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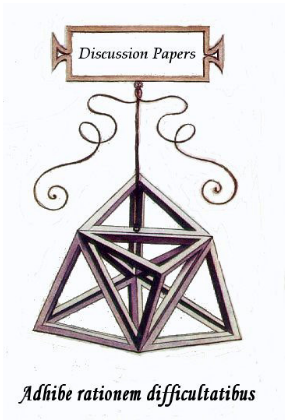

Davide Fiaschi e Tamara Fioroni

Transition to Modern Growth: the Role of Technological Progress and Adult Mortality

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Discussion Paper

n. 1

Davide Fiaschi - Tamara Fioroni

Transition to Modern Growth: the Role of Technological Progress and Adult Mortality

Abstract

This paper presents a model inspired by the Unified Growth Theory, where reductions in adult mortality together with improvements in technological progress are the deep causes of the transition from a Traditional (Malthusian) Regime to a Pre-Modern Regime, characterized by the accumulation of fixed capital only, and finally, to a Modern Regime, characterized by the joint accumulation of both fixed and human capital. A calibrated version of the model is able to reproduce the dynamics of the UK economy in the period 1541-1914, matching both the periods of transition and the pattern of main macroeconomic variables. UK growth before the mid-nineteenth century appears to be mainly due to technological progress, while thereafter, the decline in adult mortality and factors accumulation played the major role. Finally, fertility decline during the nineteenth century has only a marginal impact on growth because it is more than balanced by the increase in adult survival.

Classificazione JEL: O10, O40, I20

Keywords: Unified Growth Theory, Human Capital, Adult mortality, Nonlinear Dynamics, Endogenous Fertility, Industrial Revolution

Contents

I. Introduction

In literature there is no agreement on the main determinants of the extraordinary development in the last five centuries of Western economies and of the related phenomenon known as the "Great Divergence "(see Mokyr, 1999).

The aim of this paper is to discuss how reductions in (adult) mortality, together with improvements in technological progress, affecting the accumulation of human and fixed capital (and the dynamics of structural change), can be at the root of the long-run growth of Western countries, with the additional conjecture that the changes in both variables are driven by exogenous factors to the economy (we refer to Section II. for a detailed discussion on this point).

We propose a theoretical model in the spirit of the Unified Growth Theory proposed by Galor (2005), augmented by the presence of adult mortality, fixed (physical) capital accumulation, and two technologies differing in their inputs; one with only land and unskilled labor, the other with fixed (physical) and human capital[1], in order to match the observed structural change during the industrialization of countries (see also Hansen and Prescott, 2002 and Aghion and Howitt, 2009). We then discuss the ability of a calibrated version of the model to reproduce the dynamics of main macroeconomic variables (GDP per worker, investment rate, structural change in output composition, interest rate, investment in education, fertility rate) of the UK economy in the period 1541-1914, including the timing of the transition across different regimes (at the end of the eighteenth century for the transition from Malthusian to Pre-Modern Regime, and in the mid-nineteenth century for the transition from Pre-Modern to Modern Regime). Finally, we estimate the contribution of different factors (technological progress, adult mortality, accumulation of fixed and human capital, workforce) to the overall growth of the UK. The latter appears mainly due to technological progress before the mid-nineteenth century, while thereafter the decline in adult mortality and factors accumulation play the major role. The big change in fertility in the nineteenth century has a negligible impact on growth, because it appears more than balanced by the increase in adult mortality.

The overlapping generations model considers individuals potentially living for two periods (childhood and adulthood), with a subsistence consumption, and where the saving rate is an increasing function of wealth. Individuals arrive at adulthood with certainty and the risk of mortality occurs during adulthood. Individuals devote the first period of their life (childhood) to the acquisition of human capital (if any) and in the second period they allocate their income, given by the sum of their labor income and their (if any) bequest, between consumption and transfers to their offspring (their transfers are only positive when their income is above a certain threshold). The transfers (if positive) are invested in fixed capital and in the children's education in order to maximize the future income of children (see Galor and Moav (2004)).

An increase in adult survival has two opposite effects on transfers to the offspring: a

[1]Here we follow a classical distinction between working (circulating) capital and fixed capital, where the former consists in short-lived inputs (generally used for just one productive cycle such as raw materials), and the latter is related to those economic goods "repeatedly used in the course of a number of productive cycles "such as machinery (Cipolla, 1994, p. 80). The accelerated accumulation of fixed capital in terms of their "range and variety "is one of the most prominent phenomena of the Industrial Revolution (see, e.g., Hicks, 1969, p.142 and Cipolla, 1994, p. 80).

negative effect because it raises the lifetime consumption of the parents and a positive effect because it increases parents' income (via an increase in their labor income). [2] At low levels of income the first effect dominates, therefore an increase in adult survival leads to a reduction in the amount of resources available to future generations; the opposite occurs at high levels of income. Empirical evidence discussed in Cervellati and Sunde, 2011 (see, in particular, their Figure 5) on the U-shaped relationship between life expectancy and the growth rate of income supports this finding.[3]

The dynamic is characterized by three different regimes: (i) a *Traditional (Malthusian) Regime*, where output is produced only by a traditional technology, whose factors are unskilled labor and land; (ii) a *Pre-Modern Regime*, where an increasing share of aggregate output is produced using fixed capital and unskilled labor in an industrial sector; and, finally, (iii) a *Modern Regime* where both fixed and human capital are used in the industrial sector. The transition from the Traditional to the Pre-Modern, and, finally, to the Modern Regime can either be the result of an ongoing progress in the technological progress alone, or it can be the joint effect of an increase in technological progress *and* adult survival. On the other hand, at low levels of income (technological progress), a decrease in adult survival can alone help the transition from a Traditional to a Pre-Modern Regime (and viceversa).

The introduction of endogenous fertility does not substantially affect the main results of the paper, but just adds a possible self-reinforcing mechanism to the stability and transition from one regime to the other. The Traditional Regime assumes the typical characteristic of a Malthusian Regime (as it is denoted in the Unified Growth Theory), where increases in income are checked by increases in the population. Once a country escapes from the Malthusian Regime the decreasing fertility with respect to income further boosts the country's growth by increasing its accumulation of fixed and human capital. In the calibrated version of the model for the UK, the matching between simulated and observed fertility rate is the most problematic feature, suggesting that a more sophisticated theory is needed in line with the remark made by Easterlin (2004).

The paper is organized as follows: Section 2 discusses the related literature; Section 3 presents the model; Section 4 studies the transitions between regimes; Section 5 calibrates the model to the UK economy for the period 1541-1914; and Section 6 concludes.

[2] Many theoretical contributions find only a positive effect on growth of gains in life expectancy through various channels; see, for example, Cervellati and Sunde, 2005, Boucekkine et al., 2003, Soares, 2005, De la Croix and Licandro, 1999, Lagerlof, 2003, Weisdorf, 2004, and Bar and Leukhina, 2010). Some recent contributions argue for a positive effect on long-run growth of negative *shocks* on life expectancy (see Voigtlander and Voth (2013)).

[3] In particular, Cervellati and Sunde (2011) find a negative effect before the demographic transition (not necessarily related to the level of income), and a strong and positive effect on the next period. On the other hand, Acemoglu and Johnson (2007) argue that improvements in life expectancy, rising population growth, have a negative effect on GDP per capita. Our paper may help to reconcile these (apparently) conflicting results by adding some insight on the conditions under which improvements in life expectancy are good for the long-run growth of a country.

II. *Improvements in Technological Progress and the Decline in Adult Mortality*

The literature on deep explanation(s) for long-run growth of Western countries in the last 500 years is enormous and still increasing.

Following a long tradition in economics dating back to Joseph Schumpeter, Aghion and Howitt (2009) attribute the main explanatory factors for country growth to the economic incentives that stimulate innovation activity (such as the profits for innovators). Other scholars (see, e.g., Lucas, 1988), following a literature starting in 1960' (see, e.g., Cipolla, 1962), point to the accumulation of human capital. Moreover, other scholars point to the quality of institutions forged by environment and individual incentives (Acemoglu et al., 2001).[4] Finally, some authors relate the increases in the stock of knowledge available in the economy to the size of population, so that they identify a direct causal relationship between the size (or the growth rate) of the population and technological progress (see Kremer, 1993 and Galor, 2005). These explanations can also be seen as complementary and applicable to different periods and time scale of analysis (e.g., the Schumpterian theory applied to post First World War, the theory of knowledge accumulation applied to the very long run).[5] Common to this literature is however the search for an *endogenous* (mainly economic) explanation for countries' growth.

In this paper we take a different route, and consider the improvements in technological progress and adult survival as the result of some factors *exogenous* to the economic sphere. We argue that this approach is the most suited to our objective of explaining the transition of European countries, in particular the UK, through different regimes of growth from 1541 to 1914. As we will discuss below, the debate looks at the key source(s) of the Industrial Revolution and of the decline of adult mortality during the second half of the nineteenth century.[6]

The dynamic of European economies after 1914 is beyond the scope of the paper because they are probably related to a more sophisticated theory of innovation, accumulation of human capital, and of economic factors affecting the countries' health sector (in particular, the impact of public health systems).

II.A. Improvements in Technology Progress

In the paper we adhere to the idea that the Industrial Revolution is the result of the **cultural revolution** during the 17th century (Koyre, 1961, Cipolla, 1994, Rosenberg, 1994, Mokyr, 2002, Mokyr, 2010, Jacob and Stewart, 2004).

Starting from the perspective that "... the origins of the Industrial Revolution reach

[4]For example, Acemoglu et al. (2001) argue that the lower settler mortality in North America with respect to Center and Southern America has led to institutions being more favourable to investment.

[5]But the prevalence of one or the other has crucially different policy implications. For example, if the quality of institutions is the key factor of development, then the adoption of Western institutions (e.g. democracy) is the main policy recommendation to poor countries; in a different way, if human capital is crucial attention should be on all the factors favoring the accumulation of human capital (e.g., public expenditure in education). Finally, if incentives to innovate is crucial an efficient patent law system is needed.

[6]Mokyr (1999) provides an extensive survey of the debate on the causes and nature of the Industrial Revolution, while a survey on the causes of the decline in mortality can be found in Livi Bacci (2007).

back to that profound change in ideas, social structures, and value systems ..."(see Cipolla, 1994, p. 227) of the last part of the Middle Ages, we agree with Mokyr (2002) that the Industrial Revolution is mainly the result of a cultural revolution caused by the emergence of the new scientific method elaborated in the 17th century which particularly permeated English society during the 18th and 19th centuries : "... the interconnections between the Industrial Revolution and those parts of Enlightenment movement that sought to rationalize and spread knowledge may have played a more important role than recent writings have given them credit for ... This would explain the timing of Industrial Revolution following the Enlightenment and - equally important - why it did not fizzle out like similar bursts of macroinvention in earlier times. It might also help explain why the Industrial Revolution takes place in western Europe ... "(see Mokyr, 2002, p. 29). The same conclusion is present in Jacob and Stewart (2004), who highlight the importance of the change in approach of individuals to scientific knowledge, which "becomes a centerpiece of Western culture, a partner with industry, ..." (see Jacob and Stewart, 2004, p. 8).

We are aware that other *more materialistic explanations* have been proposed (see, e.g, Cipolla, 1994, and Allen, 2009), but *discontinuity* is the major challenge for all these explanations (Clark, 2007, p. 228).

Finally, Solow (2000, p. 97) discusses the pros and cons of the theories which aim to endogenize technological progress in contrast to the hypothesis of exogenous technological progress at the root of the traditional Solovian growth model. In particular, he argues how the development of a general and convincing framework is still to be reached, stressing how it is very reasonable to consider changes in technological progress as exogenous but not necessarily constant over time, and highlighting the importance of analyzing how the economy reacts to these changes. These same issues are also discussed in (Acemoglu, 2008, p. 414), where the issue is posed in terms "whether innovation is mainly determined by scientific constraints and stimulated by scientific breakthroughs ... or whether is, at least in part, driven by profit motives". Acemoglu concludes in favor of a role of profits in innovation (at least for developed countries after the Second World War).[7]

II.B. The Decline in Mortality

In accordance with our perspective on technological progress, decline in mortality beginning in the mid-nineteenth century is assumed to have the same exogenous root of the *Industrial Revolution* (see Mokyr, 1991 and 2010; Preston, 1975 and 1996; Easterlin, 2004; Livi Bacci, 2007; Ljungberg, 2013, Deaton, 2006). In particular, in the words of some of the most important scholars in the field "What the Mortality and Industrial Revolutions have in common is that they are both manifestations of the explosion in empirically based human knowledge, scientific and technological, that dates from the seventeenth century onward "(see Easterlin, 2004, p. 97); and, more precisely, "... the essential element in the gains was an enormous scientific breakthrough - the germ theory of disease"(see Preston, 1996, p. 6). Therefore, it is possible to conclude that "[The dominant factors of] The mortality decline of the period since 1850 ... probably include social and cultural factors

[7]From another point of view McCloskey (1995) discusses how the attempt by many economists to endogenize technological progress can be justified not in the empirical evidence in favor of such possibility, but in the current way of conducting research in the field of economics.

(methods of child rearing, personal hygiene, improved organization of markets, and so forth) in the first phase of transition ... " (see Livi Bacci, 2007, p. 124).

The sources of mortality decline have been extensively debated by historical demographers, historians of medicine and economic historians (see Schofield et al., 1991). According to McKeown (1976), the principal cause of mortality decline in England from 1838 to the current day was modern economic growth which, by increasing living standards, and particularly the nutritional level of the population, inevitably increased resistance of the population to infectious diseases.

However, subsequent research has shown empirical evidence which contradicts McKeown's theory. As Livi Bacci (2007) asserts "This theory is countered by a number of considerations which make us look to other causes. In the first place, the link between nutrition and resistance to infection holds primarily in cases of severe malnutrition; and while these were frequent during periods of want, in normal years the diet of European populations seems to have been adequate. Second the latter half of the eighteenth century and the first decade of nineteenth, the period during which this mortality transition began, do not appear to have been such a fortunate epoch."(see Livi Bacci (2007), p.71). Moreover, Livi Bacci (2007) argues that the increase in longevity was caused principally by a reduction in young and infant mortality which occurred "not because of better nutrition, but because of improved child-rearing methods and better protection from the surrounding environment."(see Livi Bacci (2007) p.71).

In his 2004 book Robert Fogel emphasizes the strict relationship between better nutrition and mortality reduction; however, he finds evidence of a very limited or even opposing relationship between economic growth and improvement in nutritional status and health during the eighteenth and nineteenth centuries in both Europe and America[8]. In particular, Fogel points out that "The overall improvement in health and longevity during this period is less than might be expected from the rapid increases in per capita income indicated by national income accounts for most of the countries in question. More puzzling are the decades of sharp decline in height and life expectancy, some of which occurred during eras of undeniably vigorous economic growth. "(see Fogel (2004) p. 18).

With respect to more recent years, i.e. from 1900 to 1960, Preston (1975) finds that economic growth explains only $10-25$ percent of the increase in life expectancy whereas the remaining $75-90$ per cent of the growth in life expectancy is attributable to factors exogenous to countries' levels of income. In particular, he emphasizes the crucial role of the widespread diffusion of medical innovation in reducing mortality: "It seems to have been predominantly broad-gauged public health programmes of insect control, environmental sanitation, health education and maternal and child health services that transformed the mortality picture in less developed areas, while it was primarily specific vaccines, antibiotics and sulphonamides in more developed areas. But the technologies were not, for the most part, indigenously developed by countries in either group. Universal values assured that health breakthroughs in any country would spread rapidly to all others where the means for implementation existed. The importance of exogenous, largely imported, health technology in the now-developed countries may have been underestimated

[8]See also Deaton (2006) "In Britain, the United States, and much of Europe, there were periods in the nineteenth century when urbanization ran ahead of the rate of public health provision and population health deteriorated during periods of rapid economic growth."(Deaton, 2006, p. 111)

for earlier periods as well."(see Preston, 1975, p. 243).

Fig.1 supports Preston's conclusion that factors not related to income explain the rise in life expectancy. The left panel of Fig.1 shows that the same level of GDP per capita is associated with different levels of life expectancy, while the right panel of Fig. 1 shows that, in a given year, the differences in life expectancy across countries are low.

