initial period* $t = 0$

i. $\frac{(\beta+b)}{\beta} < \frac{z}{\theta} < \frac{(1+b)}{\beta}$ *and* $x_0 > \underline{x}$*; or*

ii. $\frac{(\beta+b)}{\beta} \geq \frac{z}{\theta}$ *and* $x_0 > \max\{\underline{x}, x_2\}$.

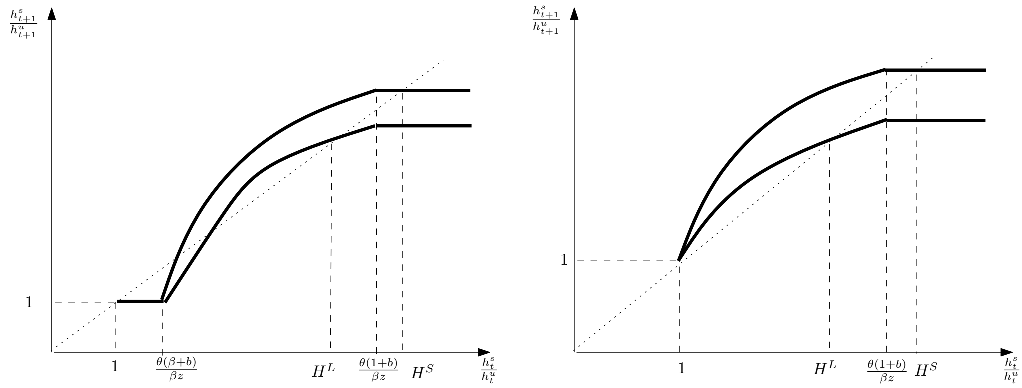
Furthermore, such long-run equilibrium is characterized by the condition $w_t^s h_t^u = w_t^u \ \forall \ t$.

The fact that the long-run equilibrium with inequality implies that $w_t^s h_t^u = w_t^u$, does not mean that the inverse implication is true. Let us assume that in period $t = \tilde{t}$ the population ratio $x_{\tilde{t}}$ reaches the threshold level $\bar{x}$. A fraction $1 - \phi_{\tilde{t}}^*$ of low-skilled workers starts to work as skilled, guaranteeing that $w_{\tilde{t}}^s h_{\tilde{t}}^u = w_{\tilde{t}}^u$ and therefore $r_{\tilde{t}}^{s*} = h_{\tilde{t}}^s / h_{\tilde{t}}^u$. Since in period $\tilde{t}-1$, $w_{\tilde{t}-1}^s h_{\tilde{t}-1}^u < w_{\tilde{t}-1}^u$, in period $t = \tilde{t}$ high-skilled workers get an income higher than in period $\tilde{t}-1$. Hence, if high-skilled workers already chose $e_{\tilde{t}-1}^s = 1$ their choices of fertility and education do not change in period $\tilde{t}$ and the equilibrium is instantaneously reached. On the contrary, if $e_{\tilde{t}-1}^s < 1$, their choices of education in period $\tilde{t}$ will increase. However, when $w_t^s h_t^u = w_t^u$, wages are constant and therefore the dynamic of $r_{t+1}^s = \frac{h_{t+1}^s}{h_{t+1}^u}$ depends only on $r_t^s = \frac{h_t^s}{h_t^u}$.[17] We provide a brief characterization of the dynamics of $r_{t+1}^s = f(r_t^s)$ that will be useful for the analysis of child labour regulation policies.

Figure 3 provides a graphical representation of the dynamics of r_{t+1}^s. Given the parameter values, the equilibrium choice can be obtained for $e^u = 0$ or $e^u = \frac{\beta z - \theta(\beta+b)}{\theta(1-\beta)} > 0$, Figure 3(a) and 3(b) respectively.

[16] This would mean that the share of low-skilled workers who work as skilled increases over time, i.e. ϕ^* decreases.

[17] We abstract from the case in which the increase in h^s is such that the increase in the supply of high-skilled work is sufficient to cover the demand for skilled labour. Even in this odd situation, the dynamics of the system will tend to re-establish the condition $w_t^s h_t^u = w_t^u$ without any change in the results.



(a) Case $e^u = 0$ (b) Case $e^u > 0$

Figure 3: The dynamic of $r^s_{t+1} \equiv \frac{h^s_{t+1}}{h^u_{t+1}} = f(r^s_t)$.

