

KEYNESIAN AND NEW-KEYNESIAN MODELS:  
THE IMPACT OF MILITARY SPENDING ON THE  
UNITED STATES ECONOMY

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**PhD. in Economics and Finance**

ACADEMIC YEAR 2010-2011

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## TABLE OF CONTENTS

1) <b>Introduction.</b>	p.1
2) <b>The effects of military spending on the US economy: an IS-MP approach.</b>	p.4
2.1) Introduction	p 4
2.2) Theory: a simple macroeconomic model	p 6
2.3) The econometric framework	p.9
2.4) Testing model implication	p. 12
2.4.1) Data	p. 12
2.4.2) Cointegration tests, estimated cointegrating vectors and policy implication	p. 15
2.5) Concluding remarks	p. 20
2.6) Appendix	p. 22
References	p. 24

### 3) How determinant are military spending shocks on the economy?

<b>A Bayesian DSGE approach for the US</b>	p. 27
3.1) Introduction	p. 27
3.2) The model	p. 30
3.2.1) Households	p. 30
3.2.2) Firms	p. 32
3.2.3) Fiscal policy	p. 34
3.2.4) Monetary policy	p. 36
3.2.5) General equilibrium and aggregation	p. 37
3.2.6) The linearized model	p. 38
4.3) Estimation results	p. 42
3.3.1) Data description	p. 42
3.3.2) Estimation methodology	p. 44
3.3.3) Priors distribution of the parameters	p. 46
3.3.4) Posterior estimates of the parameters	p. 51
3.4) Analysing the effects of fiscal shocks on the economy	p. 55

3.4.1) Model with aggregate government spending	p. 55
3.4.2) Model with non-military and military spending components	p. 58
3.5) Concluding remarks	p. 60
3.6) Appendix A: maximization problems of the model	p. 63
3.7) Appendix B: steady states and log-linearized equations	p. 70
References	p. 95
<b>4) The role of government spending components: a re-examination of the effects on private consumption</b>	p. 99
4.1) Introduction	p. 99
4.2) Specification and identification of the model	p. 103
4.3) Estimation results	p. 108
4.4) A baseline model	p. 115
4.4.1) Households	p. 116
4.4.2) Firms	p. 120
4.4.3) Monetary policy	p. 122
4.4.4) Fiscal policy	p. 122

4.4.5) General equilibrium and aggregation	p. 125
4.4.6) Linearized equilibrium conditions	p. 126
4.5) Model calibration	p. 130
4.6) The effects of government spending shocks	p. 132
4.7) Conclusions	p. 137
4.8) Appendix A: maximization problems of the model	p. 139
4.9) Appendix B: log-linearized equilibrium conditions	p. 149
4.10) Appendix C: non-competitive labour market	p. 174
References	p. 181
<b>5) Conclusions</b>	p. 185

# 1 *INTRODUCTION*

The objective of the present thesis is to analyze whether the government defence expenditure, as a component of total public spending, is able to affect the economic performance of U.S., and/or account for the potential role in explaining fiscal policy fluctuations. Broadly speaking, our work aims to answer to the following question: does military spending provide economic stimulation through higher aggregate demand for goods and services, or does military spending retard economic performance because it draws resources from more productive activities that can be devolved to the civilian sector?

The present thesis is composed by three chapters which capture different aspects about these arguments. The first chapter empirically assesses the so called "Military Keynesianism", i.e. the approach that treats the military budget as a source of aggregate demand for goods and services and, therefore, a source of economic stimulation. The military Keynesianism took centre stage in the policy debate with John Maynard Keynes, who argued that in extreme situations the government should spend on anything as a means of stimulating aggregate demand. Thus, the aim of this chapter is to empirically test the Keynesian hypothesis, by using a long-run equilibrium model for the U.S. economy. Our contribution, with respect to previous works, is twofold. First, our inferences are adjusted for structural breaks exhibited by the data concerning fiscal and monetary variables. Second, we show that the results are sensitive to sub sample choices.

In the second chapter, our goal is to disentangle the components of government spending in civilian and military expenditures into a standard DSGE new-Keynesian model and analyze their role on the U.S. economy, with particular attention on private consumption and wages. In particular, we focus on the changes in the effects of public spending components before and after a structural break that occurred in U.S. economy around 1980. We assume that this break is related to a change in

consumer behaviour, i.e. the increased asset market participation.

From a theoretical point of view, we assume a standard Dynamic Stochastic General Equilibrium Model (DSGE) with an economy with sticky prices and limited asset market participation. Moreover, we assume the existence of a fiscal policy authority that purchases consumption goods (divided in spending for military and non-military sectors), raises (lump-sum and income) taxes and issues nominal debt. Finally, we include a monetary authority, which sets its policy instrument, the nominal interest rate. We estimate the theoretical model with a Bayesian approach, the so called "strong econometric approach", which allows us to provide a full characterization of the data generating process and a proper testing specification. The latter aspect is particularly important for the fiscal shocks assessment.

In the last chapter, we focus on government spending multiplier and, in particular, on the effects of different components of public spending on the U.S. economy. We disaggregate total government spending into civilian and military expenditures and estimate, through a structural VAR approach, their effects separately on GDP and private consumption. In this chapter, we introduce three main novelties with respect to previous literature. First, we analyze the effects of public spending on the economy accounting for "within" complementarity/substitutability of military and non-military expenditures. Second, we show that the financing mechanism of the different spending components is crucial for agent's decision about consumption. Finally, we assess that crowding in/out effects of government spending components on aggregate consumption are related to the existence of a precise portion of public expenditure that stimulates/depresses a fraction of consumers. In this chapter, we also develop a simple DSGE new-Keynesian model that can potentially account for that evidence. Our framework shares many ingredients with recent dynamic optimizing sticky price models, though we improve on the assumption of the fiscal sector by introducing non-military and military spending components. This allows us to show that our empirical results can be reproduced by the theoretical model by comparing

empirical and simulated impulse response functions.



## **2 THE EFFECTS OF MILITARY SPENDING ON THE US ECONOMY: AN IS-MP APPROACH**

### **2.1 INTRODUCTION**

One of the dominant approaches to macroeconomic research in the past several decades based on policy predictions is the IS-LM model. In this framework, the debates between Keynesians and monetarists concerning the effectiveness of monetary and fiscal policy played a central role in the analysis of short-run fluctuations (Romer, 2000). One assumption largely criticized of this aggregate macroeconomic model involved in the monetary policy behaviour of the central bank concentrated on the aims targeting money supply. On the other hand, empirical policy researches have shown that the central banks mainly use the tool of the interest rate to determine the monetary policy (MP) to characterize the money supply (Taylor, 2000).

Although such a framework is useful for understanding how the monetary policy affects the economy, through a closed relationship between inflation and real interest rates, it is ill-equipped to investigate how the fiscal policy shocks impact on aggregate output through the composition of government spending. Focus on the latter was motivated by our interest in understanding how society might best avoid the distortions created by the presence of misallocation of government spending. In this paper, we provide some empirical evidence of the effects of the composition of fiscal policy on the aggregate output when the categories of defence and civilian spending are explicitly distinguished within the government sector. Firstly, we assess the model by identifying fiscal policy shocks as motivating forces for the non-stationarity of output. Indeed, equilibrium of the IS-MP framework implies that, if the shock of government spending components, namely defence and civilian spending, are unobservable shocks  $I(1)$ , these forcing variables will determine a long-run equilibrium along with output and real interest rate. Secondly, we are interested in documenting

and discussing the effects of a particular kind of government spending – the defence spending – on the long-run output since the empirical evidence does not provide a clear picture if defence spending stimulates, through higher demand and innovations, the economy or retards economic performance by crowding out effects (Gold, 2005). Thus, this paper reviews the debate in line with the new-Keynesian approach developed by Atesoglu (2002), by updating the sample of data in the U.S.

Theoretically, the well-known hypothesis of the Keynesian approach, that treats the military budget as a source of aggregate demand for goods and services, suggests that positive government spending should induce economic stimulation by means of an income multiplier effect. In the extreme case, this government economic policy is known as military Keynesianism, when the fiscal policy devotes large amounts of spending to finance the defence sector (Mintz & Hicks, 1984). The channel through which military spending can affect the economy is based on boosting utilisation of capital stock and higher employment. Positive changes in capital stock utilisation may lead to increase profit rate which, in turn, drives to higher investment in short run (Dunne, Smith, & Willenbockel, 2004; Kollias, Manolas, & Paleologou, 2004; Smith & Dunne, 2001). However, the “utilisation effect of capital stock” may have much less pronounced output effects in a longer span (Dakurah, Davies, & Sampath, 2001; Dritsakis, 2004). The defence economics literature has identified with the opportunity cost of defence spending the effects of investment crowding out which turns out to be a drag on economic take-off (Sandler & Hartley, 1995).

Focusing our attention on empirical analysis, it is known that the robustness of the aforementioned test of a long-run model effect of defence spending on aggregate output could be better obtained by working with quarterly frequency data. For the US, National Income and Product Accounts (NIPA) produces quarterly data for the categories of government defence and civilian spending.

In summary, this paper theoretically justifies and empirically tests two hypotheses: (i) the effects of defence spending on output depend on the long-run equilibrium

model that also includes the variables of monetary policy and government civilian spending; (ii) defence spending, as a component of public spending, positively and significantly impact on the long-run output. In the US, the empirical identification of a cointegrating vector shows a coherence of data with the predictions of the Keynesian model. By assessing the estimated parameters of the models, we find that the relationship between defence spending and output is strongly sample-dependent with a fall in the elasticity values in more recent years.

The structure of this paper is as follows. We discuss conceptual issues in Section 2.2. Section 2.3 provides an overview of econometric specifications. Section 2.4 presents the data, shows tests for the identification of the model and discusses the estimation results to shed some light on the Keynesian effects of defence spending on output. Concluding remarks are offered in Section 2.5.

## **2.2 THEORY: A SIMPLE MACROECONOMIC MODEL**

In this section, an IS-MP model, that identifies the policy fiscal shocks by using the defence and civilian spending components of the public budget sector, will be formulated. To organize the discussion, a stripped-down baseline model is expounded as a version of the one described by Atesoglu (2002), to characterize a number of broad principles that underlie optimal policy management. We then consider fiscal policy implications by adding various real world complications to test how the prediction from theory is linked with policy-making in practice. Specifically, it will serve as a basis for the empirical work to assess the impact of the government defence spending on economic stimulation.

Because we are interested in characterizing fiscal policy rules in terms of composition of the government budget, the model we use evolves as in Romer (2000) and Taylor (2000), and is derived by assuming that the real interest rate is predetermined by the central bank<sup>1</sup>. The main change in the monetary policy rule is that it re-

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<sup>1</sup>In the complete version of the new macroeconomic model, the real interest rate is explained by additional

places the assumption to target the money supply with a simple interest rate rule, as supported by the central bank’s behaviour in the developed countries (Taylor, 1993).

On the other hand, the importance of this assumption may depend on its applications. For example, it might be reasonable to ignore that the real interest rate may depend on aggregate output, when applied to the effects of government spending in the civilian and defence categories, if the aim is to examine their effects on aggregate output rather than to assess the new-Keynesian model<sup>2</sup>.

Below, we formally document the theoretical framework and discuss the assumption of the model. Let  $R_{jt}$  denote the measure of type- $j$  interest rate chosen as a target indicator by the central bank in period  $t$  to drive the monetary policy<sup>3</sup>. Then, the aggregate output, the amount of the final goods and services produced in the economy, is denoted as  $Y_t$ . Since the aggregate income is  $Y_t = W(r_{jt})$ , the mathematical formulation of the IS equilibrium equation requires that:

$$Y_t = -\zeta R_t + \mu_t \tag{Eq. 1}$$

where  $\mu_t$  is a stochastic term that includes shocks of fiscal policy and/or net export. The right-hand side of the IS equation describes the known inverse relationship between the (real) interest rate targeted by the central bank’s choices and aggregate output. The stochastic term of equation (Eq. 1) plays a central role in the following analysis since we will concentrate our estimations on the effects of government defence spending. It is worth keeping in mind the intuitive meaning behind it. If this specific component of fiscal policy increase(s), the shock on the IS curve generates a positive shift on output and a new equilibrium in the output-real interest rate space is produced. Let  $M$  and  $G$  denote defence and civilian components of total government spending, respectively; we will identify these shocks as “fiscal policy shocks”.

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equations.

<sup>2</sup>It is worth noting that the straightforward assumption that the central bank is able to follow a real interest rate rule makes the model Keynesian.

<sup>3</sup>See Atesoglu (2007) for a discussion on the choice rule of the interest rate target.

Let us now turn to the real interest rate. This variable is assumed to be only dependent on inflation such that the behaviour rule generates a monetary policy (MP). For the sake of simplicity, we assume that:  $r_t = \bar{\pi}$ , where  $\bar{\pi}$  is the inflation rate assumed to be predetermined (known) by the central bank. A number of implications emerge from this baseline case on which monetary policy is firmly based. Focusing on evaluating fiscal policy shocks, the main result is that the central bank adjusts the nominal short rate one-for-one with perfect foresight of (expected future) inflation. That is, it should instantaneously adjust the nominal interest rate such that it does not alter the real interest rate (and aggregate demand).