The left panel of Fig. 1 shows a marked increase in life expectancy from the end of the nineteenth century, that is the period in which, according to Easterlin (2004), the so-called Mortality Revolution started[9]. In particular Easterlin (2004) asserts "Since 1870, life expectancy at birth in many areas of the world has soared from values around 40 years or less to 70 years or more. The reduction in mortality has been accompanied by an associated improvement in health as the incidence of contagious disease has dramatically lessened. This lengthening of life and associated reduction in morbidity brought about by the Mortality Revolution has meant at least as much for human welfare as the improvement in living levels due to modern economic growth. Certainly the Mortality Revolution has substantially affected a much wider segment of the world's population. "(see Easterlin, 2004, p.84). Thus, Easterlin (2004) argues that the rise in life expectancy principally depends on the emergence and increasing importance of medical and technological innovations (see Easterlin, 2004, p. 86).[10]

III. The Model

The model is inspired by Galor and Moav (2004). Consider an economy populated by overlapping generations of people who potentially live for two periods: childhood and adulthood. They certainly live in childhood but are subject to a risk of dying during adulthood. Denote the expected length of adulthood in period t by $p_t \in (0,1)$, the total number of adults at the beginning of period t by L_t, and therefore the *actual aggregate labor supply* in the period t is equal to $p_t L_t$.

III.A. Production

In every period, the economy produces a single material good, whose price is normalized to 1. Production may take place using two different technologies: a traditional technology that employs unskilled labor and land, and an industrial technology that employs fixed capital and skilled labor. While the traditional technology is always operating, the industrial technology, as we shall see below, will become available once technology has sufficiently progressed (for the production structure we follow Aghion and Howitt, 2009). The traditional production function is given by:

$$Y_t^a = A_t^a (p_t L_t^a)^{1-\lambda} (T)^{\lambda}, \tag{1}$$

[9]The uniqueness of the development of scientific medicine in the nineteenth and twentieth centuries is also well documented in the history of medicine as, for example, by Dixon (1978), Watts (2003) and Porter (2006).

[10]In particular, he identifies three major breakthroughs which bring about mortality reduction: 1) new methods of preventing the transmission of disease, including clean water supply and education in personal hygiene; 2) new vaccines to prevent certain diseases which started in the 1890's; and 3) new drugs to cure infectious diseases (antimicrobials) which started in the late 1930's (see Easterlin, 2004, p. 104 and also Deaton, 2006, p. 110).

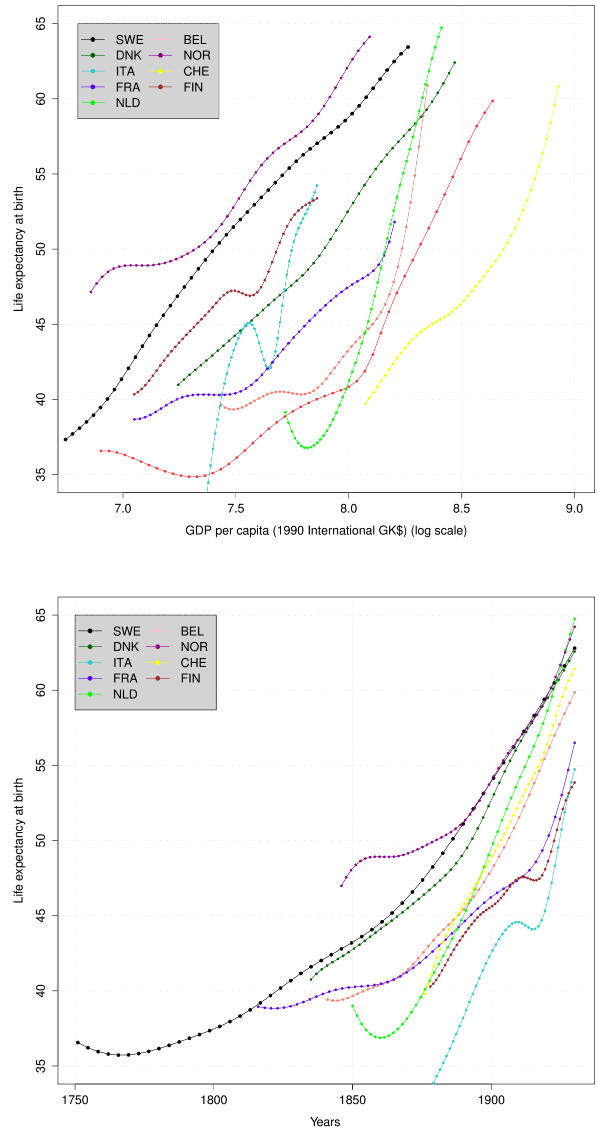


Figure 1: Life expectancy at birth versus GDP per capita and over time (1750-1930). Non-parametric kernel smoother. Sources: GDP per capita from Maddison Project Database (MPD) (update 02/2013); life expectancy at birth from the Human Mortality Database (HMD) (update 02/2013).

where $\lambda \in (0,1)$, A^a is a productivity parameter, pL_t^a is the actual amount of unskilled labor employed in the traditional sector in the period t and T is the quantity of land. The industrial production function is given by:

$$Y_t^m = A_t (p_t h_t L_t^m)^{1-\alpha} K_t^\alpha, \tag{2}$$

where $\alpha \in (0,1)$, $A > 0$ is a technological parameter, and $p_t h_t L^m$ is the actual amount of skilled labor employed in the industrial sector given by the individual level of human capital and the actual labor force $p_t L_t^m$. As established below, human capital increases with the resources invested in education, and when these resources are zero $h_t(0) = 1$ and therefore the industrial sector employs unskilled labor.

When production is conducted using only traditional technology the wage rate is given by:

$$w_t^a = (1-\lambda) A_t^a p_t^{1-\lambda} (L_t^a)^{-\lambda} T^\lambda. \tag{3}$$

When industrial technology is operating the rate of return to capital r_t and the wage rate per efficiency unit of labor w_t^m are given by:

$$r_t = \alpha A_t p_t^{1-\alpha} \left(\frac{K_t}{h_t L_t^m} \right)^{\alpha-1}; \tag{4}$$

$$w_t^m = (1-\alpha) A_t p_t^{1-\alpha} \left(\frac{K_t}{h_t L_t^m} \right)^{\alpha}. \tag{5}$$

In the early stages of development production is conducted using traditional technology while industrial technology is latent since no fixed and human capital have been accumulated yet. The economy will start employing the industrial technology together with the traditional technology when income is sufficiently high. In particular, as discussed below, the improvements in technological progress and adult survival will lead parents to leave a positive transfer to their children under the form of investments in fixed capital and education; this, in turn, will activate industrial technology.

Total output is therefore given by:

$$Y_t = Y_t^a + Y_t^m. \tag{6}$$

Workers are assumed to be perfectly mobile between the two sectors; therefore wages are equalized across the sectors in every period, i.e. $w_t^a = w_t^m h_t \ \forall t$. This implies that employment in the traditional sector is chosen in order to maximize profits (excluding the return to land), i.e. $L_t^a = \arg\max[A^a (p_t L_t^a)^{1-\lambda} (T)^\lambda - w_m h_t L_t^a]$, i.e.:

$$L_t^a = \left[\frac{A_t^a p_t^{1-\lambda} (1-\lambda)}{w_t^m h_t} \right]^{1/\lambda} T. \tag{7}$$

The amount of labor employed in the industrial sector is therefore:

$$L_t^m = L_t - L_t^a, \tag{8}$$

where L_t is the size of the working population. Assuming for simplicity that:

$$\alpha = \lambda,$$

and that the productivity in the traditional sector has the same trend as the productivity in the industrial sector, that is:

$$A_t^a = \phi A_t,$$

with $\phi < 1$, the aggregate production is given[11]:

$$Y_t = A_t p_t^{1-\alpha} L_t^{1-\alpha} \left(\tilde{T} + h_t^{\left(\frac{1-\alpha}{\alpha}\right)} K_t \right)^{\alpha}, \tag{12}$$

where $\tilde{T} = \phi^{1/\alpha} T$.

The *actual income per worker* in period t is therefore given by:

$$y_t = A_t p_t^{1-\alpha} \left(\frac{\tilde{T}}{L_t} + h_t^{\left(\frac{1-\alpha}{\alpha}\right)} k_t \right)^{\alpha}, \tag{13}$$

where $y_t \equiv Y_t / L_t$ and $k_t \equiv K_t / L_t$.

From Eq. (11) income per worker can be written as follows:

$$y_t = A_t p_t \left[\frac{\tilde{T}}{(p_t L_t)\, l_t^a} \right]^{\alpha}, \tag{14}$$

where $1/l_t^a$ is a proxy for the accumulation of fixed and human capital.

III.B. Consumption and Total Transfers

In childhood, individuals acquire education and make no decisions; in adulthood, individuals work, have n_t children, and can save to accumulate wealth for their offspring, and invest in their education.

To analyze adults' behavior it is useful to conceptualize adulthood (of length p_t) as divided into time increments (for example years or months). At each time increment individuals (born in period $t-1$) allocate their income between consumption c_t and a transfer to their offspring b_t:

$$y_t = p_t c_t + p_t b_t. \tag{15}$$

[11]Substituting Eq. (5) into Eq. (7) leads to:

$$L_t^a = \left[\frac{A_t^a p_t^{1-\lambda} (1-\lambda)(h_t L_t^m)^{\alpha}}{A_t p_t^{1-\alpha} (1-\alpha) h_t K_t^{\alpha}} \right]^{1/\lambda} T; \tag{9}$$

with $\lambda = \alpha$, it yields:

$$L_t^a = \frac{L_t^m \tilde{T}}{K_t h_t^{\frac{1-\alpha}{\alpha}}}. \tag{10}$$

Thus from Eq. (8) it follows that the labor share in the traditional sector is given by:

$$l_t^a = \frac{L_t^a}{L_t} = \frac{\tilde{T}/L_t}{k_t h_t^{\frac{1-\alpha}{\alpha}} + \tilde{T}/L_t}. \tag{11}$$

It follows that $\partial(L_t^a/L_t)/\partial h_t < 0$, $\partial(L_t^a/L_t)/\partial k_t < 0$ and $\partial(L_t^a/L_t)/\partial T > 0$.

where $p_t c_t$ is the *actual consumption* during adulthood per individual, $p_t b_t$ is the *actual transfer* which each parent gives to their children during their life.

The transfer $p_t b_t$, in turn, is allocated between the *actual spending in children's education* $p_t e_t$ and the *actual saving* $p_t s_t$ for the future wealth of their children:

$$pb_t = p_t s_t + p_t e_t. \tag{16}$$

The investment in education is devoted to increasing children's human capital. In particular, each child with a total parental investment in education $p_t e_t$ receives an amount of $\bar{e}_t \equiv p_t e_t / n_t$ and acquires:

$$h_{t+1} = h\left(\bar{e}_t\right) = \left(1 + D\bar{e}_t\right)^{\gamma}, \text{with } \gamma \in (0,1) \text{ and } D > 0, \tag{17}$$

efficiency units of human capital, where $h(0) = 1$, $h'(0) = \gamma D$ and $\lim_{\bar{e}_t \to \infty} h'\left(\bar{e}_t\right) = 0$ (see Galor and Moav, 2004, 2006). Allowing for the case $\gamma \geq 1$ implies that human capital accumulation alone can generate positive long-run growth. D is a scale parameter.

Individual preferences are defined over a consumption above a *subsistence level* $c^{\text{MIN}} > 0$ and the transfer to their children b_t. The expected utility function of altruistic individuals born in period $t-1$ is therefore:[12]

$$U = p_t[(1-\beta)\log(c_t) + \beta \log(b_t + \theta)], \tag{19}$$

where $\beta \in (0,1)$ is the discount factor and $\theta > 0$ implies that children receive a positive transfer only when parent's income is sufficiently high (see Eq. (23) below).

Parents choose the level of consumption and the transfer to the offspring so as to maximize their expected utility, that is:

$$(c_t^*, b_t^*) = \underset{c_t, b_t}{\arg\max} \{p_t[(1-\beta)\log(c_t) + \beta \log(b_t + \theta)]\}, \tag{20}$$

subject to:

$$\begin{array}{rcl} y_t & = & p_t c_t + p_t b_t; \\ c_t & \geq & c^{\text{MIN}}; \text{and} \\ b_t & \geq & 0. \end{array}$$

Assume that the following condition on parameters hold:

$$c^{\text{MIN}} < \frac{(1-\beta)\theta}{\beta}, \tag{21}$$

in order to ensure that for low levels of income optimal consumption is increasing with income per worker, while optimal bequest is zero (i.e. $y^{\text{MIN}} < y^{\text{CAP}}$). The optimal levels of

[12]Following Rosen (Rosen) we assume that the expected utility in the second period is given by the utility of the state "life" which is given by the utility from consumption and the transfer to the children, and the utility of the state "death" which is given by M and which is assumed to be equal to zero for simplicity:

$$U = p_t[(1-\beta)\log(c_t) + \beta \log(b_t + \theta)] + (1-p)M, \tag{18}$$

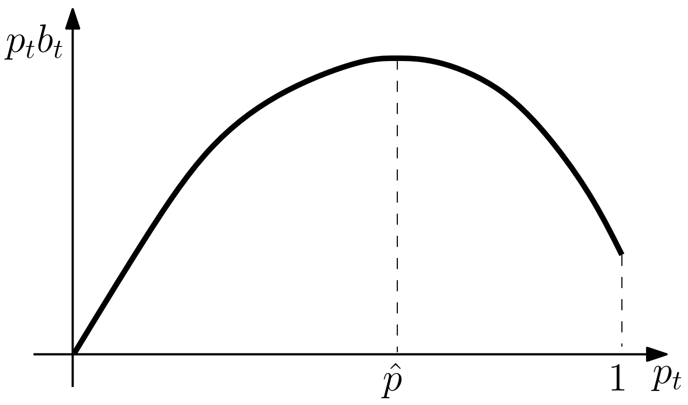


Figure 2: Actual transfer versus adult survival

consumption and transfer are given as follows:[13]

$$c_t^* = \begin{cases} \dfrac{y_t}{p_t} & \text{if } y_t \in (y^{\text{MIN}}, y^{\text{CAP}}] \\[2ex] \dfrac{(1-\beta)\,(y_t + p_t\theta)}{p_t} & \text{if } y_t \in (y^{\text{CAP}}, \infty) \end{cases} \tag{22}$$

and:

$$b_t^* = \begin{cases} 0 & \text{if } y_t \in (y^{\text{MIN}}, y^{\text{CAP}}] \\[2ex] \dfrac{\beta y_t - \theta(1-\beta)p_t}{p_t} & \text{if } y_t \in (y^{\text{CAP}}, \infty) \end{cases} \tag{23}$$

where:

$$y^{\text{MIN}} = p_t c^{\text{MIN}}, \tag{24}$$

and:

$$y^{\text{CAP}} = \frac{\theta(1-\beta)p_t}{\beta}. \tag{25}$$

III.C. Adult Survival and the Optimal Transfer

Eq.(22) shows that a rise in adult survival in period t, when income is above subsistence level, increases actual consumption in adulthood, i.e. $\partial p_t c_t / \partial p_t = \partial y_t / \partial p_t + (1-\beta)\theta > 0$.

By contrast, the actual transfer pb_t has an inverted U-shaped relationship with respect to adult survival. The decline in adult mortality $1 - p_t$ has in fact two opposing effects on transfer: on the one hand, higher longevity increases parents' consumption, thus reducing the overall transfers to their offspring; on the other hand, parents with a longer working life, experience an increase in their income thus raising the transfers to their offspring. When the initial level of income is sufficiently high, the latter effect always prevails, whereas at low levels of income the opposite is true. In particular, from Eq. (13) there exists a threshold level of adult survival rate denoted by $\hat{p}$, such that for $p_t < (>) \hat{p}$ the

[13] See Appendix A.

rise in adult survival rate positively (negatively) affects the total transfer to the offspring (see Fig. 2):

$$\hat{p} = \left[\frac{\beta(1-\alpha)A_t}{\theta(1-\beta)}\right]^{1/\alpha}\left(\frac{\tilde{T}}{L_t} + h_t^{\frac{1-\alpha}{\alpha}} k_t\right). \tag{26}$$

This threshold increases with the level of development of the country, i.e. with respect to A_t, h_t and k_t. On the other hand it decreases with respect to the size of workforce L_t.