The equilibrium levels of r^s_{t+1}, H^L and H^S in the two figures, represent the long run position. Once the economy has reached the condition $w^s_t h^u_t = w^u_t$, r^s_{t+1} can follow one of the two curves in each case. Appendix B shows the complete characterization of the two cases, and in particular under what conditions the educational choice of the high-skilled group ends up as $e^s = 1$ (i.e. H^S) or as $e^s < 1$ (i.e. H^L).

When $e^u_t = 0$ – see Figure 3(a) – the economy converges to the equilibrium H^S if $\left(\frac{b}{1+b}\right)^\beta \frac{(1+b)}{\beta} \leq \frac{z}{\theta} \leq \frac{\beta+b}{\beta}$, and to the equilibrium H^L if $1 < \frac{z}{\theta} < \left(\frac{b}{1+b}\right)^\beta \frac{(1+b)}{\beta}$.[18]

When $e^u_t > 0$ – see Figure 3(b) – there is a value of $z/\theta = \zeta$ such that if $\frac{\beta+b}{\beta} < z/\theta \leq \zeta$ the economy converges to the equilibrium H^S, and to the equilibrium H^L if $\zeta < z/\theta < \frac{1+b}{\beta}$.[19]

It is worth pointing out that if the economy follows the path described in Proposition 4.1, when the condition $w^s h^u = w^u$ holds, the level of human capital of high-skilled workers cannot decrease. Indeed the income of the high-skilled descendants cannot be lower than that of their parents. The

[18] More precisely, there is a value of $\beta = \tilde{\beta} \in (0,1)$ such that, $\left(\frac{1+b}{b}\right)^\beta = \frac{1+b}{\beta+b}$. If $\beta > \tilde{\beta}$ equilibrium H^S can emerge. See Appendix B.1 for analytical details.

[19] More precisely, Appendix B.2 shows that according to some values of β, the economy converges to H^S or H^L for any value of z/θ, since ζ does not belong to the relevant interval. Additional restrictions on parameters ensures that H^L is greater than 1.

reason is straightforward: the only variable which may change is h^s which does not affect the condition $w^s h^u = w^u$. We show in the next section that child labour regulation policies when the economy is in the long-run equilibrium can instead bring about a regressive dynamics in the human capital of high-skilled workers.

To summarize, during the transition inequality rises since the presence of a fertility differential and child labour generate an increase in the return of human capital which is captured only by high-skilled workers. Such an increase leads the economy to converge to an equilibrium with inequality where low-skilled workers are indifferent to either skilled or unskilled labour. In that equilibrium the wages of the two groups in the population are constant. However, the continuous increase in the relative size of low-skilled workers and the constant income of the two groups tend to reduce the Gini coefficient, and hence to reduce the level of inequality in the economy.[20] Finally, asymptotically the level of inequality tends to zero, since the fraction of income taken by the low-skilled group tends to 1.

5 Child Labour Regulation Policies

In our model the presence of public education is not sufficient either to eradicate child labour or to reduce inequality. Indeed, as shown in the previous section, while for skilled parents the rise in the wage differential increases children's education across generations, low-skilled dynasties do not find any incentive to send their children to school.

In this section we analyze whether policies for child labour regulation (CLR) which impose restrictions on the child labour market, can induce low-skilled adults to increase education and therefore reduce inequality in the long run. According to Doepke and Zilibotti (2005), such a policy can be equivalent to reducing the productivity of child labour – i.e. θ. When $\theta = 0$,

[20]It is straightforward to compute the Gini coefficient when wages are constant – i.e. $w^u/w^s = h^u$. The level of inequality increases when $x < \sqrt{h^s/h^u}$ and starts to decline when $x \geq \sqrt{h^s/h^u}$.

there is a total ban on child labour. Thus, given the assumption of public education, low- and high-skilled parents would send their children only to school since utility increases in the level of human capital of children.[21]

We show, not surprisingly, that a CLR policy, if applied, significantly shapes the quality-quantity trade-off by inducing an increase in education and a reduction in fertility, and therefore a lower level of inequality in the long run. However, as will become clearer, CLR is likely to run into enforcement problems. Indeed, the general equilibrium analysis provided in Section 3 allows us to investigate how CLR policies changing education and fertility choices affect the wages of low- and high-skilled workers. Such variations affect the attainable level of utility of the two groups in the short run.

5.1 CLR Policies: Short-Run Effects

Since, the results of CLR policies on welfare in the short run substantially differ according to whether the economy is in the transition or in the long-run position, we analyze the two cases separately.