To sum up, since the central bank's choice of the real interest rate is strictly predetermined by inflation rate, the real interest rate rule can be approximated by a horizontal line in the output–real interest rate space. Thus, the IS curve can be used to assess the impact of, government components of expenditure on aggregate output.

Rather than work through the details of the derivation, which are available in Appendix A, we directly introduce the key aggregate relationships by the reduced form of the model. For convenience, the theoretical framework abstracts from the way by which public expenditure was financed. This abstraction does not affect any qualitative conclusions as we will discuss. The model specification is formulated as follows:

$$Y_t = \beta_0^* + \beta_2^* M_t + \beta_3^* G_t + \beta_4^* R_t + \psi_t \quad (\text{Eq. 2})$$

where  $\psi_t$  term of equation (Eq. 2) contains net export shocks as shown in Appendix A.  $\beta^* = (\beta_0^*, \beta_2^*, \beta_3^*, \beta_4^*)$  represents the vector of parameters to be estimated. Though the model is quite simple, it nonetheless contains the main ingredients of richer frameworks that are used for policy analysis. Within the model, as in practice, the instruments of fiscal policy based on the composition of government spending, account for the short-term fluctuations. However, we would like to remark that

the presence of non-stationary (trending) variables of government expenditure might affect the long-run relationship. In the next section, a dynamic reduced form model will enable to test the presence of long-run effects of the Keynesian stimulus on the economy.

### 2.3 THE ECONOMETRIC FRAMEWORK

Given equation (Eq. 2), we discuss its specification as a cointegrated system. It firstly considers the vector autoregressive (VAR) formulation and describes the corresponding vector error correction (VECM) representation. In Section 4, this model will then be applied to test the impact of defence spending on output in the US.

Formally, we consider an extended  $VAR(p)$  specification for a  $m \times 1$  vector of variables:

$$X_t = \mu_0 + \mu_1 T + \phi_h D_{th} + \sum_{i=1}^p A_i X_{t-i} + \varepsilon_t \quad t = 1, \dots, T \quad (\text{Eq. 3})$$

with :  $\mu_i = (0, 1, \dots)$

and :  $D_{th} = \begin{cases} 0 & \text{if } t < h \\ 1 & \text{if } t \geq h \end{cases}$

where  $\mu_0$  is a  $m \times 1$  constant term,  $\mu_1$  is a  $m \times 1$  coefficient vector related to the deterministic trend,  $D_{th}$  is a  $d \times 1$  vector containing the likely presence of structural changes (shift dummies) and  $\phi_h$  the corresponding  $m \times d$  matrix of parameters<sup>4</sup>.  $A_i$  is a  $m \times m$  matrix of unknown parameters, while  $\varepsilon_t$  is a Gaussian white noise process with covariance matrix  $\Omega$  and  $p$  the lag order of the VAR. Equation (Eq. 3) can be

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<sup>4</sup>In the literature no exact definitions of structural breaks or structural changes have been given, since breaks or changes are interpreted as changes of regression parameters (Maddala and Kim, 1998). In what follows it is sufficient to refer to structural changes or structural breaks as changes of the deterministic components of the time series, such that the terms breaks and changes as equivalent.

rewritten in a *VECM* form as:

$$\Delta X_t = \mu_0 + \Pi X_{t-1}^* + \sum_{i=1}^{p-1} \Gamma_i \Delta X_{t-i} + \sum_{j=1}^{p-1} \gamma_j \Delta D_{t-j} + \varepsilon_t \quad (\text{Eq. 4})$$

$$\text{where : } \Pi = \left( \sum_{i=1}^p A_i - I_p \right)$$

$$\text{and : } \Gamma_i = - \sum_{j=t+1}^p A_j$$

$$\text{finally : } \gamma_j = -\Gamma_j \phi \text{ with } j = 1, \dots, p-1$$

The matrix of parameters  $\Pi$  ( $m \times m + 2$ ) describes the long-run relationships of the *VECM* among the variables in vector  $X_{t-1}^* = [X_{t-1}; D_t; T]'$ . A necessary condition is that the polynomial characteristics associated with the *VAR* can determine the stability of the system.  $\Gamma_i$  refers to the short-run dynamics of the system  $\Delta X_{t-i}$ , while  $\Delta D_{t-j}$  characterises the persistence of a shock of the variables included in the cointegration space by means of the vector of shift dummy variables.

Under general conditions, the *VECM* equation (Eq. 4) is  $I(1)$  and cointegrated and can be written as<sup>5</sup>:

$$\Delta X_t = \mu_0 + \alpha \beta^* X_{t-1}^* + \sum_{i=1}^{p-1} \Gamma_i \Delta X_{t-i} + \sum_{j=1}^{p-1} \gamma_j \Delta D_{t-j} + \xi_t \quad (\text{Eq. 5})$$

$$\text{where : } \beta^* = [\lambda', \eta, \theta]$$

$$\text{and : } \eta = -\lambda' \mu_1$$

$$\text{and : } \theta = -\lambda' \phi$$

In equation (Eq. 5)  $\alpha$  is a  $m \times r$  matrix,  $\beta^*$  is a  $(m+2) \times r$  matrix and  $r$  ( $0 < r < m$ ) is the cointegration rank of the system.

*VECM* equation (Eq. 5) is the extended model of this article. The residual

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<sup>5</sup>The set of the necessary and sufficient conditions so that Equation (Eq.4) is  $I(1)$  and cointegrated are: i) the roots of the characteristic polynomial are outside the unit circle; ii)  $\Pi_t = \alpha \beta'$  where  $\alpha$  and  $\beta'$  are matrices of full rank  $r'$ ,  $0 < r < m$ ; iii) the matrix obtained by multiplying the orthogonal complement of the matrix and the parameter matrix of long run is non-singular (Pesaran et al., 2000).

$r \times 1$  vector  $u_t = \beta^{*'} X_t^*$  in equation (Eq. 5) is trend-stationary and, under suitable unitary identifying normalization, can be interpreted as being a vector of deviations of observable variables from the long run equilibrium relationships.

With respect to the theoretical discussion in Section 2, we have assumed that the cointegrating rank is given by  $r = 1$ . The long run equilibrium levels are predicted by equation (Eq. 5) by identifying the block decomposition of  $X_t = (X'_{1t}, X_{2t})'$ , where  $X_{1t} = (Y_t)$  and  $X_{2t} = (M_t, G_t, R_t)$  is the  $3 \times 1$  vector containing real defence and civilian spending and the real interest rate. The deviation of estimations from observable output can therefore be obtained as:

$$u_{t-1} = \beta^* X_{t-1}^* = \begin{bmatrix} I_1, -\lambda'_2, -\vartheta, -\eta \end{bmatrix} \begin{bmatrix} X_{1t-1} \\ X_{2t-1} \\ D_t \\ T \end{bmatrix} = X_{1t-1} - \lambda'_2 X_{2t-1} - \vartheta D_t - \eta T \quad (\text{Eq. 6})$$

As we shall see in the next section, it is possible that some institutional decisions regarding monetary or fiscal policies can modify the structure of long-run patterns of time series. From an econometric point of view, their exclusion may be a cause of possible misspecifications of the model and of the inconsistency in the estimation results. In contrast, modelling structural changes influences the cointegrating rank inference. This question refers to the decision problem of whether one may still use the standard cointegration tests to avoid possible power losses and size distortions caused by modelling structural shifts or whether it is recommended to use cointegration tests that take such breaks into account. In the latter case, the proposals by Johansen, Mosconi, and Nielsen (2000) and Saikkonen and Lutkepohl (2000a) can be regarded as generalizations of the procedures by Johansen (1992, 1995) and Saikkonen and Lutkepohl (2000b), respectively.

In order to empirically test the best dynamic specification that rationalizes the data, nested models are obtained by setting  $\eta = 0$ , in which the presence of a linear



deterministic trend is excluded from equation (Eq. 6), or by setting  $\theta = 0$  where a model without a shift dummy is specified, or by a long run specification that restricts both the hypothesis tests. From the conditions to derive equation (Eq. 6), it follows that a cointegrated system is obtained by a reduced rank of the  $\Pi$  matrix. In a parsimonious long run dynamic model, inference on the number of cointegration relationships can be carried out by testing the hypothesis:

$$H(r) = \text{rank}(\Pi) \leq r \text{ against the alternative } H(m) = \text{rank}(\Pi) \leq m \quad (\text{Eq. 7})$$

for  $r = 0, 1, \dots, m - 1$ . By maximizing the log-likelihood of equation (Eq. 6) under both the null and alternative hypotheses, we derive statistics of the likelihood ratio or trace that have non standard distribution. Thus, the empirical specifications that include the presence of a constant term or linear deterministic trend uses the tabulate quantiles of the trace statistics derived by Johansen (1995), while when a break(s) is incorporated in the level of the time series the rank test is carried out by Saikkonen and Lutkepohl (2000a).

## **2.4 TESTING MODEL IMPLICATION**

### **2.4.1 DATA**

The data used for testing the model of equation (Eq. 5) for the US were obtained from different sources. Quarterly data of the government sector at current prices are classified in defence and civilian categories of government spending and were available by the NIPA, while the other macroeconomic series and deflator were taken from the International Financial Statistic (IFS) reports redacted yearly by the International Monetary Found. We transformed the original variables into real terms of logarithms by the index price for the GDP at the constant value of 2000, except the real in-

terest rate<sup>6</sup>. On the other hand, the real interest rate was set up as the difference between the nominal 3-month treasury bill rate and the annual rate of growth in the consumption price index (cpi). It is worth noting that the data available on this indicator constrained the beginning of a more extended sample: the sample for the empirical tests spans from 1957:1 to 2005:4. Fig. 1 describes the patterns of the four macroeconomic variables (in logarithm) that are included in equation (Eq. 5). As it is possible to note on the top right of the figure is reported the pattern of real defence spending in the US. What is immediately evident is that, while the long-run pattern of defence spending has remained stable enough (or slightly increasing) in the last 50 years, the profile of the graph appears to be event-driven with large cyclical spikes corresponding to wars (or threat of wars) (Gerace, 2002; Gold, 2005). The levels of real government defence spending show that a sharp first peak in the data reflects the Vietnam War, a second one is in correspondence with the worsening tensions of the Cold War during Reagan's Presidency and the first Iraq attack while, after 10 years in which the defence spending dropped, there was an upswing in response to terrorist attacks<sup>7</sup>.

In addition, the empirical studies have shown that the patterns of the US military spending and, in general, of the public sector during the 1960s are unambiguously more volatile and it might be responsible for a strong Keynesian stimulus with respect to the successive periods. This is, for example, the Gold's thesis (1997) that sustains that beginning from the 1970s, a narrower and more stable range of the US defence spending (with respect to the long-run trend of the output) generated an ineffective impact on the economy, the latter affected by the decline in output volatility over the past decades. In this regard, below we shall test the significance of defence spending

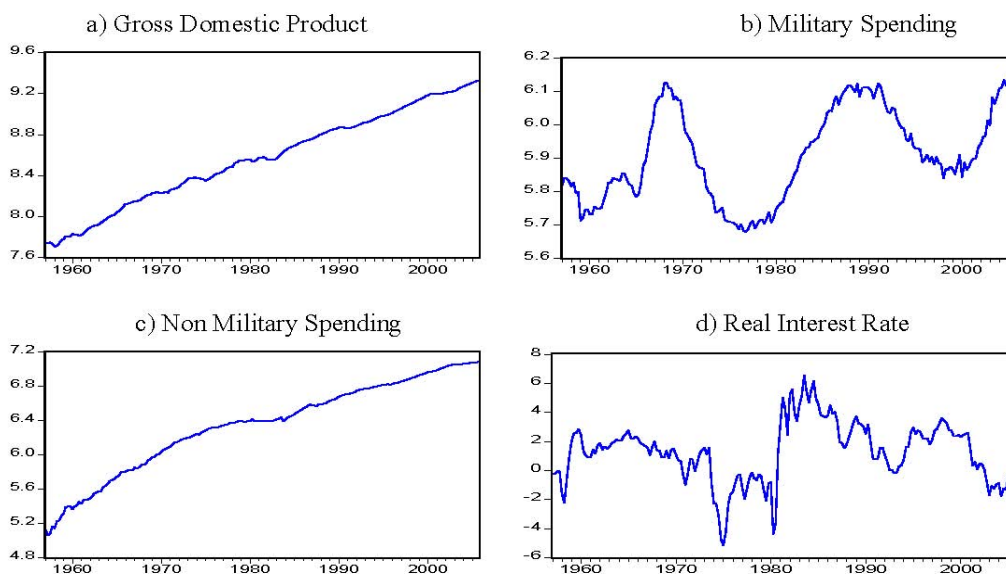
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<sup>6</sup>Gold (2005) criticizes the results of Atesoglu (2004) derived by a chained price index for GDP to deflator military spending series. The core of his criticism is that since inflation in the defence sector has tended to out pace overall inflation, this may understate defence inflation and overstate the growth in defence spending. However, the use of deflator of the GDP in the US to obtain real values of the GDP is close to the results that are possible to obtain with a chained price index (Landefeld, Moulton, & Vojtech, 2003).