III.D. Accumulation of Fixed and Human Capital

The transfer is allocated between saving, i.e. accumulation of fixed capital, and education, i.e. accumulation of human capital (see Eq. (16)). However, the economy begins to accumulate fixed capital only when parents are sufficiently rich (i.e. $y_t > y^{\text{CAP}}$, see Eq. (23)) to leave a positive transfer to their offspring; and to accumulate human capital for a still higher level of income (i.e. $y_t > y^{\text{EDU}}$, see Eq. (33) below).

The total fixed capital stock in period $t+1$ is given by the aggregate saving in period t:

$$K_{t+1} = L_t p_t s_t = L_t\left(p_t b_t - p_t e_t\right). \tag{27}$$

Given the fertility rate n_t, adult population L_t evolves according to:

$$L_{t+1} = n_t L_t. \tag{28}$$

The capital/labor ratio is therefore equal to:

$$k_{t+1} = \bar{b}_t - \bar{e}_t, \tag{29}$$

where $\bar{b}_t \equiv p_t b_t / n_t$ is the actual transfer per child, and $\bar{e}_t \equiv p_t e_t / n_t$ is the actual investment in education per child. In particular:

$$\bar{b}_t = \frac{\beta y_t - \theta(1-\beta)p_t}{n_t} \tag{30}$$

Parents choose the amount to invest in children's education in order to maximize the future income of offspring i.e. y_{t+1}. In the early stages of development, when the productivity in the industrial sector is relatively low with respect to the productivity in the traditional sector, individuals do not have the incentive to invest in the human capital of their children. However, the improvements in technological progress and the accumulation of fixed capital, pushing up the efficiency of industrial technology, will lead to a positive demand for human capital. From Eqq. (13) and (17) it follows that:

$$e_t^* = \arg\max_{e_t \in [0, b_t]}\left[A_{t+1} p_{t+1}^{1-\alpha}\left(\frac{\tilde{T}}{L_{t+1}} + \left(1 + \frac{D p_t e_t}{n_t}\right)^{\frac{\gamma(1-\alpha)}{\alpha}} k_{t+1}\right)^{\alpha}\right], \tag{31}$$

where k_{t+1} is given by Eq. (29). Eq. (23) shows that the optimal level of education is positive only if income is sufficiently high, i.e.:

$$\bar{e}_t^* = \begin{cases} 0 & \text{if } y_t \in [y^{\text{MIN}}, y^{\text{EDU}}]; \\ \\ \dfrac{\beta y_t - \theta(1-\beta)p_t - \tilde{b} n_t}{(1 + D\tilde{b})n_t} & \text{if } y_t \in (y^{\text{EDU}}, \infty) \end{cases} \tag{32}$$

where:

$$y^{\text{EDU}} \equiv \frac{\tilde{b}n_t + \theta(1-\beta)p_t}{\beta}, \tag{33}$$

and:

$$\tilde{b} \equiv \frac{\alpha}{D(1-\alpha)\gamma}. \tag{34}$$

From Eqq. (29) and (32) the capital-labor ratio in period $t+1$ is given by:

$$k_{t+1} = \begin{cases} 0 & \text{if } y_t \in [y^{\text{MIN}}, y^{\text{CAP}}]\,; \\ \dfrac{\beta y_t - \theta(1-\beta)p_t}{n_t} & \text{if } y_t \in (y^{\text{CAP}}, y^{\text{EDU}}]\,; \\ \left(\dfrac{\tilde{b}}{1+D\tilde{b}}\right)\left\{1 + D\left[\dfrac{\beta y_t - \theta(1-\beta)p_t}{n_t}\right]\right\} & \text{if } y_t \in (y^{\text{EDU}}, \infty)\,. \end{cases} \tag{35}$$

IV. *The Stages of Development*

Eqq. (23), (32) and (35) allow us to characterize the dynamic of income per worker in period $t+1$ as follows:

$$y_{t+1} = \begin{cases} \dfrac{A_{t+1}p_{t+1}^{1-\alpha}\tilde{T}^{\alpha}}{(n_t L_t)^{\alpha}} & \text{if } y_t \in (y^{\text{MIN}}, y^{\text{CAP}}]; \\ A_{t+1}p_{t+1}^{1-\alpha}\left[\dfrac{\tilde{T}}{n_t L_t} + \dfrac{\beta y_t - \theta(1-\beta)p_t}{n_t}\right]^{\alpha} & \text{if } y_t \in (y^{\text{CAP}}, y^{\text{EDU}}]; \\ A_{t+1}p_{t+1}^{1-\alpha}\left\{\dfrac{\tilde{T}}{n_t L_t} + \tilde{b}\left[\dfrac{n_t + D(\beta y_t - \theta(1-\beta)p_t)}{n_t(1+D\tilde{b})}\right]^{\frac{\gamma(1-\alpha)}{\alpha}+1}\right\}^{\alpha} & \text{if } y_t \in (y^{\text{EDU}}, +\infty). \end{cases} \tag{36}$$

The three ranges of y_t correspond to i) Traditional Regime, i.e. $y_t \in (y^{\text{MIN}}, y^{\text{CAP}})$, where production is conducted using traditional technology; ii) Pre-Modern Regime, i.e. $y_t \in (y^{\text{CAP}}, y^{\text{EDU}})$, where output is the result of using fixed capital and unskilled labor in an industrial sector; and iii) Modern Regime, i.e. $y_t > y^{\text{EDU}}$, where both fixed and human capital are jointly used in the industrial sector.[14]

Assumption 1 Income per worker is always higher than the subsistence level, i.e. $p_t c^{\text{MIN}}$. If $L_{t+1} = L_t = L\ \forall t$ and $p_{t+1} = p_t = p \forall t$, this implies that (for technical details see Appendix A):

$$A \geq A^{\text{MIN}}, \tag{37}$$

where:

$$A^{\text{MIN}} \equiv c^{\text{MIN}}\left(\frac{pL}{\tilde{T}}\right)^{\alpha}. \tag{38}$$

[14]Simple calculations show a smoothing transition from the Pre-Modern Regime to the Modern Regime, that is $\lim_{y_t \to y^{EDU-}} \partial y_{t+1}/\partial y_t = \lim_{y_t \to y^{EDU+}} \partial y_{t+1}/\partial y_t$.

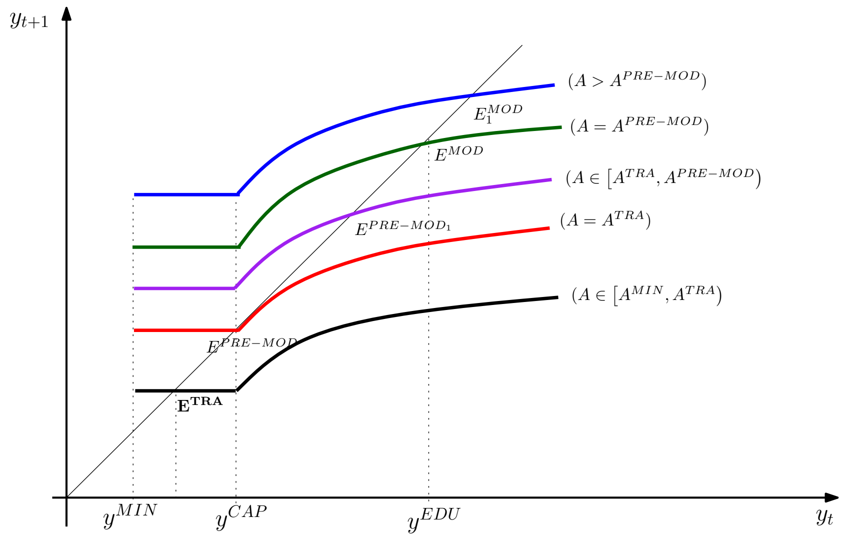


Figure 3: Regime and level of technological progress

Proposition 1 states the conditions under which there exists one or more than one equilibrium in the three regimes, and Fig. 3 provides a graphical exposition of the results contained in Proposition 1.

Proposition 1 *Suppose that Assumption 1 holds, $L_{t+1} = L_t = L$ (i.e. $n_t = 1\ \forall t$), and adult survival is constant over time, i.e $p_{t+1} = p_t = p\ \forall t$; under some (not so restrictive) conditions on the model's parameters reported in Appendix B:*

- *if $A \in [A^{\mathrm{MIN}}, A^{\mathrm{TRA}})$, then there exists one stable equilibrium in the Traditional Regime and possibly one unstable and one stable equilibrium in the Pre-Modern Regime, where:*

$$A^{\mathrm{TRA}} \equiv \frac{\theta\,(1-\beta)}{\beta}\left(\frac{pL}{\tilde{T}}\right)^{\alpha}. \tag{39}$$

- *If $A \in [A^{\mathrm{TRA}}, A^{\text{PRE-MOD}}]$, then there exists one stable equilibrium in the Pre-Modern Regime, where:*

$$A^{\text{PRE-MOD}} \equiv \frac{\tilde{b} + \theta\,(1-\beta)\,p}{\beta p^{1-\alpha}\left(\tilde{T}/L + \tilde{b}\right)^{\alpha}}. \tag{40}$$

- *Finally, if $A > A^{\text{PRE-MOD}}$ there exists just one stable equilibrium in the Modern Regime.*

Proof. See Appendix B. ■

Fig. 3 shows how as A increases the economy will pass through all three regimes.

The transition from the Traditional to the Pre-Modern Regime is driven by the increase in traditional production, which make possible positive transfers to offspring, the source of accumulation of fixed capital. The transition from Pre-Modern Regime to Modern Regime is, instead, driven by the higher accumulation of fixed capital generated by the increase of productivity in the modern sector, such accumulation, in turn, increases the return on the investment in human capital, and, therefore, incentivizes this investment. Once the accumulation of human capital starts, a self-reinforcing mechanism operates to

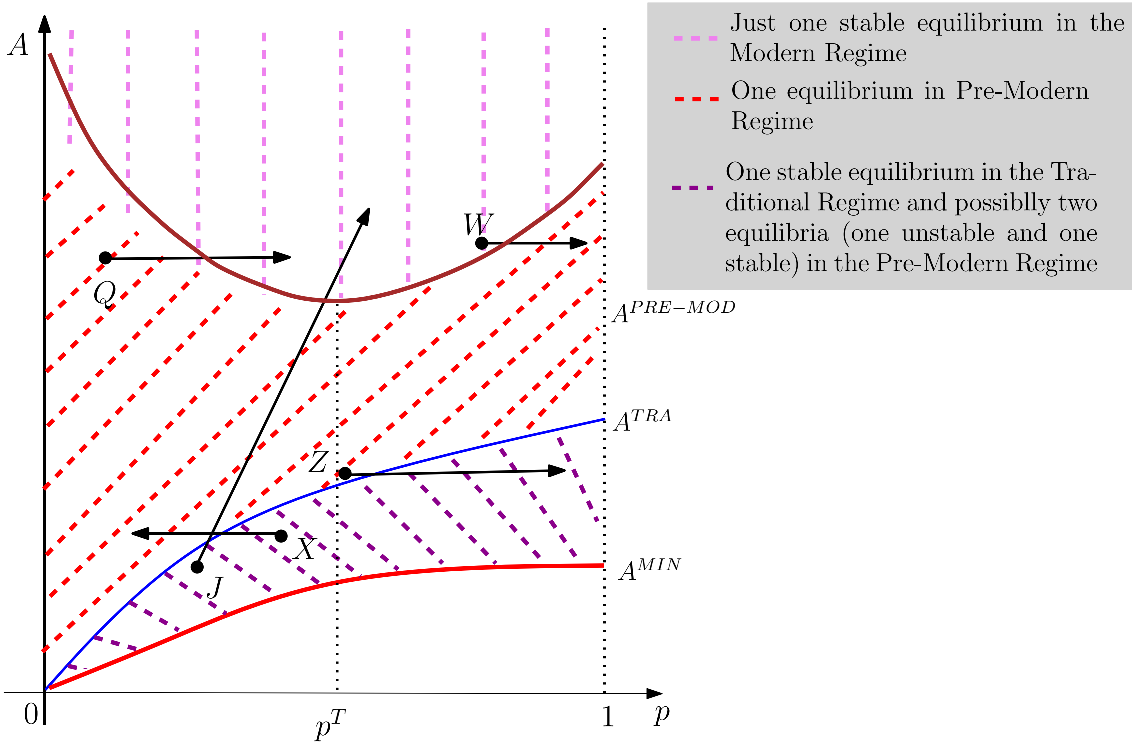


Figure 4: Technological progress, adult mortality and transitions through different regimes (no population growth)

increase further the income per worker. Decreasing returns in the accumulation of human capital (i.e. $\gamma < 1$) limit, in our model, the possibility of a positive long-run growth (see Robert E. Lucas, 2004 for a similar analysis).

Fig. 3 does not report the case of two stable equilibria for $A \in [A^{\text{MIN}}, A^{\text{TRA}})$, which would appear as a double cross of the curve in the range $(y^{\text{CAP}}, y^{\text{EDU}})$, in addition to the cross in the range $(y^{\text{MIN}}, y^{\text{CAP}}]$. This case opens the possibility of persistent effects of transitory shocks .

IV.A. The Role of Mortality Rate in the Transition between Regimes

Fig. (4) shows how different combinations of technological progress and adult mortality can lead an economy through different regimes (the figure is depicted under the assumption of no population growth, i.e. $L_t = L$ and $n_t = 1 \ \ \forall t$).

A^{MIN} and A^{TRA} are both increasing and concave with respect to p, while $A^{\text{PRE-MOD}}$ is decreasing in $[0, p^T)$ and increasing in $(p^T, 1]$.[15]

Consider the case of an economy with a low level of technological progress and a high mortality rate (i.e. point J in Fig. 4). The transition through the different regimes for this economy is driven by the simultaneous advances in technological progress and adult survival. This could be the path followed by most Western countries in the last 500 years (see Section V. below).

[15]The level of p^T is equal to $1/\theta(1-\beta)D\gamma$.

When the level of technological progress is very low, the rise in adult survival alone cannot permits the transition from Traditional to (Pre-)Modern Regime. Moreover, an economy in a Pre-Modern Regime could regress back to the Traditional Regime if the increase in adult survival is sufficiently high; this is the case of the trajectory starting from point Z.

When the level of technological progress is high enough, and the increase in adult survival is moderate, the economy can transit from a Pre-Modern to to Modern Regime (see the trajectory starting from the point Q). However, a further increase in adult survival can push the economy back to the Pre-Modern Regime (this is the case of the trajectory starting from point W). As discussed in Section III.C. the inverted U-shaped relationship between adult survival and economic growth derives from the fact that advances in adult survival have two opposing effects on intergenerational transfers. The basic motivation underlying this result is the presence of diminishing returns at low levels of income: the rise in population, due to a decline in mortality, has a less than proportional effect on output because of the presence of land. When adult survival rises above a certain threshold, at low levels of income, the rise in income is not sufficient to compensate for the rise in consumption. On the other hand, at high levels of income economy accumulates fixed and human capital, and, therefore, the rise in adult survival always allows a level of income sufficiently high to compensate for the rise in consumption. In particular, for a sufficiently high income per worker, the rise in longevity increases the return on investment in education and therefore the higher income perpetuates.

These results are in line with the empirical evidence of a non-linear relationship between life expectancy and economic growth as discussed in Cervellati and Sunde, 2011. In particular, they show that this relationship is negative before the onset of demographic transition and strongly positive thereafter.

Finally, the path starting from the point X in Fig. 4 shows a scenario in which a dramatic fall in adult survival, caused for example by an epidemic such as the Black Death, can have a positive effect on income per worker. In this case, the reduction in the actual workforce, increasing income per worker, pushes the economy from the Traditional to Pre-Modern Regime. After the shock, however, the economy will go back into Traditional Regime unless, at the original level of technological progress and adult mortality, there was also one stable equilibrium in the Pre-Modern Regime.

IV.B. Endogenous Fertility

In the following we extend the model to include endogenous fertility. Individuals preferences are now defined over consumption, the transfer to offspring, and the total number of children who survive to adulthood n_t (see Easterlin, 2004 and Galor, 2005).