Transition. It is worth distinguishing the effects of a CLR policy between the case where high-skilled workers partially send their children to work – $e^s < 1$ – and where they send their children only to school – $e^s = 1$.[22] We can interpret the first case as an economy with low inequality since the wage differential between skilled and unskilled labour is low. On the other hand, the second scenario represents an economy with high inequality.

Consider, first, an economy with low inequality, i.e. $e^s < 1$ and $0 \geq e^u < 1$. A CLR policy tends to reduce the number of children of high-skilled adults and to increase their level of education. In Appendix C we show that the derivative of n^s with respect to θ is always positive. This change has an impact on the level of wages at equilibrium. Indeed, from equation (20), a

[21] A different policy often discussed in the literature concerns compulsory education – that is, a minimum level of education $\bar{e} > 0$ is introduced into the economy. Since the results are not very different, we investigate only CLR policies on θ.

[22] Since we are analysing the effects of CLR policies in the period, for the sake of simplicity we drop the time index.

reduction in n^s implies an increase in s^*. Furthermore, from equations (2) and (3) such a variation leads to an increase in w^u and a reduction in w^s.

A CLR policy has the same effect on low-skilled adults who tend to reduce their fertility and increase the level of education. However, their choices do not affect the equilibrium level of wages, since the reduction in child labour is exactly compensated by the increase in the labour supply of the parents given that zn^u decreases.

Proposition 5.1. *When $e^s < 1$ CLR policies induce a decline in the utility of high-skilled workers. Furthermore, there exists a level $\theta = \bar{\theta} > \frac{\beta z}{1+b}$ such that, for any $\theta < \bar{\theta}$, CLR policies increase the attainable level of utility for low-skilled workers.*

Proof. Let us assume that θ decreases. From Appendix C, we know that both w^s and θw^u decrease. The effect of this change on the budget constraint of high-skilled parents is unambiguous. Indeed from equation (6) we have

$$c^s \leq (1 - zn^s)w^s h^s + (1 - e^s)\theta w^u n^s \tag{29}$$

Given the changes in w^s and θw^u, the same basket of goods is no longer purchasable. Thus the attainable level of utility shifts down.[23]

For the low-skilled parents the result is different. The budget constraint for low-skilled is

$$c^u \leq (1 - zn^u)w^u + (1 - e^u)\theta w^u n^u. \tag{30}$$

Let us consider the right hand side. If low-skilled parents do not change their choices, the first term increases (since w^u increases), and the second decreases (since θw^u decreases). However as long as θ tends to $\frac{\beta z}{1+b}$, from equation (8) e^u tends to 1 and the second term tends to zero. Thus the negative effect of a decline in θ disappears. Therefore, if before the CLR policy θ was sufficiently

[23]It is probably more intuitive to consider an increase in θ. In this case, since both w^s and θw^u increase. If the adult takes the same decisions of education and fertility, she can increase the level of consumption inducing an increase in her utility. The optimal choice will guarantee a level of utility higher than this hypothetical situation. For the same reason if θ decreases, the utility in the optimum is lower than in the initial situation.

close to $\frac{\beta z}{1+b}$, after the policy the positive effect of w^u more than compensates for the reduction in θ.[24] This implies that, keeping the choices of n^u and e^u constant, a low-skilled agent may increase consumption by increasing his/her utility. □

If the productivity of child labour is low and $e^s < 1$, a CLR policy induces an increase in the welfare of the low-skilled and a decline in the welfare of the high-skilled.

Proposition 5.2. *When $e^s = 1$ CLR policies do not affect the welfare of high-skilled parents and bring about a decline in the attainable level of utility of low-skilled workers.*

Proof. In this case the choices of high-skilled parents do not depend on θ. Thus the s ratio does not change and wages are constant. Hence the level of utility of skilled parents is the same. On the contrary, the budget constraint for low-skilled is

$$c^u \leq (1 - zn^u)w^u + (1 - e^u)\theta w^u n^u. \tag{31}$$

Since the only change in the budget constraint is the reduction in θ, the cost of children increases, and as a consequence the same basket of goods is no longer purchasable. Thus, following the same argument as Proposition 5.1, the attainable level of utility of low-skilled parents decreases. □

If the economy has a high degree of inequality and high-skilled parents choose the maximum level of education, CLR policies induce a reduction in the welfare of low-skilled and leave the welfare of high-skilled unchanged.