<sup>7</sup>The events of war or the threats of war adopted in our sample, that lead to large military buildups, are close to the political events described by Ramey and Shapiro (1998) and used in Burnside, Eichenbaum, and Fisher (2004) to identify changes in fiscal policy in the neoclassical context.

on output for the sub-sample 1970:1 to 2005:4, by assuming the presence of internal substitutability effects of the US government expenditure components (i.e. defence and civilian spending) in favour of the civilian sector. For this sub-period, long-run output responses are, therefore, expected to be greater (and statistically significant) for trended-increased civilian spending with respect to the dynamics of the military sector. It is worth remarking that the central theme of Keynesian economics, associated with the effectiveness of fiscal policy as a stabilization tool, is maintained and fluctuations are, therefore, associated with variations in the efficiency with which productive resources are used. On the other hand, the statistical hypothesis of a nonstationary data-generating process for defence spending, as well as for the other variables of the model specified in (Eq. 2), leads to the need to test the possibility of effects of the long-run on output.

*Fig. 1: Quarterly macroeconomic variables in logarithm for United States*



From a Keynesian point of view, a substantial decline in output volatility may also be attributed to better monetary policies. Martin and Rowthorn (2005) have documented that a rise or fall in the volatility of economies coincided with changes in inflation volatility, suggesting that this may have also been a contributing factor. Thus, starting from the assumption of the model in Section 2.2, we concentrate on the measure of real interest rates and its impact on monetary policies regarding output. The real interest rate pattern (bottom right of Fig. 1) reveals the presence of some volatility in the time series during the 1970s. While it is known that exogenous shocks, caused by the oil crisis in 1973, invested countries over the world and, in turn, the sharp decrease in the real interest rate due to high levels of inflation. A simple inspection shows the likely presence of a structural break, related to the fourth quarter of 1979, as a change in the manner of conducting monetary policy. In fact, Federal Reserve switched from pegging the Federal Funds' interest rate to a policy of reserve targeting, resulting in more variability in interest rates.

Finally, the drop in the real interest rate for the US economy, generated by the 2001 terrorist attacks, is in line with an exceptional active response of the FED to an unexpected negative impulse of the business cycle. As strongly suggested by Saikkonen and Lutkepohl (2000a), both of these break points will be used to assess the robustness of the long-run relationship and the estimated parameters of the theoretical model. This is what we shall do in next sub-section.

#### ***2.4.2 COINTEGRATION TESTS, ESTIMATED COINTEGRATING VECTORS AND POLICY IMPLICATION***

Given  $X_t = (X'_{1t}, X_{2t})'$ , defined as before, the unrestricted VAR (equation (Eq. 3)) was estimated over the countries' samples. The number of lags ( $p$ ) were not fixed a priori but derived by the information criteria. The parsimonious choice of lags, namely  $p = 6$  for the complete sample reveals that the disturbances of the unrestricted VAR

model can be approximated as the realisations of a white noise multivariate process.

After fixing the lags of the *VAR* (and hence of the corresponding *VECM* parameterization), the analysis is carried out by selecting the cointegration rank of the system. Consistently with equation (Eq. 6), a linear trend was restricted to belong to the cointegrated space for the US because it seems clear, on the basis of Fig. 1, that at least three of four variables contain a deterministic trend. Moreover, as suggested by the descriptive analysis, we included a shift-dummy for a first break point related to the US monetary policy change in October 1979. This institutional change determined more variability in the level of the real interest rate, leading to the issue of rank instability in the cointegrating matrix. Finally, a second shift-dummy was included in the specification model to account for the 9/11 terrorist attack, as repeatedly used in studies that assessed the (economic) effects of this unexpected event (Blomberg, Gregory, & Athanasios, 2004; Virgo, 2001).

The first column of Table 1 reports the results from the US cointegration test over the entire sample. Let  $r$  denote the number of cointegrating vectors. As shown in the methodological section, the trace test is a sequential test that moves until the null hypothesis (Eq. 7) cannot be rejected. For the entire data sample, the hypothesis of  $H_0 : r = 0$  is rejected at the 90% significant level. As for the presence of one cointegrating vector, the null hypothesis cannot be rejected at the usual significance level. Thus, in line with the previous empirical results of Atesoglu (2002) for the US economy, the data support the evidence of one cointegrating vector between endogenous variables, namely aggregate output and real interest rate, with  $I(1)$  fiscal policy shocks identified by the defence and civilian spending. This (Keynesian) evidence enables using this model to infer the relevant effects of the disaggregate measures of fiscal policy shocks.

Table 1: Cointegration tests and estimated cointegrating vectors<sup>8</sup>

US (1957:1 to 2005:4)		US (1970:1 to 2005:4)	
Cointegration Tests Statistics			
$H_0 : r = 0$	87.52	$H_0 : r = 0$	64.02
$H_0 : r = 1$	38.27*	$H_0 : r = 1$	28.21 *
$H_0 : r = 2$	21.31	$H_0 : r = 2$	12.25
$H_0 : r = 3$	8.60	$H_0 : r = 3$	0.68
Parameter Estimates			
$\beta_1^*$	-1	$\beta_1^{**}$	-1
$\beta_2^*$	0.105 (0.066)	$\beta_2^*$	0.053 (0.410)
$\beta_3^*$	0.426 (0.000)	$\beta_3^*$	1.106 (0.000)
$\beta_4^*$	-0.015 (0.001)	$\beta_4^{**}$	-0.010 (0.053)
$\vartheta_1$	0.150 (0.000)	$\vartheta_1$	0.121 (0.001)
$\vartheta_2$	-0.063 (0.053)	$\vartheta_2$	-0.019 (0.565)
$\mu$	4.636 (0.000)	$\mu$	1.062 (0.003)
$\eta$	0.005 (0.000)	$\eta$	

The estimated parameters of the US cointegrating vector are given in Table 1 (bottom of column 1). As stated above, we use a maximum likelihood estimator to obtain the estimated elements of the cointegrating vector, while normalization has no impact on the information concerning structural parameters of the model reported in Appendix A. In what follows the parameters associated with the aggregate output variable will be normalized to unity. Cointegration estimated parameters have signs consistent with those in equation (Eq. 2) and their inference reveal a statistically significant relationship among the real variables of output, defence and civilian

<sup>8</sup>The trace test statistic for cointegration and the maximum likelihood estimator for the cointegrating vector are obtained from Johansen (1995). Test statistics are adjusted for the presence of a structural break in the time series (Saikkonen & Lutkepohl, 2000a). An asterisk (\*) in the upper part of the table indicates that the null hypothesis over the rank of cointegration rejected at 90% significant level, while in the round brackets are reported the p-values of the estimated parameters.

spending and interest rate. Moreover, the data provide fairly strong support that the US monetary policy change (October 1979) and the 9/11 terrorist attacks affect the long-run equilibrium as well as it is statistically relevant to include the time trend.

Specifically, as in Atesoglu (2002), the significant effects of the US government defence spending on aggregate output seem to confirm the predictions of the theoretical model. However, taking the longest view first, our elasticity estimations are much weaker than Atesoglu (2002). The estimated value is reduced to 0.1 [with respect to Atesoglu’s estimation, 0.57]. Inspection of the full sample in Fig. 1 confirms a slight increase in the defence-spending patterns, mainly sustained by the large military spending of the aforementioned political events. The intuition to test is that the presence of  $I(1)$  shocks of the US defence spending may be responsible of the slight positive relationship with aggregate output but, linked with Gold statement, this quantitative relationship may be sensitive to the sample period. Thus, we re-estimate the long-run model for the sub-sample from 1970:1 to 2005:4. The results presented in the second column of Table 1, without the presence of a time trend<sup>9</sup>, support the hypothesis test, highlighting a much lower and insignificant defence-spending impact on the economy, while it registered a sharp increase in the impact of civilian spending (from 0.42 in the full sample to 1.10 in the sub-sample).

Though this result may solve the puzzle of the effect of defence spending generated by the gap of data inspection and empirical results (Gold, 2005), the increase of civilian spending complementarities on aggregate output needs an explanation for the central role it assumes in the economy and for its relevant fiscal policy implications. It is empirically documented that the decline in the pattern of US defence spending was substituted by an increasing civilian investment in new technology. This different government allocation shifted the military sector’s central role to one of creating spin-off and complementary relationships of demand in the economy towards the civilian

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<sup>9</sup>According to the hypothesis test, we cannot reject the hypothesis that the trend parameter  $\eta$  is not different from zero by a  $\chi^2$ -test.

sector and showed the switch of the defence to an economically mature sector<sup>10</sup>.

In conclusion, the long-run policy mechanism at work seems, therefore, to structurally sustain greater returns from civilian investments even with political events (wars or threat of wars) that increased US military spending. However, this may only be a part of the story. More convincingly, the fiscal and monetary policy responses, designed as tools for aggregate demand management, are jointly responsible for the fall in discretionary defence-spending changes in US output volatility. A source for such empirical result may be identified in the role of monetary policy as one determinant of economic and political stability. The increased credibility of inflation targets and the role of the central banks in the last two decades have been considered as being responsible for the fall in economic volatility (Martin & Rowthorn, 2005). We find confirmation of this assumption in Table 1, where the significance of the real interest rate for both empirical specifications is a strong support for the model specification and shows its relevance for policy-makers as a countercyclical tool.

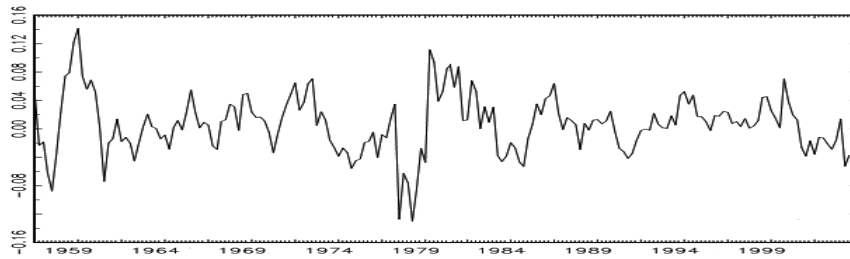
Finally, the number of cointegrating vectors corresponds to 1 in the VAR. As a confirmation, we reported the estimated vector of the error correction model both for the complete and subsample (Figs. 2 and 3). In all cases the estimated residuals range around the long-run equilibrium patterns.

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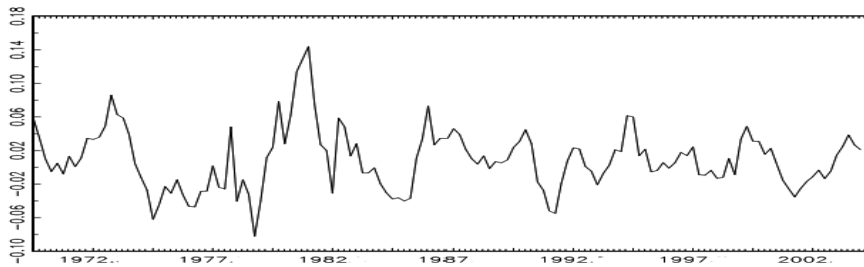
<sup>10</sup>The evidence suggests that the dynamics described made the policy makers aware of appropriateness of policies even if the spending induced by the war's lobby and defence industry are relevant components of the US economy. As an example, it is known that the reaction after the 9/11 terrorist attack to account for the predictable downturn business cycle cut the interest rate and increased the level of government defence spending. The latter policy, financed by a federal government debt was, however, perceived as a temporary event addressed to guarantee national and international security. Contrary to the expectations of government spending substitutability, was documented the constant growth of non-defence category around its equilibrium pattern justifying the leading role in a new-Keynesian perspective.



*Fig. 2: Vector Error Correction Model: Full Sample (1957:1-2005:4)*



*Fig. 3: Vector Error Correction Model: Sub-Sample (1970:1-2005:4)*



## **2.5 CONCLUDING REMARKS**

This paper aims to empirically test whether government defence spending, as a component of public spending, significantly affects the long-run aggregate output pattern. We use a Keynesian theoretical framework that explicitly account for its potential role in explaining fiscal policy fluctuations. On the other hand, since the components of fiscal policy shocks are identified as the motivating force for the non-stationarity of aggregate output, a stable long-run relationship among the macroeconomic variables is a necessary condition to accomplish their impact.

The econometric results are carried out for the US over the period 1957-2005, while a sensitivity analysis is included by estimating the theoretical model for a sub-sample and by including shift dummies to account for institutional or policy changes.

By discussing the empirical results, we found that aggregate data provides consistent evidence that defence spending, as well as civilian spending are cointegrated with output and real interest rate, in line with the theoretical suggestions for the US economy. On the other hand, answering the question whether defence spending provides economic stimulation is more complex. Although we obtain a positive and significant impact of government defence spending on output, that supports the hypothesis of a military Keynesianism, underpinning the dimension and the pattern of elasticities for the sub-sample, the hypothesis at work becomes questionable.

The estimated elasticity of government defence spending on output is really low. Even if the dimension of impact might be surprising for some, these estimates are highly in line with the descriptive evidence of the time series. More than a part of the long-run pattern between government defence spending and output, the significance of this elasticity appears linked with the persistence in event-driven government spending. Switching government priorities in favour of supplying civilian goods and services rather than financing federal defence spending may be responsible for significant fall in output elasticity.