The optimal problem of parents is therefore:

$$(c_t^*, b_t^*, n_t^*) = \arg\max_{c_t, b_t, n_t} \{p_t[(1-\beta)\log(c_t) + \epsilon \log(n_t) + \beta \log(b_t + \theta)]\}, \tag{41}$$

subject to:

$$\begin{aligned} y_t &= p_t c_t + p_t b_t + \delta n_t y_t; \\ c_t &\geq c^{\text{MIN}}; \\ b_t &\geq 0. \end{aligned}$$

where δ is the opportunity cost of raising children, that is the fraction of parents time required in order to raise each child (see Galor, 2005).

Assuming that condition (21) holds[16], the optimal levels of consumption, transfer and number of surviving children are given by:[17].

$$c_t^* = \begin{cases} c^{\text{MIN}} & \text{if } y_t \in (y^{\text{MIN}}, y^{\text{SUB}}] \\ \dfrac{(1-\beta)y_t}{(1-\beta+\epsilon)p_t} & \text{if } y_t \in (y_{\text{SUB}}, y^{\text{CAP}}] \\ \dfrac{(1-\beta)(y_t+p_t\theta)}{(1+\epsilon)p_t} & \text{if } y_t \in (y^{\text{CAP}}, \infty) \end{cases} \tag{42}$$

and:

$$b_t^* = \begin{cases} 0 & \text{if } y_t \in (y^{\text{MIN}}, y^{\text{CAP}}] \\ \dfrac{\beta y_t - \theta(1-\beta+\epsilon)p_t}{p_t(1+\epsilon)} & \text{if } y_t \in (y^{\text{CAP}}, \infty) \end{cases} \tag{43}$$

and:

$$n_t^* = \begin{cases} \dfrac{y_t - p_t c^{\text{MIN}}}{\delta y_t} & \text{if } y_t \in (y^{\text{MIN}}, y^{\text{SUB}}] \\ \dfrac{\epsilon}{\delta(1-\beta+\epsilon)} & \text{if } y_t \in (y^{\text{SUB}}, y^{\text{CAP}}] \\ \dfrac{\epsilon(y_t+\theta p_t)}{\delta(1+\epsilon)y_t} & \text{if } y_t \in (y^{\text{CAP}}, \infty) \end{cases} \tag{44}$$

where $y^{\text{MIN}} = p_t c^{\text{MIN}}$ and:

$$y^{\text{SUB}} = \frac{p_t c^{\text{MIN}}(1-\beta+\epsilon)}{1-\beta}, \tag{45}$$

and:

$$y^{\text{CAP}} = \frac{\theta(1-\beta+\epsilon)p_t}{\beta}. \tag{46}$$

Fertility is increasing with income in the first range and reaches its maximum level in the range $y_t \in (y^{\text{SUB}}, y^{\text{CAP}}]$, and then declines (see Fig. (5)). The quality/quantity trade-off for children discussed in Becker et al. (1990) starts operating at the moment parents leave a positive transfer to children.

Eq. (44) shows that fertility has also a nonlinear relationship with adult survival; when income per worker is very low such that consumption is around subsistence level, fertility decreases with adult survival (the increase in adult survival raises the total amount of resources needed to maintain consumption at subsistence level). On the contrary, when income is sufficiently high, i.e. $y_t > y^{\text{CAP}}$, fertility increases with adult survival (an increase in adult survival results in an increase in the number of children through a positive income effect).

[16]Condition (21) ensures that $y^{\text{MIN}} < y^{\text{SUB}} < y^{\text{CAP}}$.

[17]See Appendix A

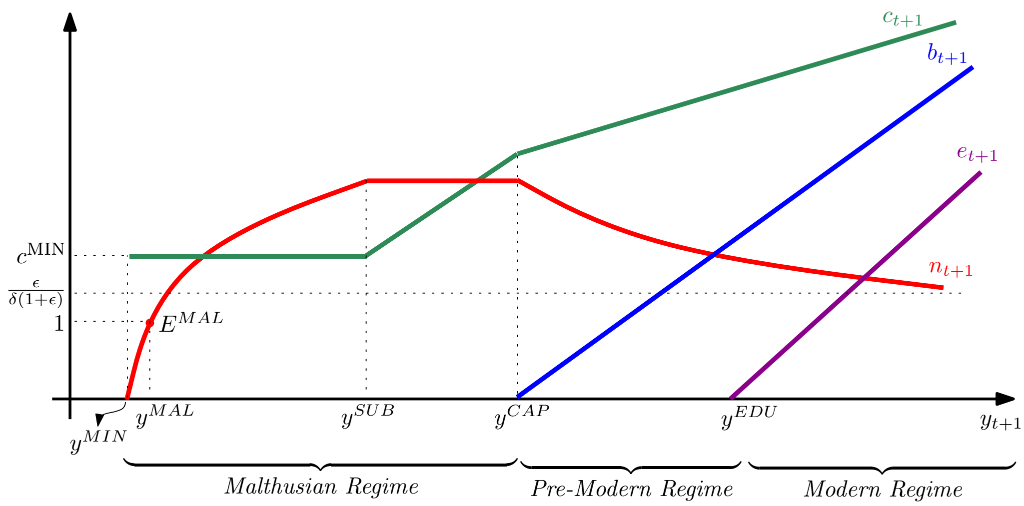


Figure 5: Optimal choices of individuals with endogenous fertility

Fig. 5 highlights how for very low levels of income per worker, i.e. $y_t < y^{\text{SUB}}$, consumption is at its subsistence level, the optimal transfer is zero, while the optimal number of children increases with respect to income per worker: the Traditional Regime therefore assumes the typical characteristics of a Malthusian Regime. In Malthusian Regime any increase in income per worker (due to improvement in technological progress and/or adult survival) results in a surge of population, which, in presence of diminishing returns to labor, leads a subsequent fall in income per worker; economy is therefore doomed to stagnate just above the subsistence level in the long run. Simple calculations show that in the range $y_t \in (y^{\text{MIN}}, y^{\text{SUB}}]$ there exists a unique stable equilibrium level of income per worker for which the population growth is zero (i.e. $n_t = 1$), that is:

$$y^{MAL} = \frac{p_t c^{\text{MIN}}}{1-\delta}, \tag{47}$$

to which correspond an equilibrium level of observed adult population[18]:

$$p_t L^{MAL} = \left[\frac{A_t(1-\delta)}{c^{\text{MIN}}}\right]^{1/\alpha} \tilde{T}, \tag{49}$$

When income increases (due to the increases in technological progress or mortality reduction) above the level y^{SUB}, this allows parents to escape from the subsistence level of consumption, thus consumption starts to increase and the fertility rate becomes constant. However, the economy is still in the Malthusian Regime since parents do not have a sufficient level of income to leave a positive transfer to their children.

If income continues to increase, the constancy of the fertility rate ensures that, at a certain point, i.e. $y_t > y^{\text{CAP}}$, the economy moves into the Pre-Modern Regime where

[18]Eq. (49) follows from:

$$A_t p_t^{1-\alpha} \left(\frac{\tilde{T}}{L_t}\right)^{\alpha} = \frac{p_t c^{\text{MIN}}}{1-\delta}. \tag{48}$$

parents start to devote a fraction of income for the future wealth of their children and the relationship between income and population growth becomes negative.

From Eqq. (43) and (44) the dynamic of income per worker can be characterized as follows:

$$y_{t+1} = \begin{cases} \dfrac{A_{t+1}p_{t+1}^{1-\alpha}\tilde{T}^{\alpha}}{L_t^{\alpha}}\left(\dfrac{\delta y_t}{y_t - p_t c^{\text{MIN}}}\right)^{\alpha} & \text{if } y_t \in (y^{\text{MIN}}, y^{\text{SUB}}]; \\ \dfrac{A_{t+1}p_{t+1}^{1-\alpha}\tilde{T}^{\alpha}}{L_t^{\alpha}}\left[\dfrac{\delta(1-\beta+\epsilon)}{\epsilon}\right]^{\alpha} & \text{if} y_t \in (y^{\text{SUB}}, y^{\text{CAP}}]; \\ A_{t+1}p_{t+1}^{1-\alpha}\left[\dfrac{\delta(1+\epsilon)y_t}{\epsilon(y_t+\theta p_t)}\right]^{\alpha}\left[\dfrac{\tilde{T}}{L_t} + \dfrac{\beta y_t - \theta(1-\beta+\epsilon)p_t}{1+\epsilon}\right]^{\alpha} & \text{if } y_t \in (y^{\text{CAP}}, y^{\text{EDU}}]; \\ A_{t+1}p_{t+1}^{1-\alpha}\left\{\dfrac{\tilde{T}}{L_t}\dfrac{\delta(1+\epsilon)y_t}{\epsilon(y_t+\theta p_t)} + \tilde{b}\left[\dfrac{\epsilon(y_t+\theta p_t) + D(\beta y_t - \theta(1-\beta+\epsilon)p_t)\delta y_t}{\epsilon(y_t+\theta p_t)(1+D\tilde{b})}\right]^{\frac{\gamma(1-\alpha)}{\alpha}+1}\right\}^{\alpha} & \text{if } y_t \in [y^{\text{EDU}}, +\infty). \end{cases} \tag{50}$$

where:

$$y^{\text{EDU}} = \frac{\tilde{b}(1+\epsilon) + \theta(1-\beta+\epsilon)p^{\text{EDU}}}{\beta}. \tag{51}$$

The following Proposition states the conditions under which we observe one or more than one equilibrium in the three regimes.

Proposition 2 *Suppose that adult survival is constant over time, i.e.* $p_{t+1} = p_t = p$, *then:*

- *If* $A \in [A^{\text{MIN}}, A^{\text{TRA}})$, *then there exists at least one equilibrium in the Traditional Regime, where* A^{MIN} *is defined in Eq. (38) and:*

$$A^{\text{TRA}} = \frac{\theta(1-\beta+\epsilon)^{1-\alpha}}{\beta}\left(\frac{\epsilon p L_t}{\delta\tilde{T}}\right)^{\alpha}. \tag{52}$$

- *If* $A \in [A^{\text{TRA}}, A^{\text{PRE-MOD}}]$, *then there exists at least one equilibrium in the Pre-Modern Regime:*

$$A^{\text{PRE-MOD}} = \frac{[\tilde{b}(1+\epsilon) + \theta(1-\beta+\epsilon)p]^{1-\alpha}[\epsilon(\tilde{b}+\theta p)]^{\alpha}}{\psi p^{1-\alpha}} \tag{53}$$

where $\psi = \beta\delta^{\alpha}(\tilde{T}/L + \tilde{b})^{\alpha}$.

- *If* $A > A^{\text{PRE-MOD}}$ *there exists at least one equilibrium in the Modern Regime.*

IV.C. Empirical Evidence on Fertility and Income

The inverted U-shaped relationship between fertility rate and income is typical of most economies. The left hand side of Fig. 6 depicts the general fertility rate [19], i.e. the number of births divided by the number of women aged 15-44, for some European countries in the period $1750 - 1920$, The right hand side of Fig. 6 depicts the general

[19] General fertility is an index of the rate of production of children, strongly correlated with the average number of children per women (see, for example, Livi Bacci, 2007)

fertility rate adjusted by the probability of surviving at 20. We are interested in the number of surviving children at 20 since parents maximize their utility with respect to the number of surviving children.

As depicted in Fig. 6 the general fertility rate, for most countries, increases at low levels of income and starts to decline when income is sufficiently high. However, in agreement with the theoretical predictions, the inverted U-shaped relationship between fertility and income is much more evident when we consider the number of surviving children.

A large body of literature has developed theoretical and empirical models to analyze this path of fertility. Becker (1960)'s seminal work argues that the main reason behind the decline in fertility was the considerable increase in income which occurred as a result of the Industrial Revolution. In particular, as income increases the number of children in a household decreases because more affluent consumers tend to choose activities which require less time, instead children require a great deal of time and energy. Moreover, richer parents choose to have fewer children so that they can dedicate more time and resources to increase the "quality "of their offspring.

However, as is apparent from Fig. 6, the fact that the level of income which reverses the relationship between fertility and income differs across countries, suggests that there are also other specific country factors which affect the relationship between fertility and income.

In this respect Clark writes "Income, however, certainly cannot by itself explain the modern decline in fertility....Had income alone been determining fertility, the rich in the preindustrial world would already have been restricting their fertility "(see Clark, 2007, p. 293).

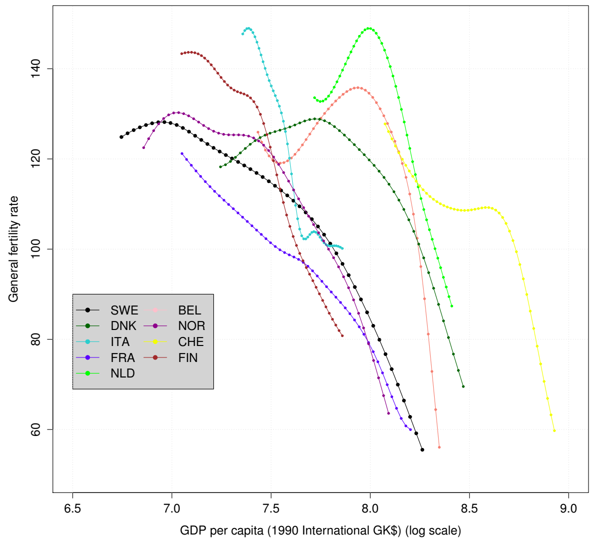
According to Clark high fertility even among the rich, in the preindustrial period, could be either due to the absence of birth control or to the fact that, in the high mortality environment of the Malthusian era, people consciously had more children in the hope of achieving a desired family size of two or three surviving children and most particularly, in richer families, a surviving son.

Thus, to explain the fertility decline we need to consider other factors beyond income, such as conscious action to limit fertility, the reduction of child mortality and the increased social status of women (see among others Livi Bacci, 2007, Easterlin, 2004).

V. A Quantitative Evaluation of the Model for the UK from 1541-1914

In this section we discuss the capacity of the model to reproduce the dynamics of the UK economy in the period $1541 - 1914$. For the purposes of our paper, the UK represents the best country because of i) the availability of long-term time series on population, health, GDP per capita, output composition, etc.; ii) many scholars argue that its advances in technological progress and health appear of the type assumed in our model (see Section II., and, e.g., Allen, 2009, Mathias, 2001, Mokyr, 2010); and, finally, iii) the implicit assumptions made in the theoretical model of the absence of any government activity and of a closed economy fit with the facts that the Industrial Revolution in the UK happened without any "conscious government policy sponsoring industrial progress 'and any role for imported capital (see Mathias, 2001, p. 4).

Fig. 7 provides a synthetic picture of the transition from stagnation to modern growth



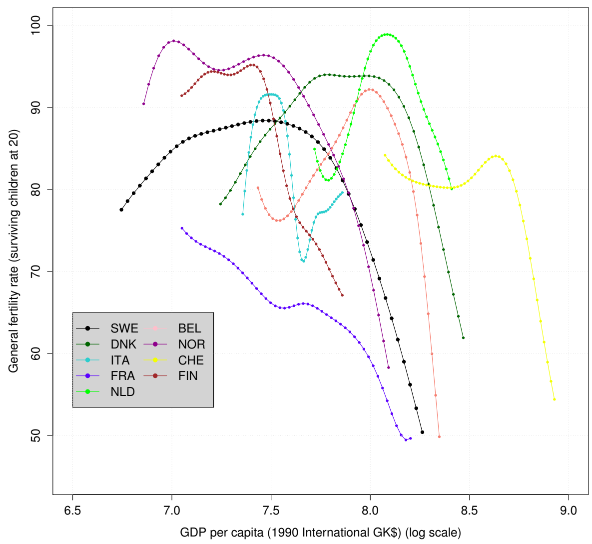


Figure 6: General Fertility rate versus Income. Europe: 1750-1930. Nonparametric kernel smoother. Sources: GDP per capita from Maddison Project Database (MPD) (update 02/2013); General fertility rate from the Human Mortality Database (HMD) (update 02/2013).

through three different regimes developed in our theoretical framework; in particular, in accordance with a large amount of literature on the issue (see, e.g., Cipolla, 1962, Galor, 2005, Clark, 2007, Livi Bacci, 2007) we will assume that the transition from the Malthusian to the Pre-Modern Regimes in the UK took place between 1770 and 1800 (the red column reported in Fig. 7); and the transition from the Pre-Modern to the Modern Regime between 1820 and 1850 (the blue column reported in Fig. 7).