Long-Run Equilibrium. As we pointed out in Section 4, the long-run equilibrium implies that $r^s = \frac{h^s}{h^u}$. In particular, since $s^* = \frac{\mu}{1-\mu}h^u$, whether $e^s = 1$ or $e^s < 1$, the wages in the economy do not change.

[24] The maximum attainable level of utility of low-skilled parents is a U shape function in θ. The minimum may also exceed $\frac{\beta z}{\beta+b}$. Hence the utility is always decreasing in θ in the relevant interval.

Nevertheless, the results are not the same in the two cases. Indeed, if a CLR policy is implemented and $e^s = 1$, as before, the utility of high-skilled is unaffected and that of low-skilled decreases. If instead $e^s < 1$, given inequality the attainable level of utility of high-skilled workers must decline.[25] Moreover, since the reduction of high-skilled fertility does not affect wages, also the utility of low-skilled will decrease.

To summarise, in the short run, a CLR policy cannot be welfare-improving. If we interpret the case in which $e^s < 1$ as a low inequality society, we find that if the degree of inequality is low the CLR policy may improve the welfare of low-skilled workers, while high-skilled workers are damaged. On the contrary, if the level of inequality is high, i.e. $e^s = 1$, the utility of high-skilled is unaffected while the welfare of low-skilled workers declines.

5.2 CLR Policies: Long-Run Effects

CLR policies have different effects in the long run according to whether the economy is in the transition or in the long-run position, i.e. whether or not the condition $w^u/w^s = h^u$ holds.

Transition. Let us assume that high-skilled parents choose the maximum level of education, $e^s = 1$, while $0 \leq e^u < 1$. Given that decision at period t, in period $t+1$ we know the supply of labour in the two markets and the equilibrium wages. In particular, if no child labour policy is implemented, the number of low-skilled adults from (9) is

$$N_{t+1}^u = n_t^u N_t^u = \max\left\{\frac{1-\alpha}{z-\theta}, \frac{(1-\alpha)(1-\beta)}{z-\theta(1+b)}\right\} N_t^u. \tag{32}$$

On the other hand, the number of high-skilled adults is

$$N_{t+1}^s = n_t^s N_t^s = \frac{1-\alpha}{z} N_t^s. \tag{33}$$

If, in $t+1$, $w^s h^u < w^u$, the supply of labour is again $L_{t+1} = \alpha N_{t+1}^u$ and

[25] The proof is equivalent to that of Proposition 5.2.

$H_{t+1} = \alpha h^s N^s_{t+1}$, then the ratio $s_{t+1} \equiv H_{t+1}/L_{t+1}$ is

$$s_{t+1} = \min\left\{\frac{z-\theta}{z}, \frac{z-\theta(1+b)}{z(1-\beta)}\right\}\frac{h^s N^s_t}{N^u_t}. \tag{34}$$

It is clear that $s_{t+1} < s_t$, hence $w^u_{t+1} < w^u_t$ and $w^s_{t+1} > w^s_t$. This is another way to show the long-run dynamics of the system discussed in Section 4.

The introduction of child labour regulation in period t affects the level of wages in $t+1$ and the level of inequality. Let $\hat{}$ be the variables at period $t+1$ if a policy against child labour is implemented. First, any reduction in θ generates an increase in the unskilled wage, that is $\hat{w}^u_{t+1} > w^u_{t+1}$, even if such a reduction in child productivity is not sufficient to induce a positive investment in education, that is if $\hat{\theta} \geq \frac{\beta z}{(\beta+b)}$, and $\hat{w}^s_{t+1} < w^s_{t+1}$.[26] Comparing the emergence of the long-run equilibrium in these two cases, we can state that the equilibrium will be the same, but the transition will be slower. Indeed, without any change in h^u the equilibrium condition $w^u = h^u w^s$ does not change, but will be reached slowly, i.e. for a $\hat{\tilde{t}} > \tilde{t}$.

If instead $\frac{\beta z}{(1+b)} < \hat{\theta} \leq \frac{\beta z}{(\beta+b)}$, then $\hat{e}^u > e^u$. As before, the policy tends to slow the divergence in the wage ratio, inducing a lower dynamic reduction in the wage of the unskilled and a lower dynamic increase in the wage of the skilled. It is easy to verify that the greater the reduction in θ the higher will be the wage of unskilled in period $t+1$. The increase in the level of education means that, in period $t+1$, $\hat{h}^u_{t+1} > h^u_{t+1}$. Thus, by comparing the long-run dynamics of the introduction of such a policy, the long-run equilibrium $w^u = \hat{h}^u w^s$ will be reached for a higher wage for the unskilled and a lower level of wage for the skilled. This means that in the long run even a low investment in education may have a significant effect on the level of inequality and poverty in the long run.