Given these dynamics, a straightforward prediction of a revised and declining role of the defence sector for the economy can be made. However, under the threat of international terrorism, new army policy initiatives (and the consequent rise in the defence spending) were announced between the end of 2001 and the middle of 2002, so that government priorities regarding international security may revitalize the procyclical effects of the military sector on aggregate output. Because of the robustness of our findings, any sample extensions are left for future work.

## 2.6 APPENDIX

In line with theoretical suggestions of Romer (2000) and Taylor (2000), the empirical specification for testing the impact of defence spending on aggregate real output in equation (Eq. 1) is build up by a new macroeconomic Keynesian model under the hypothesis that the real interest rate,  $R$ , is given. Below, we sketch the straightforward structural cross model, as in Atesoglu (2002), in which the variables are expressed in real terms:

$$Y_t = C_t + I_t + X_t + M_t + G_t$$

where:  $Y_t$  = aggregate output,  $C_t$  = consumption,  $I_t$  = investment,  $X_t$  = real net export,  $M_t$  = defence spending and  $G_t$  = civilian spending.

- Consumption function:

$$C_t = d + e(Y_t - T_t), \quad T_t = \text{real taxes}$$

- Tax function:

$$T_t = n + gY_t$$

- Investment function:

$$I_t = h - iR_t, \quad \text{real interest rate}$$

- Net export function:

$$X_t = l - mY_t - nR_t$$

The parameters of the reduced form of equation (Eq. 1) in the body of the text,  $\beta_0^*$ ,  $\beta_2^*$ ,  $\beta_3^*$  and  $\beta_4^*$ , can, therefore, be determined by substituting the aforementioned functions of the economic aggregates into the output-income equation. Formally we

obtain an extended relationship as given:

$$Y_t = d + e(Y_t - (n + gY_t)) + (h - iR_t) + (l - mY_t - nR_t) + M_t + G_t$$

Finally, solving the equation for  $Y_t$ , we obtain:

$$\begin{aligned}\beta_0^* &= \frac{d - en + h + l}{1 - e(1 + g) + m} \\ \beta_2^* &= \beta_3^* = \frac{1}{1 - e(1 + g) + m} \\ \beta_4^* &= \frac{-(i + n)}{1 - e(1 + g) + m}\end{aligned}$$

that represent the parameters to estimate in equation (Eq. 2).

## References

- [1] Atesoglu, H. S. (2002). Defense spending promotes aggregate output in the United States—Evidence from cointegration analysis. *Defence and Peace Economics*, 13(1), 55–60.
- [2] Atesoglu, H. S. (2004). Defence spending and investment in the United States. *Journal of Post Keynesian Economics*, 27(1), 163–169.
- [3] Atesoglu, H. S. (2007). Monetary policy rules and US monetary policy. *Journal of Post Keynesian Economics*, in press.
- [4] Blomberg, S. B., Gregory, D. H., & Athanasios, O. (2004). The macroeconomic consequences of terrorism. *Journal of Monetary Economics*, 51(5), 1007–1032.
- [5] Burnside, C., Eichenbaum, M., & Fisher, J. D. M. (2004). Fiscal shocks and their consequences. *Journal of Economic Theory*, 115(1), 89–117.
- [6] Dakurah, A. H., Davies, S. P., & Sampath, R. K. (2001). Defense spending and economic growth in developing countries. A causality analysis. *Journal of Policy Modeling*, 23, 651–658.
- [7] Dritsakis, N. (2004). Defense spending and economic growth: An empirical investigation for Greece and Turkey. *Journal of Policy Modeling*, 26, 249–264.
- [8] Dunne, J. P., Smith, R. P., & Willenbockel, D. (2004). Models of military expenditure and growth: A critical review. *Defence and Peace Economics*, 16(6), 449–461.
- [9] Gerace, M. (2002). US military expenditures and economic growth: Some evidence from spectral methods. *Defence and Peace Economics*, 13(1), 1–11.
- [10] Gold, D. (1997). Evaluating the trade-off between military spending and investment in the United States. *Defence and Peace Economics*, 8(3), 251–266.
- [11] Gold, D. (2005). Does military spending stimulate or retard economic performance? Revisiting an old debate. *International Affairs Working Paper*.

- [12] Johansen, S. (1992). Determination of cointegration rank in the presence of a linear trend. *Oxford Bulletin of Economics and Statistics*, 54, 383–397.
- [13] Johansen, S. (1995). Likelihood-based inference in cointegrated vector autoregressive models. *Oxford: Oxford University Press*.
- [14] Johansen, S., Mosconi, R., & Nielsen, B. (2000). Cointegration analysis in the presence of structural breaks in the deterministic trend. *Econometrics Journal*, 3, 216–249.
- [15] Landefeld, J. S., Moulton, B. R., & Wojtech, C. M. (2003). Chained-dollar indexes: Issues, tips on their use, and upcoming changes. *Survey of Current Business*, 8–16.
- [16] Maddala, G. S., & Kim, I. (1998). Unit roots cointegration and structural change. *Cambridge University Press*.
- [17] Martin, B., & Rowthorn, R. (2005). Accounting for stability. *CESifo Economic Studies*, 51(4), 649–696.
- [18] Mintz, A., & Hicks, A. (1984). Military Keynesianism in the United States, 1949–1976: Disaggregating military expenditures and their determination. *American Journal of Sociology*, 90(2), 411–417.
- [19] Pesaran, M. H., Shin, Y., & Smith, R. J. (2000). Structural analysis of vector error correction models with exogenous I(1) variables. *Journal of Econometrics*, 97(2), 293–343.
- [20] Ramey, V., & Shapiro, M. D. (1998). Costly capital reallocation and the effects of government spending. *Carnegie-Rochester Conference Series on Public Policy*, 48, 145–194.
- [21] Romer, D. (2000). Keynesian macroeconomics without the LM curve. *National Bureau of Economic Research*.
- [22] Saikkonen, P., & Lutkepohl, H. (2000a). Testing for the cointegrating rank of a VAR process with structural shifts. *Journal of Business & Economic Statistics*, 18, 451–464.

- [23] Saikkonen, P., & Lutkepohl, H. (2000b). Trend adjustment prior to testing for the cointegrating rank of a VAR process. *Journal of Time Series Analysis*, 21, 435–456.
- [24] Sandler, T., & Hartley, K. (1995). The economics of defence. *Cambridge University Press*.
- [25] Smith, R., & Dunne, P. (2001). Military expenditure growth and investment. *Mimeograph*.
- [26] Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39, 195–214.
- [27] Taylor, J. B. (2000). Teaching modern macroeconomics at the principles level. *American Economic Review*, 90(2), 90–94.
- [28] Virgo, J. (2001). Economic impact of the terrorist attacks of September 11, 2001. *Atlantic Economic Journal*, 29(4), 353–357.

### ***3 HOW DETERMINANT ARE MILITARY SPENDING SHOCKS ON THE ECONOMY? A BAYESIAN DSGE APPROACH FOR THE US***

#### ***3.1 INTRODUCTION***

One of the most prominent issues in macroeconomics concerns the effect of an increase in government spending. Even if it has been largely used the aggregate measure of government, there is no widespread agreement on the answer. At the theoretical level, macroeconomic models often differ regarding the implied effects of a rise in government spending on consumption. In that regard, the textbook IS-LM model and the standard RBC model provide a stark example of such differential qualitative predictions.

From an empirical point of view, recent empirical studies as Fatás et al. (2001), Blanchard et al. (2002), Perotti et al. (2005), Galí et al. (2007) have suggested that the transmission of fiscal policy shocks may have actually changed around the early 1980s. Indeed, these studies found a lower persistence of fiscal shocks in the more recent period. As Bilbiie et al. (2009) argued, this break is related to the role of private consumption behaviour, i.e. the increased asset market participation. In fact, retail financial markets were subject to significant restrictions until the late 1970s. Bilbiie and Straub (2006) argue that these restrictions may have effectively prevented a large fraction of households from smoothing consumption in the desired way. As Galí et al. (2007) observed, shut out from asset markets, such households would tend to exhibit an extreme version of “Keynesian” consumption behaviour. This explains the strong crowding-in effects of government spending documented for the 1960s and 1970s. At the same time, one may conjecture that the change in fiscal shocks around 1980 is critically related to the financial liberalization occurring in that



period. Specifically, deregulation and financial innovation may have widened private access to asset markets, reducing the number of households who fail to smooth their consumption profiles (in response to government spending shocks).

In this paper, we analyze all these aspects focusing our analysis on two different components on public spending. Indeed, we propose a model where the key decision is the division of the total government resources between endogenous consumer decisions of private expenditure and the allocation of military and civilian spending by public sector. Our central assumption is that spending decisions for different government components are independent. This idea is closely linked to military Keynesianism that takes centre stage in the policy debate. For example, Martin Feldstein (see, Wall Street Journal article in 2008) suggested that any DoD budget cuts were misguided. He argued that the US government recognised the need for increasing in government spending to offset the decline in consumer demand in the economy and sustained that a rise in military spending would be the best way to provide this stimulus. This view does seem to have other supporters/proponents, particularly in the US. We extend these arguments by a new-Keynesian framework to find a more general explanation to the possible sources of crowding in/out effects in consumption observed in the data. As in Galí et al. (2007) we incorporate into a RBC-model a share of households who do not have access to bonds market and who consume their current disposable income at each date<sup>11</sup>. Furthermore, price stickiness and central bank's behaviour are also determinant in the consumption dynamics in response to a government spending shock (Linnemann and Schabert, 2003).

In order to analyze the changes in fiscal shocks before and after any of the potentially important changes to financial markets, and the business cycle in general, we estimate U.S. time series data for 1954:3-1979:2 (S1) and 1983:1-2008:2 (S2). In

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<sup>11</sup> Another view based on household's preference has been stressed to explain the increase in consumption following a government expenditure expansionary shock. For instance, Linnemann (2006) argues that taking the complementarity between consumption and worked hours into account into a RBC-model, results in crowding-in effects on consumption. Indeed, the negative wealth effect resulting from the rise in government expenditure has a positive impact on labour supply, increasing the marginal utility on consumption and then consumption

particular we focus on the changes in the effects of different government spending components on private consumption and wages. In our estimates, we use five key macroeconomic time series: the inflation rate, the short-term nominal interest rate, real aggregate government spending, real expenditure for non-military and military sectors. Following recent developments in Bayesian estimation techniques (see, e.g., Geweke (1999) and Schorfheide (2000)), we estimate the model by maximising over the posterior distribution of the model parameters based on the linearized state-space representation of the DSGE model.

Our results suggest that whether an increase in government spending or its components raises or lowers consumption depends on the interaction of a number of factors. First, we find that fiscal shocks have stronger effects on consumption, wages, interest rate and inflation rate in the earlier period. Second our analysis suggests that most of the changes in fiscal policy shocks are accounted for the increased asset market participation. Moreover, we show that US economy has different responses for the increases of non-military and military expenditures. The former has a greater impact than the latter. In this context, the purpose of the estimation in this paper is twofold. First, it allows us to evaluate the ability of the new generation of new-Keynesian DSGE models to capture the empirical stochastics and dynamics in the data. Second, the estimated model is used to analyse the effects of fiscal shock on US economy. Our methodology provides a fully structural approach which makes it easier to identify the various shocks in a theoretically consistent way.

The rest of the paper is structured as follows. Section 3.2 presents the derivation of the linearized model. In Section 3.3, we, discuss the estimation methodology and present the main results. In Section 3.4, we analyse the impulse responses of the different fiscal shocks. Finally, Section 3.5 reviews some of the main conclusions that we can draw from the analysis and contains suggestions for further work.

## **3.2 THE MODEL**

In this section we present the DSGE model (see Appendix A for the full derivation) following the paper of Bilbiie et al. (2009). In particular, we assume an economy with sticky prices and limited asset market participation. A continuum of households maximize a utility function with two arguments (consumption and leisure) over an infinite life horizon. Firms produce differentiated goods, decide on labour input and set price according to the Calvo model. Moreover, a fiscal policy authority purchases consumption goods, that are divided in spending for military sector and non-military sector, and raises lump-sum taxes, income taxes and issues nominal debt. Finally, the model encompasses a central bank which sets its policy instrument, the nominal interest rate, by a Taylor rule (1993).

### **3.2.1 HOUSEHOLDS**

Let's assume a continuum of infinitely-lived households  $[0, 1]$  divided in "asset holders" and "non-asset holders". Asset holders, denoted with the fraction  $1 - \lambda$ , trade a riskless one period bond and hold shares in firms. Non-asset holders, on the  $[0, \lambda]$  interval, do not participate in asset markets and simply consume their disposable income. The distinction between households is assumed not to arise from preferences but from their actual capacity to participate in asset markets (Bilbie et al. 2009). Indeed, we assume preference homogeneity: the inverse of the Frish elasticity ( $\varphi$ ) and the inverse of the intertemporal elasticity of substitution ( $\sigma$ ) are the same for both types of households. This is consistent with the view that the only source of heterogeneity among households is their access to the asset markets, which can be limited due to exogenous institutional constraints (Mishkin, 1991).