The left panel shows how (smoothed) GDP per capita and GDP per worker display the same pattern (see Appendix C for the methodology used to calculate the GDP per worker). Until the middle of the 1600s stagnation was the dominant characteristic; afterwards a period of sustained growth started, with an acceleration at the end of the 18th century; GDP per worker continued to increase up to 1914, even though at a decelerated rate after 1890 (see Fig. 8 below). The acceleration of growth at the end of the 19th century coincides with a strong change in output structure, highlighted by the deep fall in the labor share in the agricultural sector and the land share in national income (see the second panel in Fig. 7). Accelerating growth rates, changes in output composition in favor of the modern sector and the fall in the fertility rate are (as shown in Fig. 7) coherent with the theoretical predictions of our model for the transition from Malthusian to Pre-Modern Regimes (see Fig. 5). Finally, the timing of the spur in the primary enrollment rate in 1850 as displayed in the right panel of Fig. 7 agrees with the assumed period of transition from Pre-Modern to Modern Regimes (see also Clark, 2007, p. 179).

V.A. Variables and Settings Used in the Quantitative Exercise

Table 1 lists the variables used in the quantitative exercise, the observed variables used to calculate them and the sources.

Variable	Definition	Calculated by	Source
l^a	Labor share in the traditional sector	Labor shares in agricultural sector	IHS, Broadberry et al. (2013)
pL	Employment	Cohorts age 20-70 and unemployment rate	W&S, HMD, and IHS
y	GDP per worker	GDP per capita, employment, and population	MPD, W&S
p	Probability of surviving from 20 to 70 years old	Cohorts age 20 and 70	W&S and IHS
n	Number of surviving children at adulthood	Employment and probability of surviving from 20 to 70 years old	W&S and HMD

Table 1: Time series used in the quantitative evaluation of the model. Sources: Maddison Project Database (MPD) (update 02/2013); Human Mortality Database (HMD) (update 02/2013), International Historical Statistics (IHS) (update 04/2013), Wrigley and Schofield (1989) (W&S)

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As discussed in Appendix C the calculation of the parameters' model required a set of additional information on the values taken by labor share on total output, minimum

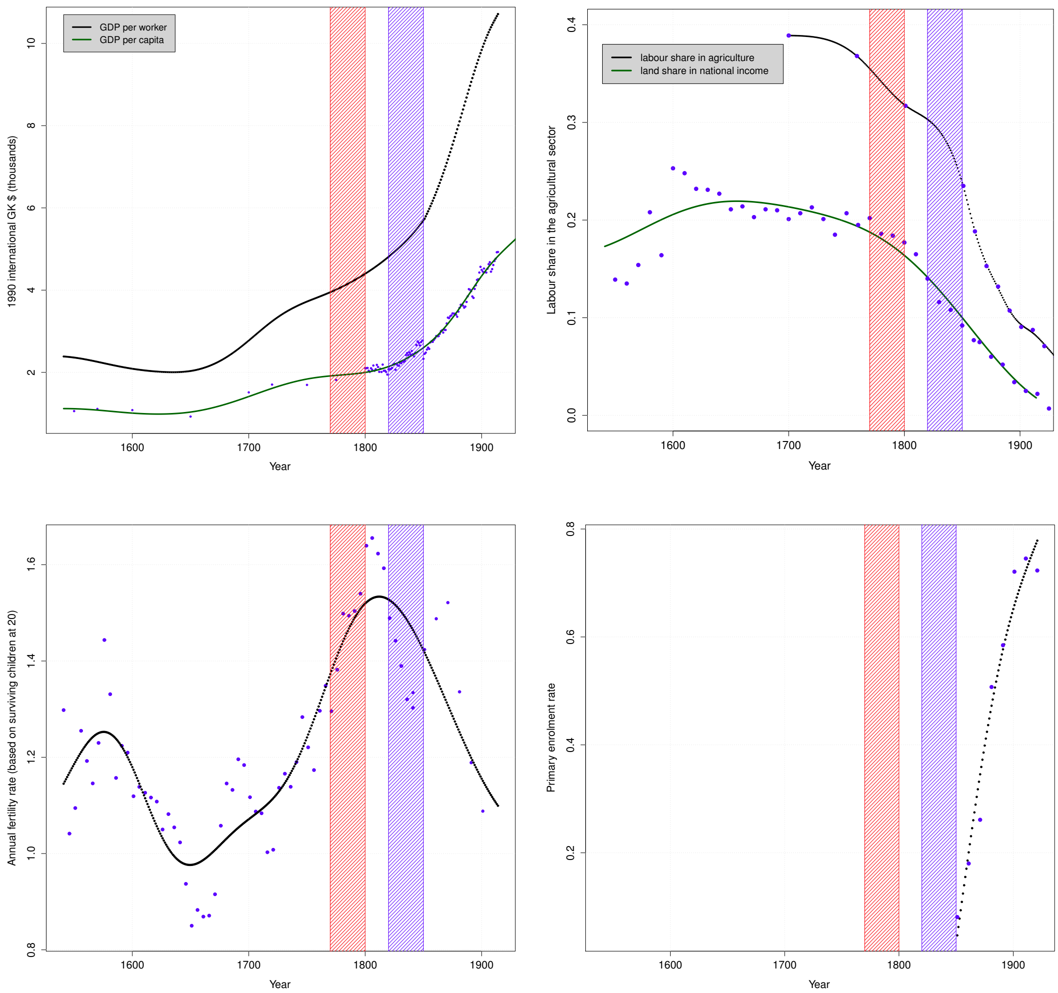

Figure 7: Transition from stagnation to modern growth: UK (England and Wales), 1541-1914. The red bar indicates the transition from Malthusian to Pre-Modern Regimes (1770-1800), while the blue bar the transition from Pre-Modern to Modern Regimes (1820-1850).

level of GDP per capita, investment rate at the beginning of the Modern Regime (set to 1850 for our exercise), and real interest rate in the Modern Regime (see again Appendix C for more details). Table 3 reports these values and their respective sources.

Parameter/variables	Definition	Value	Source
$1-\lambda, 1-\alpha$	Labor share on total output	0.6	Clark (2007), Hansen and Prescott (2002), Allen (2009)
$y^{p.c.}_{\text{MIN}}$	Minimum level of GDP per capita (1990 International GK \$)	0.67	MPD
i_{1850}	Investment rate in 1850	0.09	IHS
r_{1914}	Real interest rate in 1914	1.045	Bank of England

Table 2: Additional information needed to calculate the parameters' model. Sources: MPD, IHS, and Bank of England

.

Table 3 reports the result of the calibration of the two models specification, i.e. exogenous and endogenous fertility.

Parameter	Definition	Exogenous fertility	Endogenous fertility
c^{MIN}	Subsistence consumption	2.857	2.857
$\tilde{T}$	Fixed factor in traditional technology	14.964	14.964
β	Altruism factor	0.578	0.636
θ	Transfer factor	10.575	5.063
γ	Return to education	0.302	0.402
D	Scale parameter in education	7.438	5.588
ϵ	Taste for children		0.607
δ	Cost parameter of raising children		0.462

Table 3: Model's parameters calculated to match data for the UK reported in Tables 1 and 2

c^{MIN} and $\tilde{T}$ have exactly the same values for both models because in their calculation preferences for fertility are not present (see Eqq. (see Eqq. (100 and 102)) in Appendix C). The return on education is higher in the case of endogenous fertility, but balanced by a lower scale parameter in the production function of education. The cost parameter of raising children to 20 years old appears very high (46% of income of parents' is devoted to this factor). However, this cost should also include the consumption of children; with a children's consumption equal to 50% of adults, the opportunity cost of raising children sharply decreases to 19% of income (see Appendix C)

V.B. Growth Accounting

In this section we run a growth accounting exercise to *quantify* the role of technological progress and adult mortality in the overall growth of GDP per worker in the UK in the period 1541-1914.

From Eq. (14) the growth rate of technological progress is given by:

$$g^A = g^y - g^p + \alpha g^{pL} + \alpha g^{l^a} \tag{54}$$

where $g^A \equiv A_{t+1}/A_t - 1$, $g^y \equiv y_{t+1}/y_t - 1$, $g^p_{t+1} \equiv p_{t+1}/p_t - 1$, $g^{pL} \equiv p_{t+1}L_{t+1}/p_tL_t - 1$ and $g^{l^a} \equiv l^a_{t+1}/l^a_t - 1$ are respectively the growth rate of technological progress, GDP per worker, surviving workforce, employment and labor share in the traditional sector. The calculation of the growth rate of technological progress is therefore the same for both theoretical models.

Fig. 8 reports the individual contribution of technological progress, surviving workforce, employment, and factor accumulation to the growth rate of GDP per worker g^y, i.e. g^A, g^p, $-\alpha g^{pL}$ and $-\alpha g^{l^a}$ respectively.

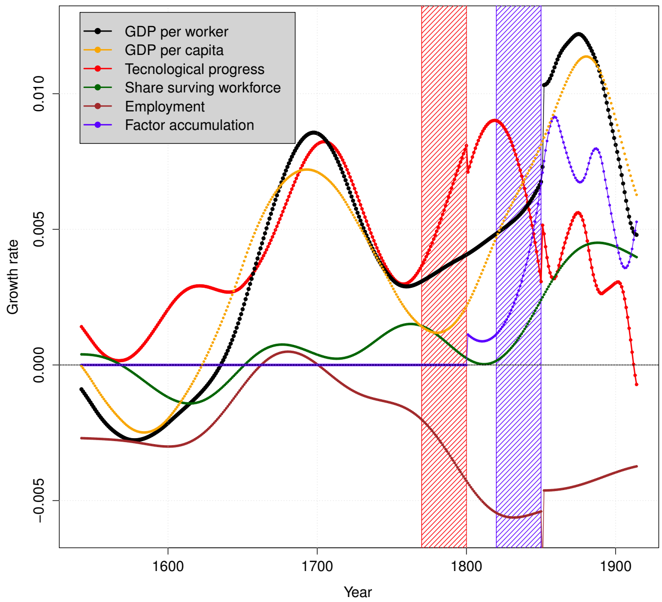

Figure 8: Growth accounting for the UK from 1541 to 1914. Contribution of different factor to GDP per worker's growth. The red bar indicates the transition from Malthusian to Pre-Modern Regimes (1770-1800), while the blue bar the transition from Pre-Modern to Modern Regimes (1820-1850).

In the Malthusian Regime (1541-1800) the growth of GDP per worker (0.24% per annum) strictly followed the growth rate of technological progress (0.39%), with a first phase up to the middle of the seventeenth century where the negative check of the growth of employment overshadowed the increasing positive growth of technological progress; and a second phase with a limited growth rate of employment (negative at the end of the seventeenth century), which allowed the growth rate of technological progress to benefit the growth of GDP per worker the most (Broadberry et al., 2010 and Broadberry et al., 2010). Changes in adult survival (the share of surviving workforce) instead, had a very limited effect.

In the Pre-Modern Regime (1800-1850) the growth rate of GDP per worker (0.52%) strongly accelerated thanks to the increase in the growth rate of technological progress (0.74%), but a remarkable contribution was given by factor accumulation (in particular fixed capital), and less by adult survival (0.24% and 0.074% respectively); the strong growth of employment (-0.54%) only partially dampened the upward trend of the growth rate of GDP per worker.

Growth of y Period		Due to growth in A	p	pL	l^a	Growth of $y^{p.c.}$
$1541-1800$	0.24%	0.39	0.018	-0.17	0	0.22
$1800-1850$	0.52%	0.74	0.074	-0.54	0.24	0.53
$1850-1914$	0.99%	0.34	0.40	-0.43	0.67	0.97

Table 4: The table shows growth rates per year for GDP per capita and GDP per worker. The entries for A, p, pL and l^a are respectively the contributions of growth of technological progress, surviving workforce, employment, and factor accumulation to the growth in GDP per worker (i.e. the entries for pL and l^a are the annual growth rates multiplied by the capital share on total output (α)).

Finally, in the Modern Regime (1850-1914) technological progress progressively contributed less and less (0.34%), while factor accumulation (fixed capital, but overall human capital) and adult survival reached the peak of their contribution to the growth of GDP per worker (0.67% and 0.40% respectively). The growth of GDP per worker increased until the end of the nineteenth century, and then strongly declined (see Mathias, 2001).

For the period 1760-1860 our estimates of the growth rate of technological progress display the same inverted U-shaped path of Antras and Voth, 2003, but with generally slightly higher values.[20] This contrasts with another strand of literature, which finds an accelerating growth rate in that period (see, e.g., Table 1 in Allen, 2009, and Table 7.1 in Temin and Voth, 2013)

Finally, for the period 1860-1900 Allen (2009) (Table 1) provides an estimate of growth rate of A substantially higher than ours (0.89% versus 0.40%): this difference can be traced to his higher estimated growth rate of GDP per worker, but overall to his exclusion from production factors of human capital.

Fig. 9 reports the estimated trajectories of technological progress and the share of surviving workforce calculated for the two models of exogenous and endogenous fertility, and the thresholds of technological progress for the transitions across regimes A^{TRA} and $A^{PRE-MOD}$. In both cases Malthusian and Pre-Modern Regimes were characterized by a strong increase in technological progress, and by very limited changes in adult survival. On the contrary, in the Modern Regime technological progress displays limited increments, and it is the surge in adult survival that played the prominent role by far. In summary, "technical progress was the prime mover behind the industrial revolution"(Allen, 2009, p. 1), while the decline in adult mortality appears to be one of the main reasons for modern growth (see Easterlin, 2004, p. 85).

V.C. The Model's Simulation

The model is simulated under the assumptions that childhood has a length of 20 years, adulthood has a length of 50 years (workforce is therefore made up of people between the

[20] Antras and Voth, 2003's estimates of the annual growth rate of technological progress (TFP) for the periods 1760-1800, 1801-1831, and 1831-1860 are equal to 0.27%, 0.54%, and 0.33%, while our estimates equal to 0.52%, 0.85% and 0.51% respectively.

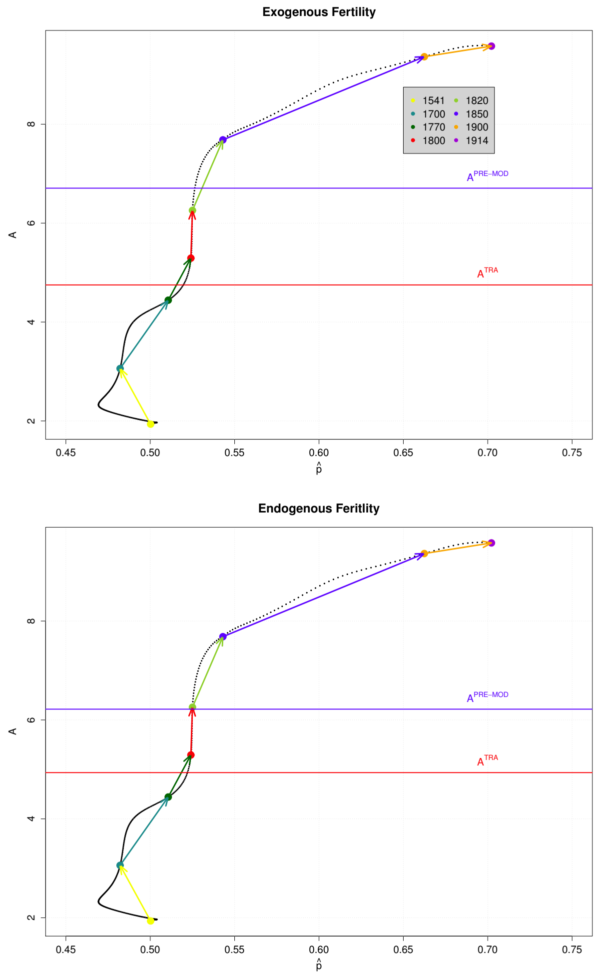

Figure 9: The estimated trajectory of technological progress and the share of surviving workforce in the UK from 1541 to 1914, and the thresholds of technological progress for the transitions across regimes A^{TRA} and $A^{PRE-MOD}$.

ages of 20 and 70), and that each time period has a length of 30 years. We start from the simulation of the model with exogenous fertility.

V.C.i. Exogenous Fertility

Eq. (36) is the base of the simulation for the case with exogenous fertility. Figure 10 displays the results of the simulation of GDP per worker with the parameters reported in Table 3.