Finally, if $\hat{\theta} \leq \frac{\beta z}{(1+b)}$, the low-skilled stop sending their children to work. Thus, in the next period inequality in the economy disappears. However, given the result of the analysis of the short run, it is very difficult for a

[26]The proof is simple. Consider equation (34), if θ decreases, $\hat{s}_{t+1} > s_{t+1}$. From equation (2), since w^u is an increasing function of s the unskilled wage increases.

government to bring about such a change abruptly, since the welfare of the first generation will be strongly reduced.

When $e^s < 1$, the result of the CLR policy in the long run is not really different. The reduction in θ leads to an increase in high-skilled education and a reduction in fertility. In period $t+1$, such changes tend to increase the income of high-skilled parents. Thus the dynamics may converge to $w^u = \hat{h}^u w^s$ faster than without a policy. However, in the long-run equilibrium, inequality and poverty decrease.

long run equilibrium. When the economy is in the long-run equilibrium, CLR policies may have a strong impact on inequality. Let us consider Figure 3(b). A reduction in θ brings about two changes: the function r^s_{t+1} shifts down and the threshold $\frac{\theta(1+b)}{\beta z}$ shifts to the left. Thus, even if the long-run equilibrium is reached for $e^s = 1$, CLR policies may tend in the long run to reduce the level of human capital of the high-skilled.[27]

The intuition is simple. As long as θ decreases, low-skilled parents increase the level of education of their children. Since wages are constant, in period $t+1$, the increase in h^u_{t+1} tends to reduce r^s_{t+1}, thus with an opposite effect with respect to the reduction in θ. If the accumulation of human capital is sufficiently concave – $\beta < 1/2$ – the effect of h^u may more than compensate for the direct effect of the change in θ. More precisely, as shown in Appendix B.2, when $\beta < 1/2$, there is a value of θ, $\hat{\theta}$ such that in the new long-run equilibrium the choice of education and the income of the high-skilled group decrease. Furthermore, if θ is set sufficiently low, the inequality in the economy disappears, since the dynamics of r^s_{t+1} converges to 1. Nevertheless, in this case, low-skilled parents choose a high level of education which is close to 1.

[27]If CLR policies induce an increase in e^u even though $e^s < 1$, the ratio r^s_{t+1} decreases. Indeed,

$$\frac{\partial r^s_{t+1}}{\partial \theta} = \left(\frac{\beta z r^s_t - \theta\beta(1+b)}{\beta z - \theta\beta(1+b)}\right)^{\beta-1} \frac{z\beta^2(1+b)}{(1-\beta)^2\theta^2}(r^s_t - 1) > 0,$$

since $r^s_t > 1$.

6 Final Remarks

This paper is built on the idea that the persistence of child labour is strictly linked to the presence of inequality within the country. For this reason we presented a model where the population is divided into two groups endowed with two different levels of human capital and studied how this initial heterogeneity affected the distribution of income in the long run. The crucial result of this analysis is that the increase in the return of human capital is not sufficient to induce a transition to a high-skilled economy. The presence of two groups, with different levels of initial human capital, generates a continuous increase in the income of high-skilled workers with respect to those endowed with a low level of human capital. The presence of endogenous fertility induces the low income group to have more children. Thus, child labour will increase. The substitutability between adult and child labour increases the resilience of this result: the economy is trapped in an equilibrium with a high fraction of the population with low income and low human capital. In other words, there is a vicious cycles between child labour and inequality.

Moreover, we find that there is a long-run equilibrium with inequality where fertility and education choices do not change. In the long-run equilibrium a fraction of the low-skilled starts to work as skilled, ensuring market clearing and the stability of wages. However, the presence of differential fertility continues to increase the relative size of the low-skilled group, reducing the average level of human capital in the society. According to our findings, in the transition path inequality increases, while in the long-run equilibrium, after a certain threshold in the relative size of the two groups, it tends asymptotically to reduce.

Public policies may reduce the degree of child labour. We investigated how child labour regulation (CLR) policies which reduce the productivity of children in the labour market can induce a reduction in child labour and their effects on the welfare of the two groups in the short run and inequality in the long run. We find that in the short run, if inequality is low – that is high-skilled parents do not choose the maximum level of education – CLR policies reduce the welfare of high-skilled and can increase the welfare of low-skilled, if child labour productivity is low. On the contrary, if high-skilled parents choose the maximum level of education, CLR policies always reduce the welfare of the unskilled group and leave the welfare of high-skilled households unchanged.