**ASSET HOLDERS.** Let  $C_{A,t}$ ,  $L_{A,t}$  and  $B_{A,t+1}$  denote, respectively, consumption, leisure and nominal bond holdings for each asset holder on the  $[\lambda, 1]$  interval. These households face the following intertemporal problem:

$$\max_{\{C_{A,t}, L_{A,t}, B_{A,t+1}\}} E_t \sum_{t=0}^{\infty} \beta^t \frac{(C_{A,t} L_{A,t}^\varphi)^{1-\sigma}}{1-\sigma} \quad (\text{Eq. 1})$$

where  $\beta \in (0, 1)$  denotes the discount factor. We note that the utility function is non separable in consumption and leisure and belongs to the King-Plosser-Rebelo (1988) class.

The asset holder intertemporal budget constraint is given by:

$$R_t^{-1} B_{A,t+1} + P_t C_{A,t} + P_t T_t = B_{A,t} + (1 - \tau) (W_t N_{A,t} + P_t D_{A,t}) \quad (\text{Eq. 2})$$

We assume that the income tax rate ( $\tau$ ) is constant, and the real lump-sum taxes ( $T_t$ ) are adjusted to a rule specified below. We denote  $R_t$  as the gross nominal return on bonds purchased in period  $t$ , whereas  $P_t$  is the price level,  $W_t$  the nominal wage, and  $D_{A,t}$  represents real dividend payments to households who own shares in the monopolistically competitive firms. Finally, the hours worked by the asset holder are denoted by  $N_{A,t}$ . We assume that time endowment is normalized to one, thus we have:  $N_{A,t} = 1 - L_{A,t}$ .

Combining the First Order Conditions of consumption and nominal bond holdings we obtain:

$$R_t^{-1} = \beta E_t [\Lambda_{t,t+1}] \quad (\text{Eq. 3})$$

where  $\Lambda_{t,t+s}$  denotes the stochastic discount factor of asset holders for real  $s$ -period ahead payoffs:

$$\Lambda_{t,t+s} = \beta^s \left( \frac{C_{A,t}}{C_{A,t+s}} \right)^\sigma \left( \frac{L_{A,t+s}}{L_{A,t}} \right)^{\varphi(1-\sigma)} \frac{P_t}{P_{t+s}} \quad (\text{Eq. 4})$$

The labour decision equation is given by:

$$\frac{C_{A,t}}{L_{A,t}} = \frac{(1-\tau)W_t}{\varphi P_t} \quad (\text{Eq. 5})$$

**NON-ASSET HOLDERS.** We denote consumption and hours worked by non-asset holders, respectively, as  $C_{N,t}$  and  $N_{N,t}$ . In each period  $t$ , these households solve the following intratemporal problem:

$$\max_{\{C_{N,t}, L_{N,t}\}} \frac{\left(C_{N,t} L_{N,t}^\varphi\right)^{1-\sigma}}{1-\sigma} \quad (\text{Eq. 6})$$

subject to the following budget constraint:

$$P_t C_{N,t} = (1-\tau)W_t N_{N,t} - P_t T_t \quad (\text{Eq. 7})$$

According to expression (Eq. 7), non-asset holders consumption equals their net income.

The first order condition of this problem is given by:

$$\frac{C_{N,t}}{L_{N,t}} = \frac{(1-\tau)W_t}{\varphi P_t} \quad (\text{Eq. 8})$$

### 3.2.2 FIRMS

Final good is produced by competitive firms using the aggregation technology of the CES form:

$$Y_t = \left( \int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (\text{Eq. 9})$$

where  $\varepsilon$  denotes the constant elasticity of substitution, whereas  $Y_t(i)$  indicates the quantity of intermediate good  $i \in [0, 1]$ , at time  $t$ , used as input.

Profit maximization of the final good firms is given by:

$$\max_{\{Y_t(i)\}} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$

where  $P_t$  is the price index for the final good and  $P_t(i)$  denotes the price of the intermediate good  $i$ . From the first order condition for  $Y_t(i)$  we obtain the downward sloping demand for each intermediate input:

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t \quad (\text{Eq. 10})$$

that implies a price index equal to:

$$P_t = \left[ \int_0^1 (P_t(i))^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}$$

The intermediate good,  $Y_t(i)$ , is produced by monopolistically competitive producers that face a production function that is linear in labour and subject to a fixed cost  $F$ :

$$Y_t(i) = N_t(i) - F, \text{ if } N_t(i) > F, \text{ otherwise, } Y_t(i) = 0 \quad (\text{Eq. 11})$$

thus, real profits for these firms correspond to:

$$O_t(i) \equiv \left[ \frac{P_t(i)}{P_t} \right] Y_t(i) - \left[ \frac{W_t}{P_t} \right] N_t(i)$$

The intermediate-good firms are subject to Calvo style price-setting frictions (Calvo (1983) and Yun (1996)). Thus, we assume that intermediate firms can re-optimize their prices with probability  $(1 - \alpha)$ , whereas with probability  $\alpha$  they keep their prices constant as in a given period. In particular, a firm  $i$ , resetting its price

in period  $t$ , solves the following maximization problem:

$$\max_{\{P_t^*(i)\}} E_t \sum_{s=0}^{\infty} \alpha^s \Lambda_{t,t+s} [P_t^*(i) Y_{t,t+s}(i) - W_{t+s} Y_{t,t+s}(i)]$$

subject to the demand function:

$$Y_{t+s}(i) = \left( \frac{P_t^*(i)}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s}$$

where  $P_t^*(i)$  is the optimal price chosen by firms resetting prices at time  $t$ . Moreover, we note that in last equation appears  $\Lambda_{t,t+s}$ , i.e. the stochastic discount factor characterizing asset holders, who own the firms. Solving this maximization problem we obtain the following first order condition:

$$E_t \sum_{s=0}^{\infty} \alpha^s \Lambda_{t,t+s} \left[ P_t^*(i) - \frac{\varepsilon}{\varepsilon - 1} W_{t+s} \right] = 0 \quad (\text{Eq. 12})$$

Finally, the expression for price law of motion is equal to:

$$P_t = \left[ \alpha (P_{t-1})^{1-\varepsilon} + (1 - \alpha) (P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \quad (\text{Eq. 13})$$

### 3.2.3 FISCAL POLICY

The government purchases consumption goods, raises distortionary ( $\tau$ ) and lump-sum taxes ( $T_t$ ), and issues debt ( $B_{t+1}$ ), consisting of one-period nominal discount bonds. Its budget constraint corresponds to the following expression:

$$R_t^{-1} B_{t+1} = B_t + P_t [G_t - \tau Y_t - T_t] \quad (\text{Eq. 14})$$

Since our analysis focuses on the impact of public spending on the economy, we distinguish two different cases: first, we analyze the case of total government spending,

second, we split public expenditure in non-military and military components.

**TOTAL GOVERNMENT SPENDING.** We assume that total government spending is one of the exogenous  $AR(1)$  processes that drives the economy:

$$\log(G_t) = \rho^G \log(G_{t-1}) + \epsilon_t^G \quad (\text{Eq. 15})$$

$$\text{where: } \epsilon_t^G \sim N(0, \sigma_G^2)$$

where  $\rho^G$  indicates the persistence of total government spending and  $\epsilon_t^G$  is a i.i.d. distributed error term that captures the shock volatility.

**NON-MILITARY AND MILITARY SPENDING.** According to the additive principle, total public expenditure can be seen as the sum of its different components. Thus, government spending is divided into civilian sector spending ( $NM_t$ ) and military sector spending ( $M_t$ ):

$$G_t = NM_t + M_t \quad (\text{Eq. 16})$$

We assume that civilian and military expenditure levels are independent and exogenous  $AR(1)$  processes:

$$\log(NM_t) = \rho^{NM} \log(NM_{t-1}) + \epsilon_t^{NM}, \quad (\text{Eq. 17})$$

$$\text{where: } \epsilon_t^{NM} \sim N(0, \sigma_{NM}^2)$$

$$\log(M_t) = \rho^M \log(M_{t-1}) + \epsilon_t^M, \quad (\text{Eq. 18})$$

$$\text{where: } \epsilon_t^M \sim N(0, \sigma_M^2)$$

where  $\rho^{NM}$  and  $\rho^M$  are, respectively, the persistence parameters of the civilian and military shocks, while  $\epsilon_t^{NM}$  and  $\epsilon_t^M$  are, respectively, the stochastic civilian and military terms that are i.i.d. distributed.



**FINANCING MECHANISM OF PUBLIC EXPENDITURE.** We define the government primary deficit as total non-interest spending less the revenues, formally:

$$D_t = G_t - \tau Y_t - T_t \quad (\text{Eq. 19})$$

Moreover we assume that government incurs to a structural deficit ( $D_{s,t}$ ), which is equal to (see Appendix A for details):

$$D_{s,t} = D_t + \tau(Y_t - Y) = G_t - T_t - \tau Y \quad (\text{Eq. 20})$$

i.e. the primary deficit adjusted for automatic responses of tax revenues resulting from deviations on output from its steady state value ( $Y$ ).

### 3.2.4 MONETARY POLICY

In our baseline model the Central Bank is assumed to set the nominal interest rate every period according the following empirical monetary policy reaction function:

$$\begin{aligned} R_t = & \rho^R R_{t-1} + (1 - \rho^R) \{ \bar{\pi}_t + r_\pi (\pi_{t-1} - \bar{\pi}_t) + r_y (Y_t - Y) \} \\ & + r_{\Delta\pi} (\pi_t - \pi_{t-1}) + r_{\Delta y} ((Y_t - Y) - (Y_{t-1} - Y)) + \epsilon_t^R \end{aligned} \quad (\text{Eq. 21})$$

where  $\pi_t$  denotes the inflation rate.

The monetary authority follow a generalised Taylor rule by gradually responding to deviations of lagged inflation from an inflation objective (normalised to be zero) and the lagged output gap defined as the difference between actual and steady state output (Rabanal et. al, 2001). We include an interest rate smoothing parameter,  $\rho_R$ , following recent empirical work (as in Clarida et al. (1998)). In addition, there is also a short-run feedback from the current changes in inflation and the output gap.

Finally, we assume that there are two monetary policy shocks: the first is a

persistent shock to the inflation objective ( $\bar{\pi}_t$ ) which is assumed to follow a first order autoregressive process:

$$\begin{aligned} \log(\bar{\pi}_t) &= \rho^{\bar{\pi}t} \log(\bar{\pi}_{t-1}) + \epsilon_t^{\bar{\pi}} & (\text{Eq. 22}) \\ \text{where } : & \epsilon_t^{\bar{\pi}} \sim N(0, \sigma_{\bar{\pi}}^2) \end{aligned}$$

The second shock is a temporary i.i.d. normal interest rate shock that will also be denoted as monetary policy shock:

$$\epsilon_t^R \sim N(0, \sigma_R^2)$$

### 3.2.5 GENERAL EQUILIBRIUM AND AGGREGATION

A dynamic stochastic general equilibrium is a set of values for prices and quantities such that the representative household's and firm's optimality conditions, and the market clearing conditions are satisfied.

In this case, the final good market is in equilibrium if production equals demand by total household consumption and government spending:

$$Y_t = C_t + G_t \quad (\text{Eq. 23})$$

where aggregate consumption is:

$$C_t = \lambda C_{N,t} + (1 - \lambda) C_{A,t} \quad (\text{Eq. 24})$$

The labour market is in equilibrium when the wage level is such that firms' demand for labour equals total labour supply:

$$N_t = \lambda N_{N,t} + (1 - \lambda) N_{A,t} \quad (\text{Eq. 25})$$

Finally, the share market is in equilibrium if the households hold all outstanding equity shares and all government debt be held by asset holders:

$$B_{t+1} = (1 - \lambda) B_{A,t+1} \quad (\text{Eq. 26})$$

### 3.2.6 THE LINEARIZED MODEL

For the empirical analysis of section 2.3 we linearize the model equations described above around the non-stochastic steady state (see Appendix B for details). Below we summarize the resulting linear rational expectations equations. We denote by small letters the log deviation of a variable from its steady-state value, while for any variable  $X_t$ ,  $X$  stands for its steady-state value and  $X_Y$  its steady-state share in output,  $X/Y$ .

**HOUSEHOLDS.** The log-linearized Euler equation for asset-holders (Eq. 3) relates consumption dynamics with real balances and with hours growth multiplied by steady state taxes and government spending shares with respect to output:

$$c_{A,t} = E_t c_{A,t+1} - \frac{1}{\sigma} (r_t - E_t \pi_{t+1}) + \left( \frac{1}{\sigma} - 1 \right) \left( 1 + \frac{T_Y}{1 - G_Y} \right) (E_t n_{A,t+1} - n_{A,t}) \quad (\text{Eq. 27})$$

When  $\sigma > 1$  the elasticity of consumption growth ( $E_t c_{A,t+1} - c_{A,t}$ ) to hours growth ( $E_t n_{A,t+1} - n_{A,t}$ ) is positive. We note that the elasticity of consumption to the real interest rate is given by  $1/\sigma$ .

The log-linearization of the labour decision equation for asset holders (Eq. 5) is given by:

$$\frac{N}{1 - N} n_{A,t} = w_t - c_{A,t} \quad (\text{Eq. 28})$$

According to this intratemporal optimality condition, asset holders choose optimally their labour supply taking wages as given by the firms.