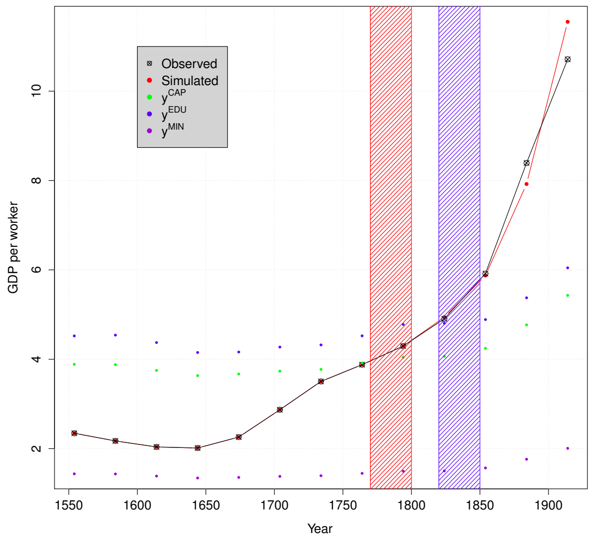

Figure 10: Observed (black) versus simulated (red) GDP per worker of UK for the period 1541-1914. Violet points are the minimum level of GDP per worker y^{MIN}, green points are the thresholds of GDP per worker y^{CAP} to move to the Pre-Modern Regime, and blue points the thresholds y^{EDU} to move to the Modern Regime.

The model accurately reproduces the actual path of GDP per worker, with the timing of transitions across regimes perfectly matched. Given the assumption of exogenous fertility, the capacity of the model to replicate the observed pattern of growth should be evaluated only by focusing on the Pre-Modern and Modern Regimes, where the *endogenous* accumulation of fixed and human capital is one source of growth. In the Traditional Regime, instead, the method used to calculate A directly drives to the equality between observed and simulated GDP per worker.

The time path of the accumulation of fixed and human capital reported in Fig. (11) confirms the timing of transition across different regimes. The decreasing consumption rate (the ratio between consumption and income) after 1800 mirrors the increasing transfer rate (the ratio between transfer and income); such transfers are firstly devoted to the accumulation of fixed capital, and only after 1820 to the accumulation of human capital (the education rate, i.e. the ratio between education expenditure and income, increases over time peaking at 8% in 1914). The simulated saving rate in 1820 is almost equal to the observed one (orange point in the picture). Real interest rate increases over time thanks to the increments in technological progress and the accumulation of human capital,

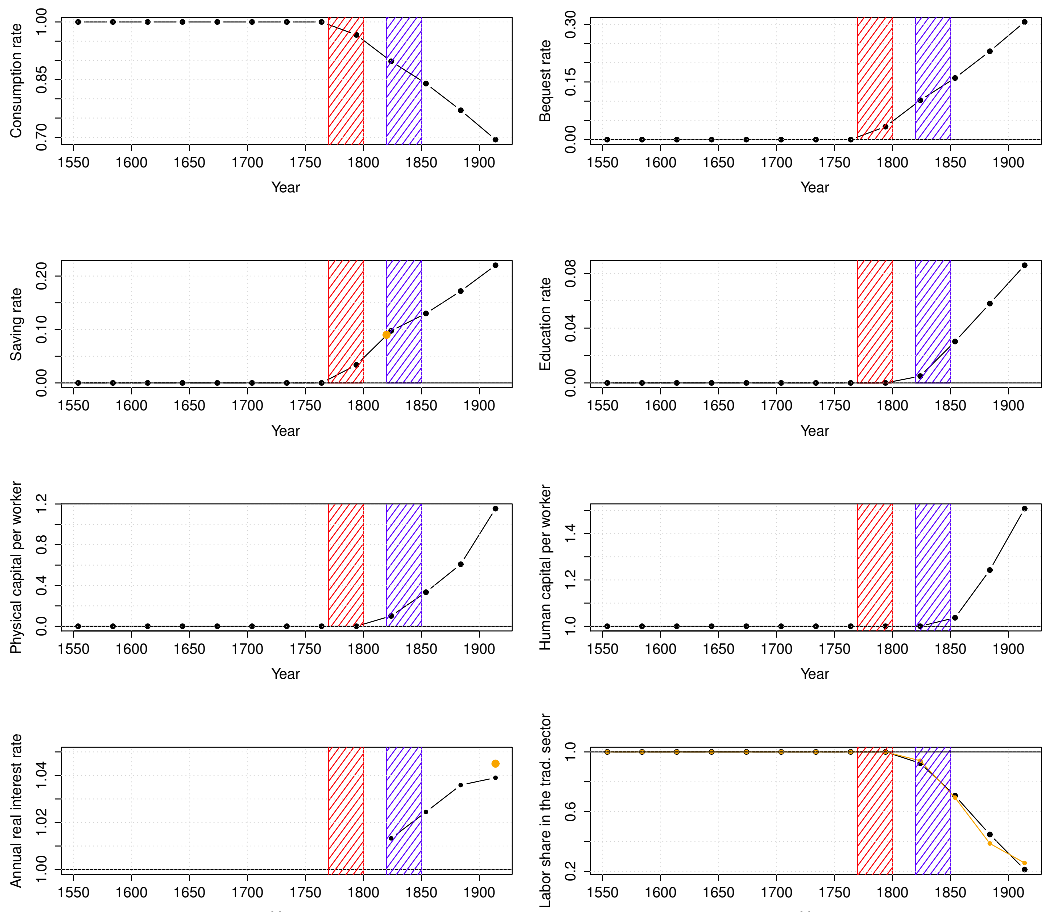

Figure 11: Time path of the main variables of the model (black points), and of relative observed values where available (orange points).

stabilizing at around 4% at the end of the period (slightly below the interest rate observed in 1914). Finally, the dynamics of output composition proxied by the labor share in the traditional sector appears completely replicated by the model.

Overall the dynamics of variables appear credible, with the exception of the level of the saving rate, which peaks at 20% at the end of the period (instead, the observed saving/investment rate is about 9% on average with no evidence of trend, see Fig. 22). We will see that the simulation with endogenous fertility provides a partial correction to this feature.

V.C.ii. Endogenous Fertility

Fig. 12 shows how in the case of endogenous fertility the replication of observed GDP per worker is not as perfect as the one of exogenous fertility, in particular for the Malthusian Regime. The overestimate of GDP per worker up to the end of the eighteenth century is caused by the underestimate of employment, which is, in turn, the outcome of the difference between the simulated and observed annual fertility rate (see Fig. 13). When, around 1800, the observed and simulated employment again become very close, the same happens for the GDP per worker. After 1800, the differences between simulated and observed fertility appear less marked, and, overall, the dynamics of GDP per capita in that period strongly depend more on the factor accumulation and less on employment growth.

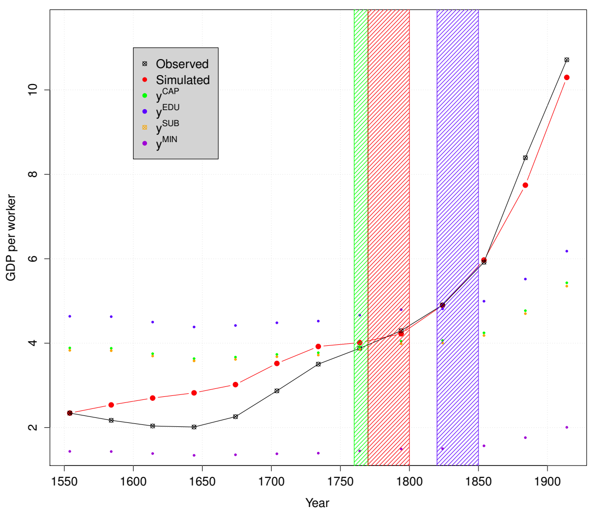

Figure 12: Observed (black) versus simulated (red) GDP per worker of the UK for the period 1541-1914. Violet points are the minimum level of GDP per worker y^{MIN}, green points are the thresholds of GDP per worker y^{CAP} to move to the Pre-Modern Regime, and blue points the thresholds y^{EDU} to move to the Modern Regime. The green bar indicates the transition to a consumption above the subsistence level.

The timing of transition from Pre-Modern to Modern Regimes is the one expected, while the transition from Malthusian to Pre-Modern Regime happens about 30 years before we expected.

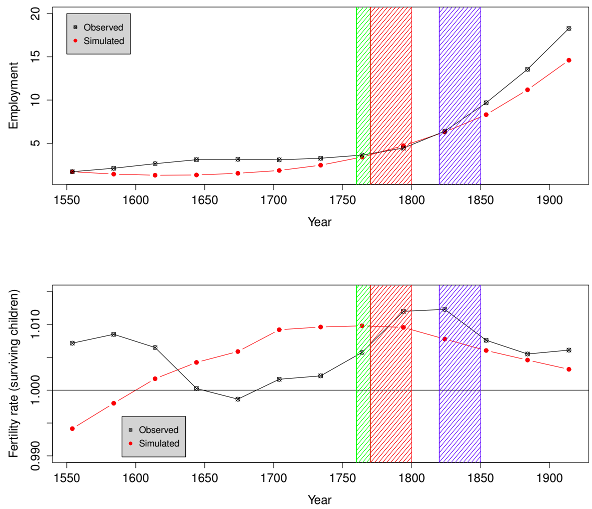

Figure 13: Observed (black) versus simulated (red) employment and annual fertility rate (surviving children at 20 years) of the UK for the period 1541-1914.

As expected, the consumption rate appears to decrease over time (see Fig. 14); until the end of the eighteenth century this decline is the outcome of a constant consumption at subsistence level and of an increasing GDP per worker; in particular, the not-consumed resources are devoted to increase fertility (see 13). The decline of the consumption rate after 1800 is motivated by the increasing saving and education rates; simulated saving rates, differently from the exogenous fertility case, display a more plausible pattern. Finally, the simulated real interest rate in 1914 closely matches the observed one, as well as the dynamics of labor share in the traditional sector.

The main drawback of the endogenous fertility case seems, indeed, to be that of the fertility choices of individuals. While the decreasing trend in fertility starting around 1800 is well captured (apart from a slight lag in the observed turning point of the fertility rate), the dynamics before 1800 seem hard to replicate with the actual theoretical model.

VI. Concluding Remarks

This paper contributes to the literature on the role of mortality reduction on economic growth by accounting for its differential effects during the various regimes of economic development. The rise in technological progress always favors the transition from a Malthusian to a Modern Regime. Instead, the mortality decline can have opposite effects on the transition. At low levels of income such reduction can impede the transition or, worse, push back an economy to Malthusian Regime. This result is driven by the presence of a fixed factor of production (land) in the traditional sector, which implies diminishing returns of labor. A decrease in mortality in Pre-Modern Regime, indeed, increasing the workforce, pushes the income of workers downward, and, finally, leading to a decrease in the intergeneration transfer of resources (in the worst case of no transfers the economy returns to a Malthusian Regime). On the contrary, at high levels of income it favors the transition, by fostering investment in fixed and human capital. The presence of endoge-

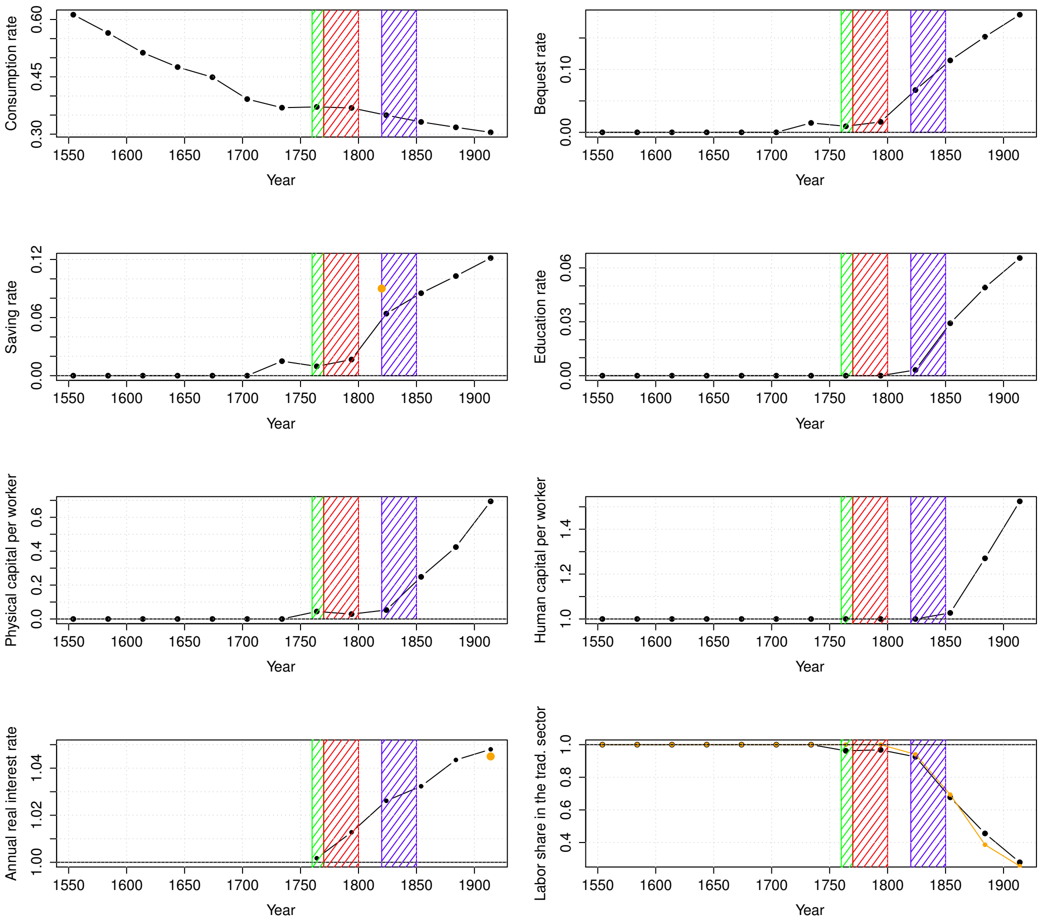

Figure 14: Time path of the main variables of the model (black points), and of relative observed values when available (orange points).

nous fertility dampens the effects analyzed above but does not qualitatively change the key findings.

The quantitative exercise shows that the model, when fertility is exogenous, is able to reproduce the observed transition from stagnation to growth. By simulating the model, indeed, we show that the long-run behavior of income per worker follows the empirical evidence. By contrast, the model with endogenous fertility partially reproduces the observed path of income per worker and fertility. This result suggests the need for a more complex theory of fertility to take into account the possible natural and social constraints on theoretical fertility.

Finally, the introduction of endogenous mortality should not affect the qualitative results of the paper but should merely add a possible self-reinforcing mechanism to the transition from stagnation to growth. In particular, some authors argue that the acquisition of human capital has the side-effect of increasing the diffusion of the best practises in personal hygiene, leading, in particular, to a fall in infant mortality (Easterlin, 2004; Mokyr, 1993; Ljungberg, 2013).

Affiliation and Address of the Authors
Davide Fiaschi, Dipartimento di Economia e Management, Università di Pisa. Address: Via Ridolfi 10, 56124 Pisa, Italy Tel: +39 050 2216208. Email: dfiaschi@ec.unipi.it

Tamara Fioroni, Dipartimento di Scienze Economiche, Università di Verona. Address: Vicolo Campofiore 2, 37129 Verona, Italy. Tel.+39 0458425457. Email: tamara.fioroni@univr.it.

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A *Optimal Choices*

AA. Exogenous Fertility

The individual's maximization problem is given:

$$(c_t^*, b_t^*) = \arg\max_{c_t, b_t}\{p_t[(1-\beta)\log(c_t) + \beta\log(b_t+\theta)]\}, \tag{55}$$

subject to:

$$y_t = p_t c_t + p_t b_t;$$

$$c_t \geq c^{\text{MIN}};$$

$$b_t \geq 0.$$

The Lagrangian for problem (55) is given by:

$$\mathscr{L} = p\left[(1-\beta)\log\left(\frac{y_t - p_t b_t}{p_t}\right) + \beta\log(b_t+\theta)\right] + \lambda b_t + \mu\left(\frac{y_t - p_t b_t}{p_t} - c^{\text{MIN}}\right) \tag{56}$$

and the first order conditions are:

$$\frac{\partial \mathscr{L}}{\partial b_t} = p_t\left[-\frac{(1-\beta)p_t}{y_t - p_t b_t} + \frac{\beta}{b_t+\theta}\right] + \lambda - \mu = 0.$$

$$\lambda b_t = 0$$

$$\mu\left(\frac{y_t - p_t b_t}{p} - c^{\text{MIN}}\right) = 0$$

Thus we can have different cases:

1. $c_t = c^{\text{MIN}}$ and $b_t = 0$.