In the long run, CLR policies slow the transition path to the long-run equilibrium, and increase the level of income of the unskilled dynasties whilst reducing that of the high-skilled. If the policy is implemented once the economy is in the long-run equilibrium, in order to reduce the degree of child labour significantly in the low-skilled group, the level of human capital of the high-skilled may decrease. This is due to the fact that, in the long-run equilibrium, educational choices of the high-skilled depend positively on the ratio of the level human capital between the two groups. If the level of human capital of the low-skilled increases, high-skilled households find it optimal to reduce the choice of education of their children and send them partially to work.

In conclusion, this study of the effects of CLR policies shows the emergence of various conflicts of interests between the low- and high-skilled group. Such friction may help explain the difficulties of many governments in dealing with the issue of child labour.

Appendices

A Fertility Differential and Transition

In this Appendix, we show that r_t^s is always an increasing function of x_t. In order to simplify the notation we denote: $A = 2\theta\alpha\mu$, $B = \theta(1-\alpha\mu)$, $C = z\alpha\mu$, $D = 4\theta(1-\alpha)z\alpha\mu$. Given this simplification we can rewrite equation (22) as follows

$$r_t^{s*} = \begin{cases} \frac{Ax_t}{B+Cx_t-\sqrt{(Cx_t-B)^2+Dx_t}} & \text{if } x_1 \leq x_t \leq x_2 \\ \frac{A(1+b)x_t}{B(1+b)+Cx_t-\sqrt{[Cx_t-B(1+b)]^2+D(1-\beta)x_t}} & \text{if } x_2 \leq x_t \leq x_3 \\ \frac{\mu x_t}{1-\mu} & \text{if } x_3 \leq x_t \leq x_4 \end{cases} \quad (35)$$

Thus the derivative of the first line in equation (35) w.r.t. x_t is positive if

$$2B\sqrt{(Cx_t - B)^2 + Dx_t} > 2B(Cx_t - B) - Dx_t. \tag{36}$$

By simplifying equation (36), we get

$$\alpha(1-\mu) > 0, \tag{37}$$

which always holds. The derivative of the second line in equation (35) w.r.t. x_t is positive if

$$2B(1+b)\sqrt{(Cx_t - B(1+b))^2 + D(1-\beta)x_t} > 2B(1+b)(Cx_t - B(1+b)) - D(1-\beta)x_t. \tag{38}$$

By simplifying equation (38), we get

$$b(1-\alpha\mu) + \alpha(1-\mu) + \beta(1-\alpha) > 0, \tag{39}$$

which always holds. Finally, it is straightforward to verify that the derivative of the third line in equation (35) w.r.t. x_t is always positive. Hence, r_t^s is always an increasing function of x_t.

B Long run Dynamics

B.1 Case $e^u = 0$

When $e_t^u = 0$ – i.e. $\frac{z}{\theta} \leq \frac{(\beta+b)}{\beta}$) – the dynamic of r_{t+1}^s is given by

$$r_{t+1}^s = \begin{cases} 1 & \text{if} \quad 1 \leq r_t^s \leq \frac{\theta(\beta+b)}{\beta z}, \\ \left[\frac{r_t^s \beta z - \theta\beta(1+b)}{\theta(1-\beta)b}\right]^\beta & \text{if} \quad \frac{\theta(\beta+b)}{\beta z} \leq r_t^s \leq \frac{\theta(1+b)}{\beta z}, \\ \left(\frac{1+b}{b}\right)^\beta & \text{if} \quad r_t^s \geq \frac{\theta(1+b)}{\beta z}. \end{cases} \tag{40}$$

Note, from Figure 3(a), that the economy converges to the equilibrium H^S if, for $r_t^s = \frac{\theta(1+b)}{\beta z}$,

$$r_{t+1}^s = \left(\frac{1+b}{b}\right)^\beta \geq \frac{\theta(1+b)}{\beta z}, \tag{41}$$

that is if

$$\frac{z}{\theta} \geq \left(\frac{b}{1+b}\right)^\beta \frac{(1+b)}{\beta}. \tag{42}$$