Similarly, the log-linearized labour decision equation for non-asset holders is obtained from expression (Eq. 8) and is equal to:

$$\frac{N}{1-N}n_{N,t} = w_t - c_{N,t} \quad (\text{Eq. 29})$$

The consumption for non-asset holders is obtained log-linearizing their budget constraint (Eq. 7) and is given by:

$$(1 - G_Y)c_{N,t} = (1 - \tau)(w_t + n_{N,t}) - T_Y t_t \quad (\text{Eq. 30})$$

From the last two relations, we obtain a reduced-form labour supply for non-asset holders. Specifically we have:

$$n_{N,t} = \frac{\varphi}{1 + \varphi} \left[ \frac{-T_Y}{1 - G_Y + T_Y} \right] (w_t - t_t) \quad (\text{Eq. 31})$$

From this condition, we can observe that, since  $-T_Y > 0$ , hours of non asset holders respond positively to increases in the real wage,  $w_t$ , and taxes relative to their steady state value,  $T_Y t_t$ .

We simplify the analysis assuming that steady state labour is the same across household types,  $N_A = N_N = N$ . Thus, the log-linearized expression for aggregate hours (Eq. 25) is given by:

$$n_t = \lambda n_{N,t} + (1 - \lambda) n_{A,t} \quad (\text{Eq. 32})$$

Because of preference homogeneity, the assumption of equal labour levels in the steady state implies that  $C_A = C_N = C$  (see Appendix B). Thus, the log-linearized expression for aggregate consumption (Eq. 24) is given by:

$$c_t = \lambda c_{N,t} + (1 - \lambda) c_{A,t} \quad (\text{Eq. 33})$$

**FIRMS.** The log-linearized aggregate production function (Eq. 11) is given by:

$$y_t = (1 + F_Y) n_t \quad (\text{Eq. 34})$$

We note that the share of the fixed cost  $F$  in steady-state output governs the degree of increasing returns to scale.

Combining the log-linearized expressions of (Eq. 12) and of price level dynamics equation (Eq. 13), yields the familiar new-Keynesian Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha} w_t \quad (\text{Eq. 35})$$

We note that current inflation depends positively on expected future inflation and on the marginal cost, i.e. the wage rate.

**FISCAL POLICY.** In both cases of aggregate government spending and disaggregation of non-military and military sectors, the linearization of the budget constraint (Eq. 14) around a steady state with zero debt and a balanced primary budget gives the following expression:

$$\beta \hat{b}_{t+1} = \hat{b}_t + G_Y g_t - T_T t_t - \tau y_t \quad (\text{Eq. 36})$$

**NON-MILITARY AND MILITARY EXPENDITURES.** Distinguishing the different components of public spending gives an additional condition. Indeed, log-linearized public expenditure composition is obtained from expression (Eq. 16) divided by output:

$$g_t G_Y = N M_Y n m_t + M_Y m_t \quad (\text{Eq. 37})$$

We note that total government spending is the sum of civilian and military component, respectively weighted by their shares with respect to output.

**FINANCING MECHANISM OF PUBLIC EXPENDITURE.** The log-linearized structural primary deficit (Eq. 20) is given by:

$$\hat{d}_{s,t} = G_Y g_t - T_Y t_t \quad (\text{Eq. 38})$$

We assume that the structural deficit is adjusted according to the following rule:

$$\hat{d}_{s,t} = \eta \hat{d}_{s,t-1} + \phi_g G_Y g_t + \phi_b \hat{b}_t \quad (\text{Eq. 39})$$

Rules of this type have been studied extensively, including by Bohn (1998) and Galí and Perotti (2003). The parameter  $\eta$  captures the possibility that budget decisions are autocorrelated. As regards the parameters  $\phi_g$  and  $\phi_b$  they measure the response of structural deficit to changes in government spending and debt, respectively. In particular,  $\phi_b$  captures a “debt stabilization” motive, thus a low value implies that deficit is adjusted in order to stabilize outstanding debt.

**MONETARY POLICY.** The monetary policy rule (Eq. 21) is already linearized:

$$\begin{aligned} R_t = & \rho_R R_{t-1} + (1 - \rho_R) \{ \bar{\pi}_t + r_\pi (\pi_{t-1} - \bar{\pi}_t) + r_y (Y_t - Y) \} \\ & + r_{\Delta\pi} (\pi_t - \pi_{t-1}) + r_{\Delta y} ((Y_t - Y) - (Y_{t-1} - Y)) + \epsilon_t^R \end{aligned} \quad (\text{Eq. 40})$$

**MARKET CLEARING CONDITION.** The log-linearized good market clearing condition (Eq. 23) can be written as:

$$y_t = g_t G_Y + c_t (1 - G_Y) \quad (\text{Eq. 41})$$

Our aim is to analyze the stochastic behaviour of the system of linear rational expectations equations. According to the different modelling of fiscal sector, we dis-

tinguish two models: first we assume that the whole economy is driven by three exogenous shock variables: one arising from total government spending ( $\epsilon_t^G$ ) and two from monetary policy ( $\epsilon_t^{\bar{\pi}}, \epsilon_t^R$ ). The second case corresponds to the disaggregation of total public spending into non-military and military sectors. In this context, the exogenous processes governing the economy are four: two coming from non-military and military spending components ( $\epsilon_t^{NM}, \epsilon_t^M$ ) and two from monetary policy ( $\epsilon_t^{\bar{\pi}}, \epsilon_t^R$ ). As discussed above, all the shock variables are assumed to follow an independent first-order autoregressive stochastic process, except from the interest rate shock that is assumed to be i.i.d. independent process.

### **3.3 ESTIMATION RESULTS**

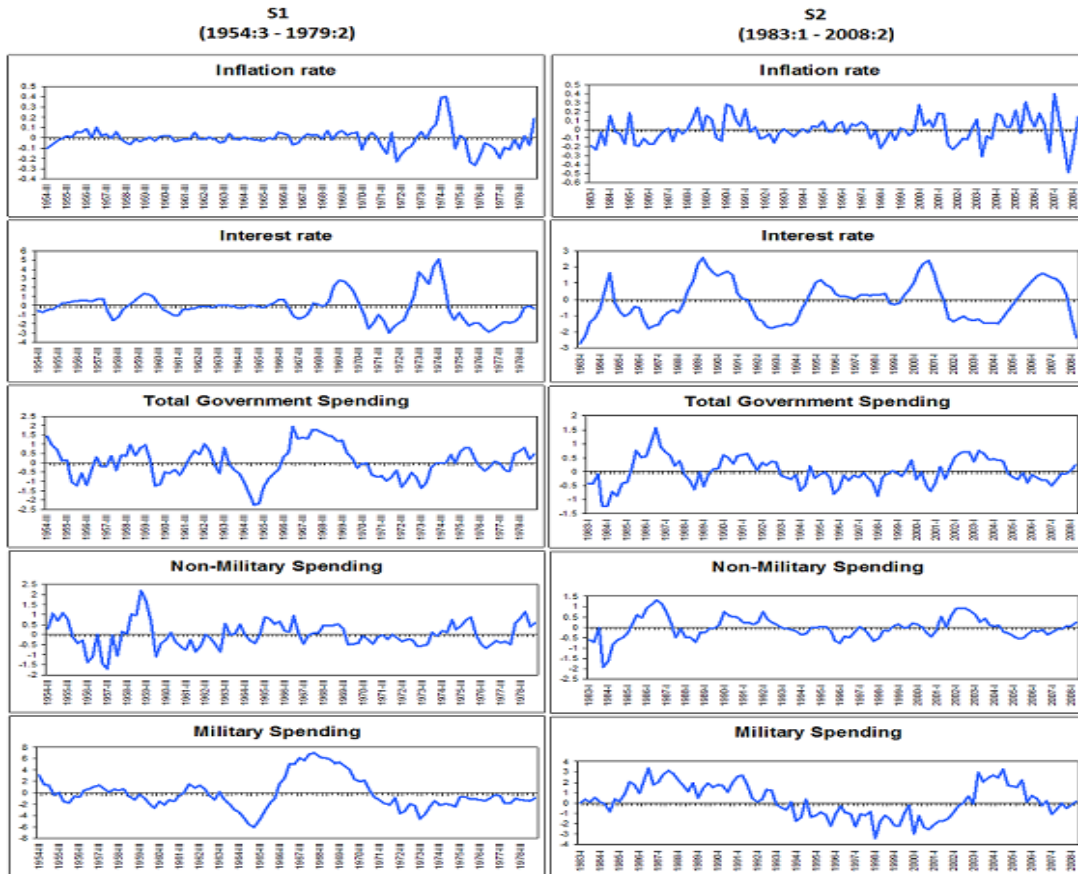
In this section we, first, describe the data used in order to assess the theoretical model. Secondly, we discuss how we estimate the structural parameters and the processes governing the structural shocks. Finally, we present the main estimation results.

#### **3.3.1 DATA DESCRIPTION**

We concentrate on U.S. data for two samples 1954:3–1979:2 (S1) and 1983:1–2008:2 (S2). This choice reflects the hypothesis of a structural break in the early 1980s. Thus, we follow the assumption that the effects of fiscal shocks changed substantially in the early 1980s as a consequence of the financial liberalization occurring in that period, as argued by Bilbiie et al (2009). We assume that the investigator observes the inflation rate, the short-term nominal interest rate, total government spending, non-military and military expenditures. The inflation rate corresponds to the quarterly growth rate of the GDP price index. For the short-term nominal interest rate we consider the effective federal funds rate expressed in quarterly terms (averages of

monthly values, in percentage terms). The source of these data is the Federal Reserve Bank of St. Louis' website.

Figure 1: Interest Rate, Inflation rate, Total Government, Non-Military and Military Expenditures.





As regards total government spending and expenditures for non-military and military sectors, we collect data from the Bureau of Economic Analysis, National Economic Accounts. Military spending corresponds to national defence data, whereas non-military spending is obtained from the difference between government consumption expenditures and gross investment data and national defence data. These variables are deflated by respective deflators and are expressed in log per capita terms. Despite these transformations, some series still display an upward trend. Thus, we separately eliminate it from their log using a Hodrick-Prescott filter, as in Canova et al. (2010) and Castelnovo (2010). While the resulting fluctuations display long period of oscillation, they are overall stationary as the model assumes (see Figure 1).

### 3.3.2 ESTIMATION METHODOLOGY

Geweke (1999) distinguishes between weak and strong econometric approaches in order to estimate the parameters of a linearized DSGE model. Smets and Wouters (2003) argue that the weak interpretation is closest in spirit to the original RBC programme developed by Kydland and Prescott (1982, 1996). The parameters of a DSGE model are calibrated in such a way that selected theoretical moments given by the model match as closely as possible those observed in the data. One way of achieving this, is by minimising some distance function between the theoretical and empirical moments of interest. Among the others Rotemberg and Woodford (1998), Christiano, Eichenbaum and Evans (2001) have estimated the parameters in monetary DSGE models by minimising the difference between an empirical and the theoretical impulse response to a monetary policy shock.

In contrast, as observed by Smets and Wouters (2003), the strong econometric interpretation attempts to provide a full characterisation of the observed data series. Among the others, Sargent (1989) has estimated the structural parameters of his DSGE model using classical maximum likelihood methods<sup>12</sup>. These maximum

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<sup>12</sup>See the references in Ireland (1999).

likelihood methods usually consist of four steps. In the first step, the linear rational expectations model is solved for the reduced form state equation in its predetermined variables. In the second step, the model is written in its state space form. This involves augmenting the state equation in the predetermined variables with an observation equation which links the predetermined state variables to observable variables. In this step, the researcher also needs to take a stand on the form of the measurement error that enters the observation equations<sup>13</sup>. The third step consists of using the Kalman filter to form the likelihood function. In the final step, the parameters are estimated by maximising the likelihood function.

Alternatively within this strong interpretation, Smets and Wouters (2003) show a Bayesian approach combining the likelihood function with prior distributions for the parameters of the model, to form the posterior density function. This posterior is optimised with respect to the model parameters through Monte-Carlo Markov-Chain (MCMC) sampling methods.

According to Smets and Wouters (2003), the attractions of the strong econometric interpretation are clear. When successful, it provides a full characterisation of the data generating process and allows for proper specification testing and forecasting. In particular, its attractiveness arises from three reasons. First, the dynamics of various DSGE models are able to match not only the contemporaneous correlations in the observed data series, but also the serial correlation and cross-covariances. Moreover, if one allows for a sufficiently rich stochastic structure, the singularity problem can be avoided and a better characterisation of the unconditional moments in the data is achieved. Second, as pointed out by Geweke (1999), the weak econometric interpretation of DSGE models is not necessarily less stringent than the strong interpretation: in spite of the focus on a restricted set of moments, the model is assumed to account for all aspects of the observed data series and these aspects are used in

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<sup>13</sup>Ireland (1999) has suggested a way of combining the power of DSGE theory with the flexibility of vector autoregressive time-series models by proposing to model the residuals in the observation equations (which capture the movements in the data that the theory cannot explain) as a general VAR process.

calculating the moments of interest. Third, computational methods have improved so that relatively large models can be solved quite efficiently.

In this paper, we follow the strong econometric interpretation of DSGE models. In particular we follow the same estimation methodology used by Smets and Wouters (2003, 2007). As in papers by Geweke (1998), Landon-Lane (2000), Otrok (2001), Fernandez-Villaverde and Rubio-Ramirez (2001) and Schorfheide (2000), we apply Bayesian techniques for two reasons. First, this approach allows one to formalise the use of prior information coming either from micro-econometric studies or previous macro-econometric studies and thereby makes an explicit link with the previous calibration-based literature. Second, from a practical point of view, the use of prior distributions over the structural parameters makes the highly non-linear optimisation algorithm more stable.