2. $c_t > c^{\text{MIN}}$ and $b_t = 0$. Thus we have that $c_t = \frac{y_t}{p_t}$. This implies that $c_t > c^{\text{MIN}}$ if $y_t > p_t c^{\text{MIN}}$.

3. $c_t > c^{\text{MIN}}$ and $b_t > 0$. Thus solving the first order conditions we get:

$$c_t^* = \frac{(1-\beta)(y_t + p_t\theta)}{p_t} \tag{57}$$

$$b_t^* = \frac{\beta y_t - \theta(1-\beta)p_t}{p_t} \tag{58}$$

4. We do not consider the case $c_t = c^{\text{MIN}}$ and $b_t > 0$.

AB. Endogenous Fertility

When fertility is endogenous the agent's maximization problem is given:

$$(c_t^*, b_t^*, n_t^*) = \underset{c_t, b_t, n_t}{\arg\max}\{p_t[(1-\beta)\log(c_t) + \epsilon\log(n_t) + \beta\log(b_t+\theta)]\}, \tag{59}$$

subject to:

$$y_t = \delta y_t n_t + p_t c_t + p_t b_t;$$

$$c_t \geq c^{\text{MIN}};$$

and

$$b_t \geq 0.$$

The Lagrangian for this optimization problem is given as follows:

$$\mathscr{L} = p_t\left[(1-\beta)\log c_t + \epsilon\log\left(\frac{y_t - p_t(c_t+b_t)}{\delta y_t}\right) + \beta\log(b_t+\theta)\right] + \lambda b_t + \mu\left(c_t - c^{\text{MIN}}\right) \tag{60}$$

and the first order conditions are:

$$\frac{\partial\mathscr{L}}{\partial c_t} = \frac{(1-\beta)}{c_t} - \frac{\epsilon p_t}{y_t - p_t(c_t+b_t)} + \mu = 0. \tag{61}$$

$$\frac{\partial\mathscr{L}}{\partial b_t} = -\frac{\epsilon p_t}{y_t - p_t(c_t+b_t)} + \frac{\beta}{b_t+\theta} + \lambda = 0. \tag{62}$$

$$\lambda b_t = 0$$

$$\mu\left(c_t - c^{\text{MIN}}\right) = 0$$

Thus we can have different cases:

1. $c_t = c^{\text{MIN}}$ and $b_t = 0$. Thus given $c_t = c^{\text{MIN}}$, from the budget constraint we get:

$$n_t^* = \frac{y_t - p_t c^{\text{MIN}}}{\delta y_t} \tag{63}$$

2. $c_t > c^{\text{MIN}}$ and $b_t = 0$. Given that $\mu = 0$, we solve the first order condition (61) with respect to c_t:

$$c_t^* = \frac{(1-\beta)y_t}{(1-\beta+\epsilon)} \tag{64}$$

Substituting this solution into the budget constraint, the optimal number of children, is given by:

$$n_t^* = \frac{\epsilon}{\delta(1-\beta+\epsilon)} \tag{65}$$

3. $c_t > c^{\text{MIN}}$ and $b_t > 0$ Thus given $\mu = 0$ and $\lambda = 0$, from the first order conditions (61), (62) and the budget constraint we get:

$$c_t^* = \frac{(1-\beta)(y_t + p_t\theta)}{1+\epsilon} \tag{66}$$

$$b_t^* = \frac{\beta y_t - \theta(1-\beta+\epsilon)p_t}{1+\epsilon} \tag{67}$$

$$n_t^* = \frac{\epsilon(y_t + \theta p_t)}{\delta(1+\epsilon)y_t} \tag{68}$$

4. We do not consider the case $c_t = c^{\text{MIN}}$ and $b_t > 0$.

AC. Optimal Education

The maximization of Eq. (31) yields the following solution for the optimal education:

$$\bar{e_t}^* = \begin{cases} 0 & \text{if } \bar{b_t} \in [0, \tilde{b}]; \\ \dfrac{\bar{b_t} - \tilde{b}}{1 + D\tilde{b}} & \text{if } \bar{b_t} \in (\tilde{b}, \infty) \end{cases} \tag{69}$$

where:

$$\tilde{b} \equiv \frac{\alpha}{D(1-\alpha)\gamma}. \tag{70}$$

From Eqq.(30) and (69) the level of income such that $\bar{b}_t = \tilde{b}$, when fertility is exogenous, is given as follows:

$$y^{\text{EDU}} \equiv \frac{\tilde{b}n_t + \theta(1-\beta)p_t}{\beta} \tag{71}$$

When fertility is endogenous, y^{EDU} is calculated using Eq. (43):

$$y^{\text{EDU}} \equiv \frac{\tilde{b}(1+\epsilon) + \theta(1-\beta+\epsilon)p_t}{\beta}. \tag{72}$$

AD. Thresholds

From Eqq. (13) when production is conducted using traditional technology per-worker income at time $t+1$ is given by:

$$y_{t+1} = A_{t+1}p_{t+1}^{1-\alpha}\left(\frac{\tilde{T}}{L_{t+1}}\right)^{\alpha} \tag{73}$$

Thus, from Eq. (22), income per worker when production is conducted using traditional technology ensures an income at least equal to the subsistence level (i.e. $p_t c^{\text{MIN}}$) if:

$$A_{t+1}p_{t+1}^{1-\alpha}\left(\frac{\tilde{T}}{L_t}\right)^{\alpha} \geq p_t c^{\mathrm{MIN}} \tag{74}$$

which implies that:

$$A_{t+1} \geq A^{\mathrm{MIN}} \equiv \frac{p_t c^{\mathrm{MIN}}}{p_{t+1}}\left(\frac{p_{t+1}L_{t+1}}{\tilde{T}}\right)^{\alpha}. \tag{75}$$

If p and L are constant over time we get:

$$A^{\mathrm{MIN}} \equiv c^{\mathrm{MIN}}\left(\frac{pL}{\tilde{T}}\right)^{\alpha}. \tag{76}$$

B Proof Proposition 1

In what follows we assume that $p_{t+1} = p_t = p$, $L_{t+1} = L_t = L$ (thus $n_t = 1$).
From Eq.(36) when $y_t \in (y^{\mathrm{EDU}}, \infty)$, :

$$\frac{\partial y_{t+1}}{\partial y_t} = \frac{A_{t+1}p^{1-\alpha}\alpha\beta}{n_t}\left[B(C+Qy_t-F)^{-\frac{\alpha}{\gamma}} + \tilde{b}(C+Qy_t-F)^{1-\gamma}\right]^{\alpha-1}, \tag{77}$$

where for simplicity we set $B = \tilde{T}/L$, $C = 1/(1+D\tilde{b})$ $Q = D\beta/(1+D\tilde{b})$, $F = \theta(1-\beta)p/(1+D\tilde{b})$. From Eq.(77), simple calculations show that:

$$\lim_{y_t\to\infty}\frac{\partial y_{t+1}}{\partial y_t} = 0. \tag{78}$$

Given this condition, the economy shows one stable equilibrium in the Traditional Regime and possibly one unstable and one stable equilibrium in the Pre-Modern Regime if the following conditions hold:

$$\lim_{y_t\to y^{\mathrm{CAP}}} y_{t+1} \leq y^{\mathrm{CAP}} \tag{79}$$

$$\lim_{y_t\to y^{\mathrm{EDU}}} y_{t+1} \leq y^{\mathrm{EDU}}. \tag{80}$$

- The first condition holds if $A_{t+1} < A^{\mathrm{TRA}}$:

$$A^{\mathrm{TRA}} \equiv \frac{\theta(1-\beta)}{\beta}\left(\frac{pL}{\tilde{T}}\right)^{\alpha} \tag{81}$$

  where $A^{\mathrm{MIN}} < A^{\mathrm{TRA}}$ if assumption 21 holds.

- The second condition holds if:

$$A \leq A^{\mathrm{PRE\text{-}MOD}} \equiv \frac{\tilde{b}+\theta(1-\beta)p^{\alpha}}{\beta p^{1-\alpha}\left(\frac{\tilde{T}}{L}+\tilde{b}\right)^{\alpha}} \tag{82}$$

where $\lim_{p\to 0} A^{\text{PRE-MOD}} = \infty$ and $\partial A^{\text{PRE-MOD}}/\partial p < 0$ if:

$$p < p^T = \frac{(1-\alpha)\tilde{b}}{\theta(1-\beta)\alpha}. \tag{83}$$

An economy shows one stable equilibrium in the Pre-Modern Regime if:

$$\lim_{y_t \to y^{\text{CAP}}} y_{t+1} \geq y^{\text{CAP}}, \tag{84}$$

$$\lim_{y_t \to y^{\text{EDU}}} y_{t+1} \leq y^{\text{EDU}}, \tag{85}$$

- The first condition holds if $A \geq A^{\text{TRA}}$
- The second condition holds if $A \leq A^{\text{PRE-MOD}}$

An economy shows one stable equilibrium in the Modern Regime if:

$$\lim_{y_t \to y^{\text{CAP}}} y_{t+1} \geq y^{\text{CAP}}, \tag{86}$$

$$\lim_{y_t \to y^{\text{EDU}}} y_{t+1} \geq y^{\text{EDU}}, \tag{87}$$

- The first condition holds if $A > A^{\text{TRA}}$
- The second and third conditions hold if $A > A^{\text{PRE-MOD}}$

C Quantitative Exercise

CA. Details on Table 1

All variables are estimated using a nonparametric method. In what follows we describe the procedures used to compute the variables listed in Table 1.

- *The labor share in the traditional sector* l_t^a is estimated using the data on the labor share in the agricultural sector from Broadberry et al. (2013) for the period 1700 to 1851. For the period 1852-1914 the data is taken from the IHS. In particular, we adjust the data on the labor share in the agricultural sector to match the labor share in our model which is set equal to one until the end of the Malthusian Regime (i.e. 1800, see the black curve in Fig. 15). This assumption is supported by the pattern of the land share in national income which is nearly constant until the end of the eighteenth century and then start to decline. The data on the land share in the traditional sector are taken from Clark (2010).
- *The employment* pL. The labor force, according to theoretical model is given by people between the ages of 20 and 70 (i.e. Pop_t^{20-70}) and it is adjusted to take into account both the unemployment rate and the fact that some individuals do not belong to the labor force (15%) (see Fig. 16). In particular, the population

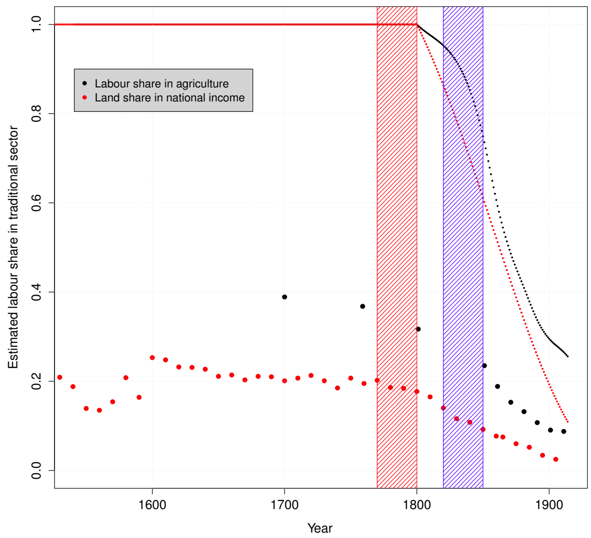

Figure 15: labor share in the traditional sector (black curve) and land share on national income (red curve).

between the ages 20 and 70 for the period $1541-1851$, is calculated using the data on cohorts from Wrigley and Schofield (1989), and for the period $1851-1914$, the data from the IHS.

Regarding the unemployment rate the first data available dates from 1855. Thus, the unemployment rate in the period 1541 to 1854 is assumed equal to the average unemployment rate for the period $1855-1880$. From 1855 to 1914, instead, we consider a smoothed unemployment rate. Fig.16 shows the results of our estimate for total employment.

- *The GDP per worker* y_t is calculated using the data on income per capita, i.e. $y_t^{p.c}$, from MP (2014). In particular the data until 1850 is for England and Wales only, and after 1850 is for the United Kingdom (defined as England, Wales and Scotland). In particular GDP per worker is calculated as:

$$y_t = \frac{y_t^{p.c} L_t^{TOT}}{p_t L_t}, \tag{88}$$

where L_t^{TOT} is the observed total population. The data on L_t^{TOT} is taken, for the period 1541 to 1850 (England and Wales) from Wrigley and Schofield (1989) and, for the period 1850 to 1914 (England, Wales and Scotland) from the IHS.

- *The probability of surviving from 20 to 70 years* p, is calculated as the ratio between the population 70 years old in a given period (for example 1591), i.e. Pop_{t+1}^{70}, and the population 20 years old 50 years before (for example 1541),i.e. Pop_t^{20}:

$$\hat{p} = \frac{Pop_{t+50}^{70}}{Pop_t^{20}} \tag{89}$$

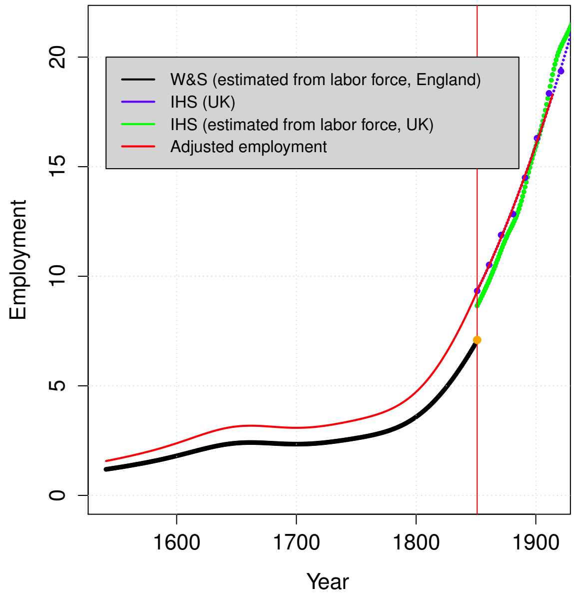

Figure 16: Employment 1541-1914

The data for the period 1541 to 1821 is taken from Wrigley and Schofield (1989) and for the later period (England, Wales and Scotland) from the IHS and HMD (see Fig. 17).

The green curve in Fig. 17 represents the adjusted adult survival rate in order to have a smoothed adult survival for the whole period.

If we consider that the workforce is given by cohorts between the ages of 20 and 70, then a more careful definition of p implies that it could be considered as the share of the surviving workforce. Thus to compute this variable we use the following procedure.

We assume to have 50 cohorts of individuals in the range of age $20-50$ and that in each period $\tilde{L}$ new individuals reach 20 years old. If we define as ρ the probability of a 20 year old agent of surviving to the next year, the total observed population in the range of age $20-50$ is:

$$Pop^{20-70} = \rho\tilde{L} + \rho^2\tilde{L} + ... + \rho^{50}\tilde{L}, \tag{90}$$

i.e.:

$$Pop^{20-70} = \rho\tilde{L}\left(\frac{1-\rho^{50}}{1-\rho}\right). \tag{91}$$

Thus if $\rho = 1$ (i.e. no deaths in adulthood) we have the maximum workforce:

$$Max\_Pop^{20-70} = 50\tilde{L}. \tag{92}$$

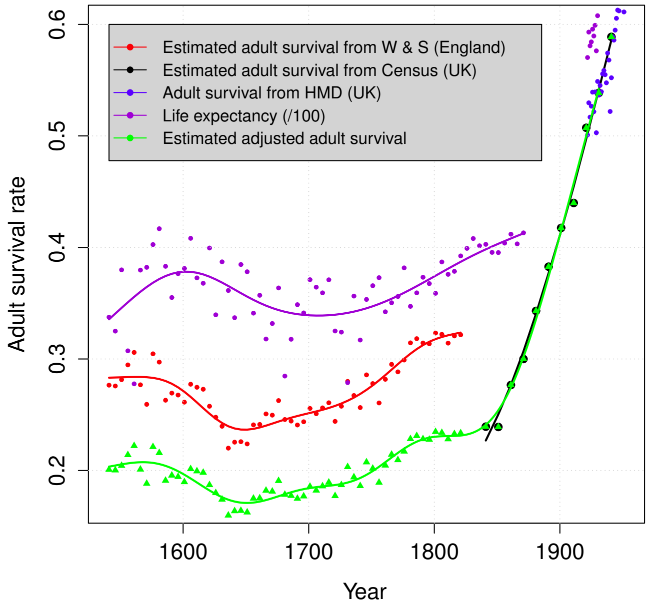

Figure 17: Adult Survival.