Since, when $e^u = 0$, $\frac{z}{\theta} \leq \frac{\beta+b}{\beta}$, the condition (42) can be satisfied only if

$$\left(\frac{1+b}{b}\right)^\beta > \frac{1+b}{\beta+b}. \tag{43}$$

The LHS of equation (43) is increasing in β, it takes the value 1 if $\beta = 0$ and the value $\left(\frac{1+b}{b}\right)$ if $\beta = 1$. The RHS is decreasing in β, it takes the value 1 if $\beta = 0$ and the value

$\left(\frac{1+b}{b}\right)$ if $\beta = 1$. Thus, there is a value of β, $\tilde{\beta} \in (0,1)$ such that if $\beta > \tilde{\beta}$, the economy converges to H^S if inequality (42) holds, while, if $\beta < \tilde{\beta}$ the economy converges to the equilibrium H^L for any value of $\frac{z}{\theta}$.

B.2 Case $e^u > 0$

When $e^u_t = \frac{\beta z - \theta(\beta+b)}{\theta(1-\beta)} > 0$,

$$r^s_{t+1} = \begin{cases} \left[\frac{r^s_t z - \theta(1+b)}{z - \theta(1+b)}\right]^\beta & \text{if} \quad 1 \leq r^s_t \leq \frac{\theta(1+b)}{\beta z}, \\ \left[\frac{(1+b)(1-\beta)\theta}{\beta z - \beta\theta(1+b)}\right]^\beta & \text{if} \quad r^s_t \geq \frac{\theta(1+b)}{\beta z}. \end{cases} \tag{44}$$

First, let us consider Figure 3(b). Note that the economy converges to the equilibrium H^S if, for $r^s_t = \frac{\theta(1+b)}{\beta z}$,

$$r^s_{t+1} = \left[\frac{(1+b)(1-\beta)\theta}{\beta z - \theta\beta(1+b)}\right]^\beta > \frac{\theta(1+b)}{\beta z}. \tag{45}$$

Let us define $\xi \equiv \frac{z}{\theta}$. It is useful to rewrite inequality (45) as

$$\left[\frac{\beta(\xi - 1 - b)}{(1+b)(1-\beta)}\right]^\beta < \frac{\xi\beta}{(1+b)}, \tag{46}$$

where the LHS is an increasing concave function in ξ, and the RHS is an increasing line. Given that $e^u > 0$, the relevant interval of ξ is $\frac{\beta+b}{\beta} < \xi < \frac{1+b}{\beta}$. Moreover, both the sides of inequality (46) assume value 1 at $\xi = \frac{1+b}{\beta}$. Thus, if the derivative of the LHS is greater than the derivative of the RHS w.r.t. ξ, in $\xi = \frac{1+b}{\beta}$, inequality (46) is always satisfied for any value of ξ and the economy converges to equilibrium H^S in Figure 3(b). That is

$$\left.\frac{\partial \left[\frac{\beta(\xi-1-b)}{(1+b)(1-\beta)}\right]^\beta}{\partial \xi}\right|_{\xi=\frac{1+b}{\beta}} = \frac{\beta^2}{(1+b)(1-\beta)} > \frac{\beta}{1+b}. \tag{47}$$

Hence if $\beta > \frac{1}{2}$ then (45) holds for any $\frac{\beta+b}{\beta} < \theta < \frac{1+b}{\beta}$.

If, on the contrary, the LHS of inequality (46) is greater than the RHS for $\xi = \frac{\beta+b}{\beta}$, then the economy converges to the equilibrium H^L for any value of ξ in the relevant interval. This happens when

$$\left(\frac{1+b}{b}\right)^\beta < \frac{1+b}{\beta+b} \tag{48}$$

Since the LHS and the RHS are the same as inequality (43), there is a value of $\beta = \tilde{\beta} \in (0,1)$ such that for every $\beta \leq \tilde{\beta}$ inequality (48) holds. Thus if $\beta \leq \tilde{\beta}$ for any value of ξ the economy converges to the equilibrium H^L.

In the case in which $\tilde{\beta} < \beta < \frac{1}{2}$, we find that there exists a threshold of ξ, i.e. $\zeta \in (\frac{\beta+b}{\beta}, \frac{1+b}{\beta})$, such that inequality (45) holds if, and only if, $\xi < \zeta$.