In order to estimate the parameters of the DSGE model presented in Section 3.2 we proceeded with the following steps. First, we estimated the mean of the posterior distribution by maximising the log posterior function, which combines the prior information on the parameters with the likelihood of the data. In a second step, the Metropolis-Hastings algorithm was used to get a complete picture of the posterior distribution and to evaluate the marginal likelihood of the model<sup>14</sup>.

### **3.3.3 PRIOR DISTRIBUTION OF THE PARAMETERS**

Before discussing the estimation results we first discuss the choice of the prior distributions. In that regard we distinguish two groups of parameters. The first group is kept as fixed and these parameters can be viewed as a very strict prior because they can be directly related to the steady-state values and are not identifiable from the data we use. For these values we follow the parameterization of Galí et al. (2007) and Bilbiie et al. (2009). Table 1 and 2 display the choice of fixed parameters in the

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<sup>14</sup>See Smets and Wouters (2003) for a more elaborate description of the methodology. All the estimations are done with Dynare (<http://www.cpremap.cnrs.fr/dynare>).

two sub samples for aggregate government spending model and disaggregate public expenditure components model, respectively.

As we can note from Table 1, the share of government expenditure in GDP ( $G_Y$ ) in S1 (0.28) is higher than the one in S2 (0.18). This innocuous assumption reflects that fact that average of public spending decreased during the period considered. In addition, in the case of disaggregation between public components, Table 2 shows the share of spending in GDP for non-military sector ( $NM_Y$ ) and military sector ( $M_Y$ ) in S1 and S2. Also in this case we assume that the non-military and military expenditures as share of GDP decreased, respectively from 0.18 to 0.12 and from 0.10 to 0.06.

*Table 1: Fixed Parameters for the Model  
with Total Government Spending*

Parameter	S1 (1954:3-1979:2)	S2 (1983:1-2008:2)
$\beta$	0.99	0.99
$G_Y$	0.28	0.18
$\tau$	0.30	0.30
$\alpha$	0.75	0.75
$\sigma$	2.00	2.00
$\psi$	2.63	2.63
$T_Y$	-0.02	-0.12
$F_Y$	6.00	6.00
$N$	0.25	0.25

*Table 2: Fixed Parameters for the Model  
with Non-Military and Military Expenditures*

Parameter	S1 (1954:3-1979:2)	S2 (1983:1-2008:2)
$\beta$	0.99	0.99
$G_Y$	0.28	0.18
$M_Y$	0.10	0.06
$NM_Y$	0.18	0.12
$\tau$	0.30	0.30
$\alpha$	0.75	0.75
$\sigma$	2.00	2.00
$\psi$	2.63	2.63
$T_Y$	-0.02	-0.12
$F_Y$	6.00	6.00
$N$	0.25	0.25

For the remaining fixed parameters we use same values for both sub samples and in both the models (see Tables 1-2). The discount factor ( $\beta$ ) is calibrated to be 0.99, which implies an annual steady state real interest rate of 4%. The steady state tax rate ( $\tau$ ) is set to 0.3. Together with the assumption that the steady-state share of debt is zero, these parameters pin down lump-sum transfers in steady state. The elasticity of substitution ( $\varepsilon$ ) is chosen such that the mark-up in steady state equals 20%. Moreover, we assume that, in steady state, agents spend one-fourth of their time endowment working. We fix  $\alpha$ , the probability that prices are not changed in a given period, at 0.75, a value in the middle of the range reported for different specifications by Galí and Gertler (1999), who apply single equation estimation techniques to the New Keynesian Phillips Curve. Lastly, we assume a conventional value of 2 for the inverse of the elasticity of substitution, as in Bilbiie et al. (2009).

The second group of parameters are estimated using the Bayesian method. Table 3-4 show the priors of the stochastic processes in the model with aggregate government spending and in the model with non-military/military expenditures, respectively. Since the main objective of the paper is to compare the effects of the different

shocks on the fiscal side, we choose to take same prior distributions for monetary policy parameters in both sub samples and for both the models. As well as, in order to compare coherently the estimated results, we choose the general assumption that the priors for deficit rule parameters are the same in both sub samples and for both the models.

In particular, standard errors of the innovations are assumed to follow an inverse-gamma distribution with two degrees of freedom for both models (see Tables 3-4), which corresponds to a rather loose prior (as argued by Smets and Wouters, 2003). After the inspection analysis of the observed time series, we choose mean values of the stochastic processes that are slightly higher than the ones set by Smets and Wouters (2007). For aggregate government spending model (Table 3), we fix the mean of the fiscal shock equal to 0.4, whereas monetary shocks' means are assumed equal to 0.2. In the case of disaggregate public spending components (Table 4) we choose both the means for non-military and military shocks equal to 0.4, whereas for both inflation objective and interest rate shocks equal to 0.2. In both models (Table 3-4) the persistence of the  $AR(1)$  processes is beta distributed as in Smets and Wouters (2007). We fix the prior standard deviation of these parameters equal to 0.1. In the case of total government spending (Table 3), the mean of the fiscal shock persistence is set equal to 0.25, whereas for both monetary policy shocks equal to 0.65. In the case non-military and military model (Table 4), we fix  $\rho^M$ ,  $\rho^{\bar{\pi}_t}$  and  $\rho^R$  all equal to 0.65, except for  $\rho^{NM}$  that is equal to 0.25.

Table 3: Priors and Posteriors of Shock Processes  
for the Model with Total Government Spending

	Prior distribution	Prior mean	Prior standard deviation	Posterior mean	Confidence intervals		Posterior mean	Confidence intervals	
				S1 (1954:3-1979:2)			S2 (1983:1-2008:2)		
$\sigma_g$	inverse gamma	0.4	2	0.5889	0.4783	0.6829	0.3710	0.3109	0.4333
$\sigma_\pi$	inverse gamma	0.2	2	0.3192	0.1646	0.4640	0.8586	0.4168	1.2543
$\sigma_r$	inverse gamma	0.2	2	0.4926	0.4036	0.5712	0.9544	0.7200	1.1898
$\rho_g$	beta	0.25	0.1	0.5411	0.4151	0.6906	0.5700	0.4497	0.6859
$\rho_\pi$	beta	0.65	0.1	0.9226	0.8880	0.9654	0.9257	0.8883	0.9663
$\rho_r$	beta	0.65	0.1	0.5283	0.4444	0.6302	0.6627	0.5735	0.7503

Note: The posterior distributions is obtained using the Metropolis-Hastings algorithm

Table 4: Priors and Posteriors of Shock Processes  
for the Model with Non-Military and Military Expenditures

	Prior distribution	Prior mean	Prior standard deviation	Posterior mean	Confidence intervals		Posterior mean	Confidence intervals	
				S1 (1954:3-1979:2)			S2 (1983:1-2008:2)		
$\sigma_{nm}$	inverse gamma	0.4	2	0.6327	0.5362	0.7368	0.4090	0.3388	0.4775
$\sigma_m$	inverse gamma	0.4	2	1.0038	0.8529	1.1568	0.7967	0.6658	0.9277
$\sigma_\pi$	inverse gamma	0.2	2	0.3273	0.1592	0.4748	0.8697	0.4879	1.2135
$\sigma_r$	inverse gamma	0.2	2	0.5034	0.4104	0.5975	0.9430	0.6948	1.1622
$\rho_{nm}$	beta	0.25	0.1	0.4166	0.2620	0.5565	0.5653	0.4471	0.6870
$\rho_m$	beta	0.65	0.1	0.7946	0.7051	0.8684	0.7276	0.6272	0.8257
$\rho_\pi$	beta	0.65	0.1	0.9194	0.8815	0.9619	0.9278	0.8905	0.9639
$\rho_r$	beta	0.65	0.1	0.5216	0.4228	0.6187	0.6865	0.6078	0.7647

Note: The posterior distributions is obtained using the Metropolis-Hastings algorithm

Tables 5-6 display the prior distributions of the other eight estimated parameters for both S1 and S2 in both the model with total public spending and the model with disaggregate government components. As before, these prior distributions are the same in S1 and S2. In addition, we assume that this group of priors is identical in both models. We begin describing fiscal policy parameters. We choose that  $\phi_b$  and  $\phi_g$  are both gamma distributed with standard error equal to 0.1 and mean 0.3. Both these mean values are within the range of estimated parameters reported in Galí et al. (2007). The parameter  $\eta$  captures the possibility that budget decisions are autocorrelated. We impose it to be gamma distributed with mean 0.5 and standard error equal to 0.1. The mean value of this parameter is in line with Bilbiie et al. (2009).

The parameters describing the monetary policy rule are based on a standard Taylor rule and in line with the values of Smets and Wouters (2007). In particular, we choose the short and long term coefficients on inflation as normal distributed with standard errors equal to 0.1. Prior mean of  $r_\pi$  is set equal to 1.5 in order to guarantee a unique solution path when solving the model, as stated by Smets and Wouters (2003). The prior of the short and long term coefficients on output are gamma distributed with means equal to 0.05 and 0.1, respectively, and standard errors equal to 0.05 for both parameters.

Finally, we focus on the share of non-asset holders, i.e.  $\lambda$ . As we discuss below, this parameter is crucial for the crowding in/out effects of total public spending and its different components on private consumption. Here, we follow seminal papers of Galí et al. (2007) and Mankiw (2000) assuming their same mean value. Thus, we choose that  $\lambda$  has a gamma prior distribution with mean 0.5 and standard error 0.1.

### **3.3.4 POSTERIOR ESTIMATES OF THE PARAMETERS**

Tables 3-6 show also the results of the parameter estimates in the two sub sam-



ples for the model with aggregate government spending and the model with non-military/military expenditures model. In particular, we report the mean, and the 5 and the 95 percentiles of the posterior distribution for parameters obtained through the Metropolis-Hastings sampling algorithm<sup>15</sup>. The latter is based on 100,000 draws<sup>16</sup>.

A number of observations are worth making with regard to the estimated processes for the exogenous shock variables. First, it appears that the data are quite informative as indicated by the lower variance of the posterior distributions relative to the prior distributions. In the model with aggregate government spending (Table 3) we find that the fiscal shock volatility ( $\sigma_G$ ) is 0.59 in S1 and 0.37 in S2. Interestingly, these estimates are very close to those reported by Smets and Wouters (2007) in their sub samples (1966:1-1979:2 - 1984:1-2004:4). Fiscal shock persistence is higher in S1 than in S2. Also this result is line with Smets and Wouters (2007).

*Table 5: Priors and Posteriors of Structural Parameters  
for the Model with Total Government Spending*

	Prior distribution	Prior mean	Prior standard deviation	Posterior mean	Confidence intervals		Posterior mean	Confidence intervals	
				S1 (1954:3-1979:2)			S2 (1983:1-2008:2)		
$\phi_b$	gamma	0.3	0.1	0.3014	0.1394	0.4520	0.2964	0.1392	0.4350
$\phi_\varepsilon$	gamma	0.3	0.1	0.3066	0.1455	0.4613	0.2968	0.1344	0.4258
$\eta$	gamma	0.5	0.1	0.4864	0.3297	0.6435	0.4941	0.3316	0.6336
$r_\pi$	normal	1.5	0.1	1.5333	1.3723	1.6675	1.5194	1.3654	1.6689
$r_{\Delta\pi}$	normal	0.5	0.1	0.5228	0.3614	0.6741	0.5256	0.3589	0.6750
$r_\gamma$	gamma	0.1	0.05	0.1028	0.0226	0.1870	0.0773	0.0163	0.1342
$r_{\Delta Y}$	gamma	0.05	0.05	0.1480	0.0003	0.3036	0.2656	0.0796	0.4531
$\lambda$	gamma	0.5	0.1	0.4304	0.3156	0.5629	0.2729	0.2150	0.3384

*Note: The posterior distributions is obtained using the Metropolis-Hastings algorithm*

<sup>15</sup>See Landon-Lane (1998) and Otrok (2001) for earlier applications of the MH algorithm to DSGE models and Geweke (1998) for a discussion of the various sampling algorithms.

<sup>16</sup>A sample of 100,000 draws was sufficient to ensure the convergence of the MH sampling algorithm (see Smets and Wouters, 2003).

Table 6: Priors and Posteriors of Structural Parameters  
for the Model with Non-Military and Military Expenditures

	Prior distribution	Prior mean	Prior standard deviation	Posterior mean	Confidence intervals		Posterior mean	Confidence intervals	
				S1 (1954:3-1979:2)			S2 (1983:1-2008:2)		
$\phi_b$	gamma	0.3	0.1	0.2918	0.1471	0.4317	0.3132	0.1478	0.4624
$\phi_g$	gamma	0.3	0.1	0.3029	0.1539	0.4663	0.2996	0.1423	0.4460
$\eta$	gamma	0.5	0.1	0.4952	0.3331	0.6494	0.4997	0.3381	0.6553
$\Gamma_\pi$	normal	1.5	0.1	1.5119	1.3637	1.6906	1.5233	1.3779	1.6551
$\Gamma_{\Delta\pi}$	normal	0.5	0.1	0.5242	0.3569	0.6757	0.5149	0.3570	0.6852
$\Gamma_Y$	gamma	0.1	0.05	0.0906	0.0284	0.1643	0.0762	0.0240	0.1321
$\Gamma_{\Delta Y}$	gamma	0.05	0.05	0.1809	0.0004	0.3650	0.2745	0.0957	0.4827
$\lambda$	gamma	0.5	0.1	0.4266	0.2998	0.5384	0.3436	0.2669	0.4294

Note: The posterior distributions is obtained using the Metropolis-Hastings algorithm

As concerns monetary policy, we find that a more active monetary authority in S2 with respect to S1. These estimates confirms the results of Boivin and Giannoni (2006), which find that a stronger central bank response to inflation in the second sub period can account for a smaller output response to monetary policy shocks estimated in identified VARs.