The ratio between Pop^{20-70} and Max_Pop^{20-70} measures the impact of adult mortality on the size of the workforce:

$$\frac{Pop^{20-70}}{Max\_Pop^{20-70}} = \frac{\rho}{50}\left(\frac{1-\rho^{50}}{1-\rho}\right). \tag{93}$$

As specified above the probability of surviving from 20 to 70 is given:

$$\hat{p} = \frac{Pop^{70}}{Pop^{20}} = \frac{\rho^{50}\tilde{L}}{\rho\tilde{L}} = \rho^{49}. \tag{94}$$

Therefore, the ratio between Pop^{20-70} and Max_Pop^{20-70} has a (strong) concave relationship with $\hat{p}$:

$$\frac{Pop^{20-70}}{Max\_Pop^{20-70}} = \frac{\hat{p}^{1/49}}{50}\left(\frac{1-\hat{p}^{50/49}}{1-\hat{p}^{1/49}}\right). \tag{95}$$

If we consider an arbitrary q number of cohorts (e.g. every 1 month) then:

$$\frac{Pop^{20-70}}{Max\_Pop^{20-70}} = \frac{\hat{p}^{1/(q-1)}}{q}\left(\frac{1-\hat{p}^{q/(q-1)}}{1-\hat{p}^{1/(q-1)}}\right). \tag{96}$$

In the limit of $q \to \infty$ (i.e. the continuous time case) the share of surviving workforce is given by:

$$\frac{Pop^{20-70}}{Max\_Pop^{20-70}} = \frac{\hat{p}-1}{\log(\hat{p})}. \tag{97}$$

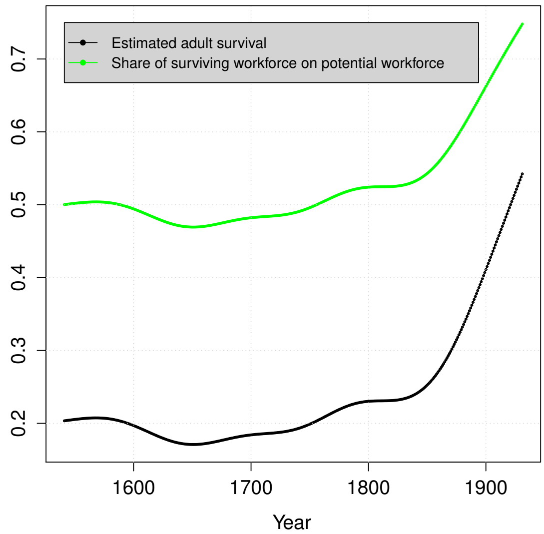

Figure 18: Estimated adult survival vs the share of surviving workforce

Fig.(18) shows the path of the probability of surviving from 20 to 70 and of the share of surviving workforce.

- *Fertility* n_t, using Eq. (28) can be calculated as the growth rate of employment, that is:

$$n_t = \frac{p_{t+1}L_{t+1}}{p_t L_t}\frac{p_t}{p_{t+1}} \tag{98}$$

where pL is the employment calculated as specified before.

In Fig.(19) we compare the fertility rate calculated as the growth rate of employment and the fertility rate calculated as the growth rate of total population between age $15-24$:

CB. Details on Table 2

- *labor share on total output* $1-\alpha = 0.6$ is chosen to be equal to its average value in the period $1541-1914$ (see red line in Fig. 20). Moreover, Clark (2007) (p.138), Hansen and Prescott (2002), Allen (2009) use the same value.

- *The minimum level of GDP per capita* set equal to 0.67 is chosen close to the value of 0.6 (in thousands of 1990 international GK \$) as reported in the MPD for England in year 1 .

- *The real interest rate in 1914* is set equal to 4.5% according to the estimates of the Bank of England as shown in Fig. 21.[21]

[21] According (Mathias, 2001, p. 133) the interest rate in the nineteen century was about 5% for personal loans and businessmen.

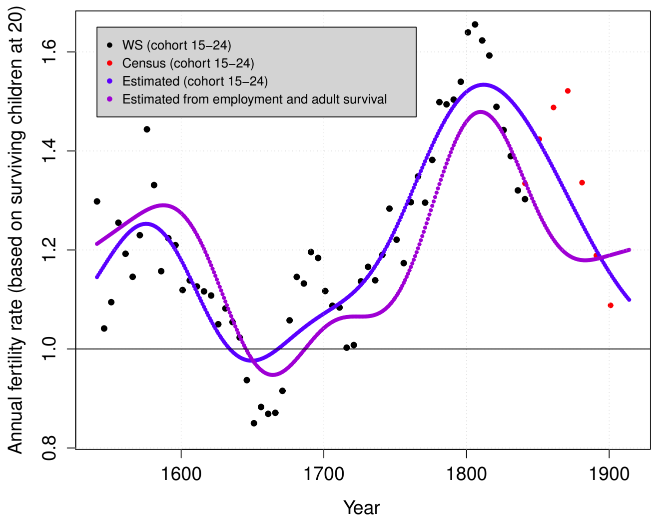

Figure 19: Fertility rate

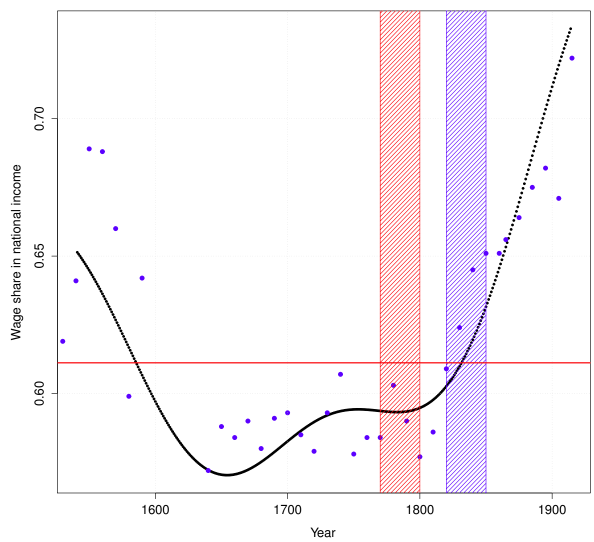

Figure 20: Wage share in national income in 1541 – 1914. Source: Clark (2010, Tables 13, 34)

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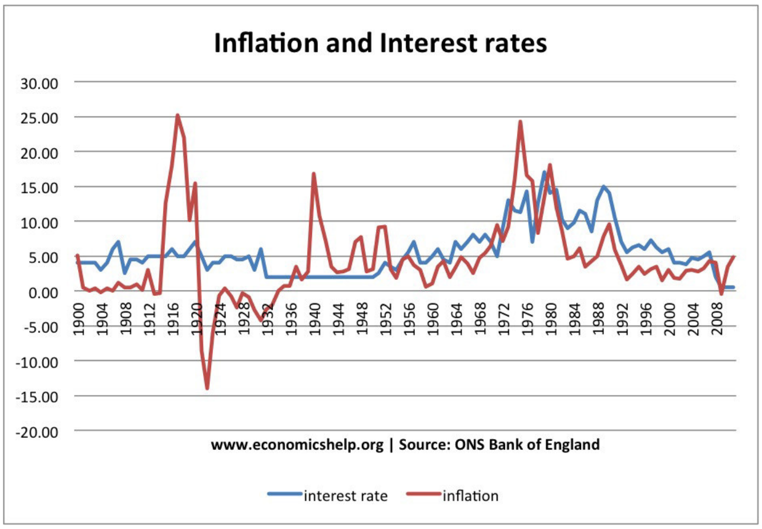

Figure 21: Nominal interest rates and inflation rates in UK in 1900-2008. Source:http://www.economicshelp.org/wp-content/uploads/blog-uploads/2012/01/inflation-interest-rates-1900-2011.png

- *The investment rate* $i = 0.09$ *in* 1850 is set equal to its average value in the period $1840 - 1900$ as shown in Fig. 22.

CC. Details on parameters'calculation reported in Table 3

Henceforth CAP denotes the year in which the economy enters the Pre-Modern regime (i.e. 1800) and EDU the year in which the economy enters the modern regime (i.e. 1850).

- *Subsistence Consumption.* To calculate the subsistence consumption c^{MIN} we first calculate the minimum level of GDP per worker y^{MIN}. It is calculated from the value of minimum GDP per capita and the ratio population/employment in 1541, i.e.:

$$y_{\text{MIN}} \equiv \frac{y_{\text{MIN}}^{p.c.} L_{1541}^{\text{TOT}}}{p_{1541} L_{1541}} \tag{99}$$

  The subsistence consumption is thus given as:

$$c^{\text{MIN}} = \frac{y^{\text{MIN}}}{p^{\text{MIN}}} \tag{100}$$

- *Fixed factor in traditional technology.* From Eq. (11) $\tilde{T}$ can be calculated as a function of $\tilde{b}$ since in the transition's year from the Pre-Modern Regime to Modern Regime, i.e. $EDU = 1850$, $k_{t+1} = k_{1850} = \tilde{b}$. Thus we first calculate $\tilde{b}$ as a function of the investment rate, income per worker and fertility in 1850:

$$\tilde{b} = \frac{i_{1850} y_{1850}}{n_{1850}}, \tag{101}$$

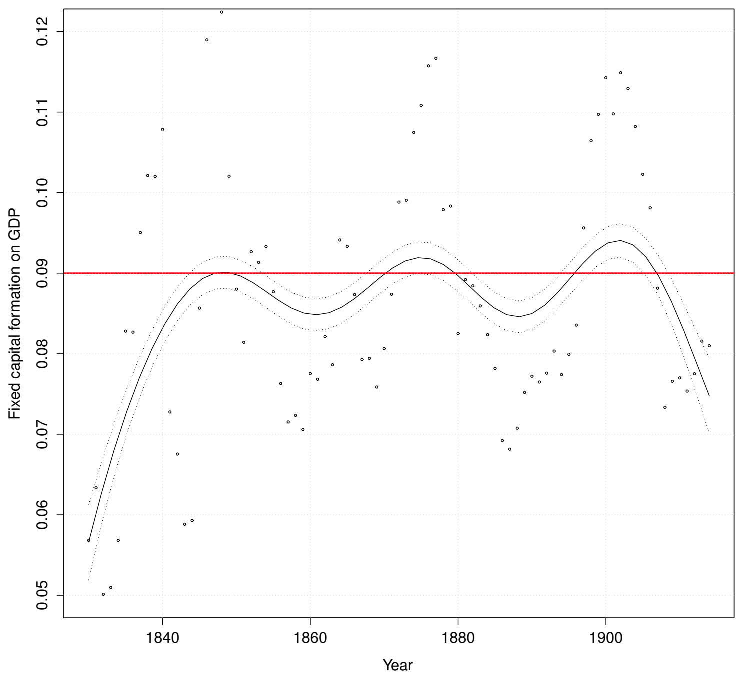

Figure 22: Fixed capital formation in UK in 1830-1914. Source: IHS (update 4/2013)

thus:

$$\tilde{T} = \tilde{b} L_{1850} \left(\frac{l^a_{1850}}{1 - l^a_{1850}} \right), \tag{102}$$

CC.i. Exogenous Fertility

- *Altruism factor.* From Eqq. (25) and (33) β can be calculated as follows:

$$\beta = \frac{\tilde{b} n^{\mathrm{EDU}}}{y^{\mathrm{EDU}} - \dfrac{y^{\mathrm{CAP}} p^{\mathrm{EDU}}}{p^{\mathrm{CAP}}}}. \tag{103}$$

- *Bequest factor.* From Eqq. (25) and (103) the parameter θ can be calculated as follows:

$$\theta = \frac{y^{\mathrm{CAP}} \beta}{(1-\beta) p^{\mathrm{CAP}}}. \tag{104}$$

- *Return to education.* To compute the parameter γ we use the value of the real interest rate in 1914. In particular from Eqq. (4) and (13) we get:

$$h_{t+1} = \left[\frac{\left(\dfrac{y_{t+1}}{A_{t+1} p_{t+1}^{1-\alpha}} \right)^{1/\alpha} - \frac{\tilde{T}}{L_{t+1}}}{p_{t+1}(1 - l^a_{t+1}) \left(\dfrac{\alpha A_{t+1}}{r_{t+1}} \right)^{1/(1-\alpha)}} \right]^{\alpha}. \tag{105}$$

From eqq. (17), (32) and (34), h_{t+1} it is also given as follows:

$$h_{t+1} = \left\{ \frac{\gamma\tilde{b}(1-\alpha)n_t + \alpha[\beta y_t - \theta(1-\beta)p_t]}{\tilde{b}n_t[(1-\alpha)\gamma + \alpha]} \right\}^{\gamma}. \tag{106}$$

Thus substituting all variables at time $t+1 = 1914$ and at time $t = 1880$ (remember that the length of a generation is 30 years) in Eqq. (105) and (110) we can compute the parameter γ.

- *Scale parameter in education.* The scale parameter D is then calculated using Eq.(34):

$$D = \frac{\alpha}{\tilde{b}(1-\alpha)\gamma} \tag{107}$$

CC.ii. Endogenous fertility

- *Altruism factor.* From Eqq. (45), (46), (51), β can be calculated as follows:

$$\beta = \frac{\tilde{b}y^{\text{SUB}}/p^{\text{SUB}}c^{\text{MIN}}}{y^{\text{EDU}} - y^{\text{CAP}}p^{\text{EDU}}/p^{\text{CAP}} + \tilde{b}(y^{\text{SUB}} - p^{\text{SUB}}c^{\text{MIN}})/p^{\text{SUB}}c^{\text{MIN}}} \tag{108}$$

- *Bequest factor.* From Eqq. (46) and (108) the parameter θ can be calculated as follows:

$$\theta = \frac{y^{\text{CAP}}\beta}{(1-\beta+\epsilon)p^{\text{CAP}}}. \tag{109}$$

- *Return to education.* To compute the parameter γ we use the value of the real interest rate in 1914. In particular, we use Eq. (105) and the human capital production function given as follows:

$$h_{t+1} = \left\{ \frac{\gamma\tilde{b}(1-\alpha)n_t(1+\epsilon) + \alpha[\beta y_t - \theta(1-\beta+\epsilon)p]}{\tilde{b}(1+\epsilon)n_t[(1-\alpha)\gamma + \alpha]} \right\}^{\gamma}. \tag{110}$$

where n_t is given by Eq. (44).

- *Taste for children.* From Eqq. (45) and (108) the parameter ϵ is calculated as follows:

$$\epsilon = \frac{(1-\beta)(y^{\text{SUB}} - p^{\text{SUB}}c^{\text{MIN}})}{p^{\text{SUB}}c^{\text{MIN}}}. \tag{111}$$

- *Cost parameter of raising children.* The parameter δ is calculated using the equilibrium level of income in the Malthusian regime (1690). Thus from Eq. (47):

$$\delta = 1 - \frac{pc^{\text{MIN}}}{y^{MAL}} \tag{112}$$

A more careful definition of the cost of raising children should also include their consumption. In other words, at the Malthusian equilibrium (i.e. $n = 1$):

$$y^{\text{MIN}} = c^{\text{MIN}} + \bar{\delta}y^{\text{MIN}} + \eta c^{\text{MIN}} \tag{113}$$

where η is the fraction of parent's minimum consumption as consumed by children. Thus $\delta y^{\text{MIN}} = \bar{\delta} y^{\text{MIN}} + \eta c^{\text{MIN}}$. Therefore if we include the consumption of children we get:

$$\bar{\delta} = 1 - \frac{(1+\eta) c^{\text{MIN}}}{y^{\text{MIN}}}. \tag{114}$$

Discussion Papers
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