Finally, we may consider the case in which the function r^s_{t+1} is always below the 45° line. This happens if

$$\left.\frac{\partial r^s_{t+1}}{\partial r^s_t}\right|_{r^s_t=1} \leq 1, \tag{49}$$

We have that

$$\left.\frac{\partial r^s_{t+1}}{\partial r^s_t}\right|_{r^s_t=1} = \frac{\beta\xi}{\xi-(1+b)} \leq 1. \tag{50}$$

Thus (49) holds if

$$\xi \geq \frac{1+b}{1-\beta}. \tag{51}$$

It is possible that $\xi = \frac{1+b}{(1-\beta)}$ is not in the relevant interval $(\frac{\beta+b}{\beta}, \frac{1+b}{\beta})$. We find that if $\beta \geq 1/2$, $\frac{1+b}{1-\beta} \geq \frac{1+b}{\beta}$, thus $\frac{\partial r^s_{t+1}}{\partial r^s_t} > 1$ for every $\xi \in (\frac{\beta+b}{\beta}, \frac{1+b}{\beta})$. On the other hand, if $\beta = \hat{\beta} \leq (b^2+b)^{1/2} - b < 1/2$, $\frac{1+b}{1-\beta} \leq \frac{\beta+b}{\beta}$, thus $\frac{\partial r^s_{t+1}}{\partial r^s_t} \leq 1$ for every $\xi \in (\frac{\beta+b}{\beta}, \frac{1+b}{\beta})$.

When $\beta \in (\hat{\beta}, 1/2)$, $\frac{\partial r^s_{t+1}}{\partial r^s_t} \leq 1$ if, and only if, $\xi \in [\frac{1+b}{1-\beta}, \frac{1+b}{\beta})$.

C Effects of CLR policies on the short run

In this Appendix we prove that if $e^s < 1$ then any policy inducing a reduction in θ diminishes the level of utility of high-skilled agents.

Let us consider an economy in the short-run equilibrium characterized by $x_2 \leq x_t \leq x_3$. Given equations (9) and (22),

$$n^{s*} = \frac{(1-\alpha)(1-\beta)2\alpha\mu x_t}{z\alpha\mu x_t - \theta(1+b)(1-\alpha\mu) + \sqrt{\Delta_2(x_t)}}. \tag{52}$$

In this case $\frac{\partial n^{s*}}{\partial\theta} \geq 0$. Indeed

$$\frac{\partial n^{s*}}{\partial\theta} = \frac{2\alpha\mu x(1-\alpha)(1-\beta)[(1+b)(1-\alpha\mu) - \frac{1}{2}\Delta_2^{-\frac{1}{2}}\frac{\partial\Delta_2(x)}{\partial\theta}]}{[\alpha\mu z x - \theta(1+b)(1-\alpha) + \sqrt{\Delta_2}]^2}, \tag{53}$$

where

$$\Delta_2 = [z\alpha\mu x - \theta(1+b)(1-\alpha\mu)]^2 + 4\theta(1+b)(1-\beta)(1-\alpha)z\alpha\mu x. \tag{54}$$

In order to simplify the notation we define $A = z\alpha\mu x$, $B = (1+b)(1-\alpha\mu)$ and $C = (1+b)(1-\beta)(1-\alpha)$. Thus we can rewrite Δ_2 as follows:

$$\Delta_2 = (A-\theta B)^2 + 4\theta CA, \tag{55}$$

from which:

$$\frac{\partial\Delta_2}{\partial\theta} = -2B(A-\theta B) + 4CA, \tag{56}$$

Thus $\frac{\partial n^{s*}}{\partial\theta} \geq 0$ if:

$$B[(A-\theta B)^2+4\theta CA]^{1/2} > -B(A-\theta B)+2CA \tag{57}$$

which may be simplified as follows:

$$A(B-C)>0, \tag{58}$$

by substituting for B and A and simplifying yields:

$$\mu < \frac{1-(1-\alpha)(1-\beta)}{\alpha}. \tag{59}$$

which always holds since the right hand side is always greater than 1. Thus $\frac{\partial n^{s*}}{\partial \theta} > 0$.

Using $\frac{\partial n^{s*}}{\partial \theta} > 0$ we can show that $\frac{\partial \theta w^u}{\partial \theta} > 0$. From equation (9) we know that if $e^s < 1$ for a high-skilled workers:

$$n^s = \frac{(1-\alpha)(1-\beta)}{z-(1+b)\frac{\theta w^u}{w^s h^s}}. \tag{60}$$

since $\frac{\partial w^s}{\partial \theta} > 0$ then $\frac{\partial n^{s*}}{\partial \theta} > 0$ if and only if $\frac{\partial \theta w^u}{\partial \theta} > 0$.

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