Now, we turn to estimated results for the exogenous shocks in the model with non-military and military expenditure (Table 4). The most remarkable differences are both the volatility and the persistence of these two public spending components. Indeed,  $\sigma_M$  is around the double of  $\sigma_{NM}$  in both S1 (1.0 against 0.6) and S2 (0.8 against 0.4). This confirms that military spending shocks have a greater impact on the economy. Also the persistence of military expenditure shock is higher than the one of civilian spending shock (in S1, 0.8 and 0.4, respectively, whereas in S2, 0.7 and 0.6, respectively). Interestingly, we note that the persistence of non-military shock increased in the second sub sample. This can be explained by the rise of the resources devolved to civilian sector with respect to military sector.

The estimates of fiscal policy parameters are very similar for the model with total government spending and the model with non military/military expenditures (first three lines of Tables 5-6). First, we note that the estimates for  $\phi_b$  of 0.30 in both samples S1 and S2 are in the range of the ones reported by Galí et al. (2007). As Bilbiie et al. (2009) argue these values imply a tendency toward debt stabilization: in both samples, in response to a higher level of debt the structural deficit is reduced. The second fiscal policy parameter,  $\phi_g$ , indicates the response of structural deficit to changes in government spending. Again, we observe a substantial identical value across samples of 0.30, as found by Galí et al. (2007). Bilbiie et al. (2009) observes that this value suggests a great reliance on deficits to finance an extra spending unit. As concerns the autoregressive parameter  $\eta$  is estimated 0.5 in both S1 and S2, implying great persistence of deficits in both sub samples. These values are in line with the ones reported by Bilbiie et al. (2009).

Estimated parameters for monetary policy are shown in the last rows of Tables 5-6. We obtain the same the results both for the model with aggregate government spending and for the model with non-military/military expenditures. First, we note a high value for the long run response of interest rate to inflation. Indeed, the parameter  $r_\pi$  is estimated at 1.5 both in S1 and in S2. These estimates are fairly close to those reported by Smets and Wouters (2007). The long run response of interest rate to output gap decreases from around 0.10 in S1 to 0.07 in S2. Similar values were found by Smets and Wouters (2007).

Finally, we analyze the results concerning the parameter indicating the share of non-asset holders, i.e.  $\lambda$ . First, we note that asset market participation differs considerably across periods. Specifically, for the model with aggregate government spending, the share of consumers who do not smooth consumption by trading in assets is estimated at 0.43 in S1 and at 0.27 in S2. As concerns the model with disaggregate public spending components  $\lambda = 0.43$  in S1 and  $\lambda = 0.34$  in S2. These results are perfectly in line with the ones reported by Bilbiie et al. (2009) and imply

that access to asset markets widened with the important institutional changes in the early 1980s. As discussed by Mishkin (1991), financial liberalization was caused by “Regulation Q,” which imposed tight restrictions on the interest paid by commercial banks; a reduced minimum denomination of Treasury bills; the emergence of money market mutual funds; a sharp decrease in trading costs; and a rise in private shareholding. As we discussed above, fiscal policy shocks changed before and after this period.

### **3.4 ANALYZING THE EFFECTS OF FISCAL SHOCKS ON THE ECONOMY**

In this Section we use the estimated DSGE model to analyse the impulse responses of the fiscal structural shocks. We first describe the results of the model concerning total government spending. Successively, we turn to discuss the effects in the model with the disaggregation of public spending into non-military and military sectors.

#### **3.4.1 MODEL WITH AGGREGATE GOVERNMENT SPENDING**

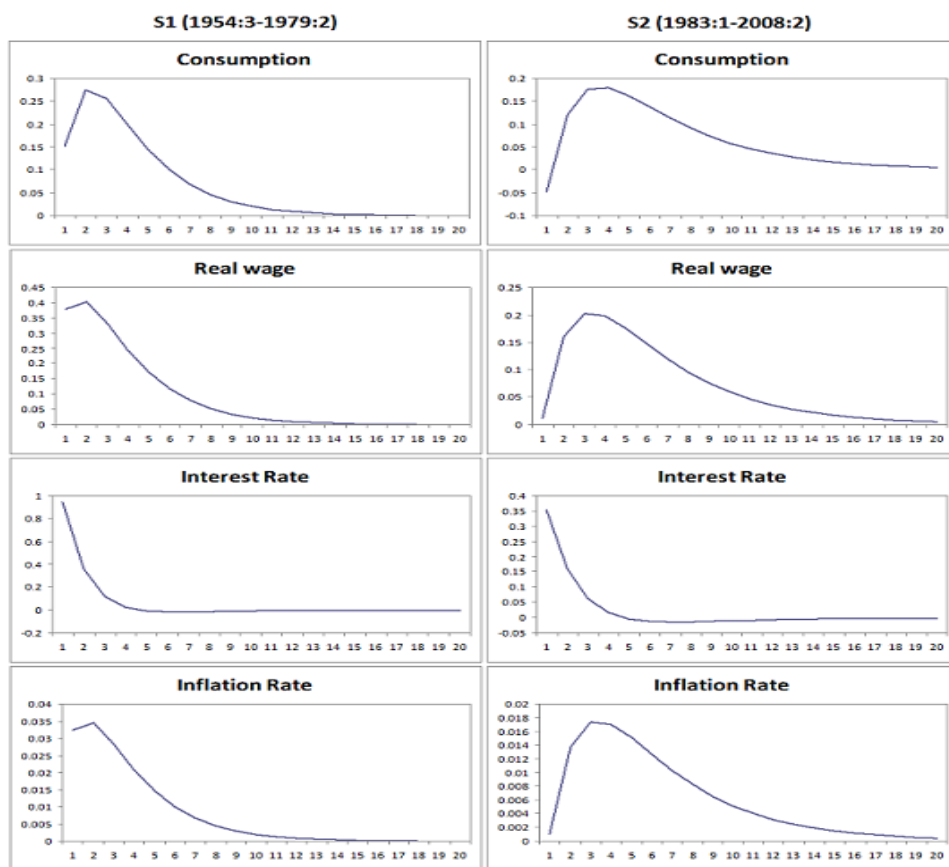
Figure 2 plots the impulse responses to a positive government spending shock with the 5 and 95 percentiles. The variable responses are expressed as shares of aggregate public spending by multiplying the response from the Bayesian estimates (which are expressed in logs) by the respective sub sample average share of total government expenditure in GDP (as in Monacelli and Perotti, 2006). The first row depicts the response of consumption. As we can note, the patterns confirm the earlier findings of Mihov (2003) and Perotti (2005) regarding a weaker response of private consumption in S2 relative to S1. The response of the real wage is reported in the second row. Here, we observe a much higher increase in the first sample with respect to second one. This pattern is in line with other studies that cover the same sample period, as Galí et al. (2007) and Bilbiie et al. (2009). In the third row, the response of nominal

interest rate can be seen to display a higher magnitude in S1 than in S2. The last set of panels pertains to the response of inflation rate. The price level seems to rise more consistently in S1 than in S2.

Our results are in line with theoretical explanations stated by Galí et al. (2007) and Bilbiie et al. (2009). In particular, the essential condition for the crowding-in of private consumption (i.e. an increase in consumption in response to a rise in government spending) is a strong enough rise in the real wage. Indeed, a higher real wage induces an increase in the consumption of non-asset holders, which may eventually more than offset the fall in consumption of asset holders. It is straightforward that labour demand and supply determine the response of real wage. It is also well known that a positive government spending shock increases the demand for goods. In our case, prices are not flexible because we are in the presence of sticky prices à la Calvo. This has an effect on labour demand: firms that cannot change their price will adjust quantities, hence shifting labour demand at a given wage, whereas the rest of the firms will increase their prices, creating inflation.

Meanwhile, labour supply shifts for two different reasons. First, non-asset holders will work more as tax burden increases (the so-called wealth effect). This change is mitigated by deficit-financing of public expenditure because the taxation dynamics matters for non-asset holders. Second, asset holders also increase labour supply for a given wage: this is due both to wealth effect and to intertemporal substitution. The latter effect occurs if an increase in inflation causes a rise in the real interest rate, thus providing incentives for asset holders to postpone consumption. When the shift in labour demand dominates the shift in labour supply, the real wage may increase enough to raise aggregate consumption.

Figure 2: Total Government Spending Impulse Responses

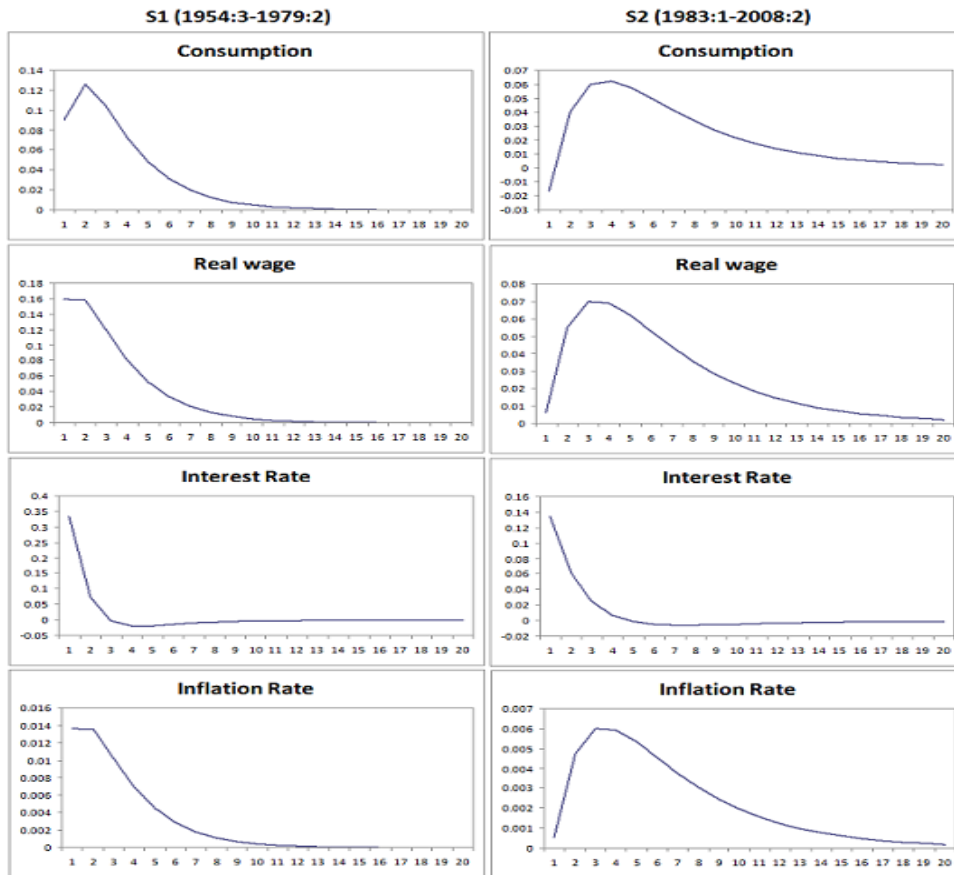


Finally, it is important to note that we assumed the case in which utility is non separable ( $\sigma \neq 1$ ). Specifically, since  $\sigma = 2$ , hours and consumption will co-move positively: thus, for a given increase in the real wage, asset holders substitute out of leisure into consumption. Thus, the negative wealth effect that induces an increase in hours worked can also induce an increase in consumption. Moreover,  $\sigma = 2$  implies a lower elasticity of intertemporal substitution. As a result, asset holders have weaker incentives to postpone consumption for a given increase in real interest rate.

### 3.4.2 MODEL WITH NON-MILITARY AND MILITARY SPENDING COMPONENTS

Figures 3-4 show the responses of the variables, with the 5 and 95 percentiles, to non-military and military spending shocks, respectively. As before, the variable responses are expressed as shares of non-military (military) spending by multiplying the response from the Bayesian estimates by the respective sub sample average share of non-military (military) expenditure in GDP.

Figure 3: Non-Military Spending Impulse Responses



First, we note a substantial change in the effects of both non-military and military spending shocks from S1 to S2. In particular, we note that the responses of consumption (first rows of Figures 3-4) are lower in the post-1980 sample. Likewise, we observe that the rise in real wages is double in S1 with respect to S2 (second rows of Figures 3-4). From third rows of Figures 3-4, it is evident that interest rate itself also shows much higher increase in S1. Lastly, the responses of inflation rate indicate a more intense effect of non-military and military spending shocks in S1 (fourth rows of Figures 3-4).

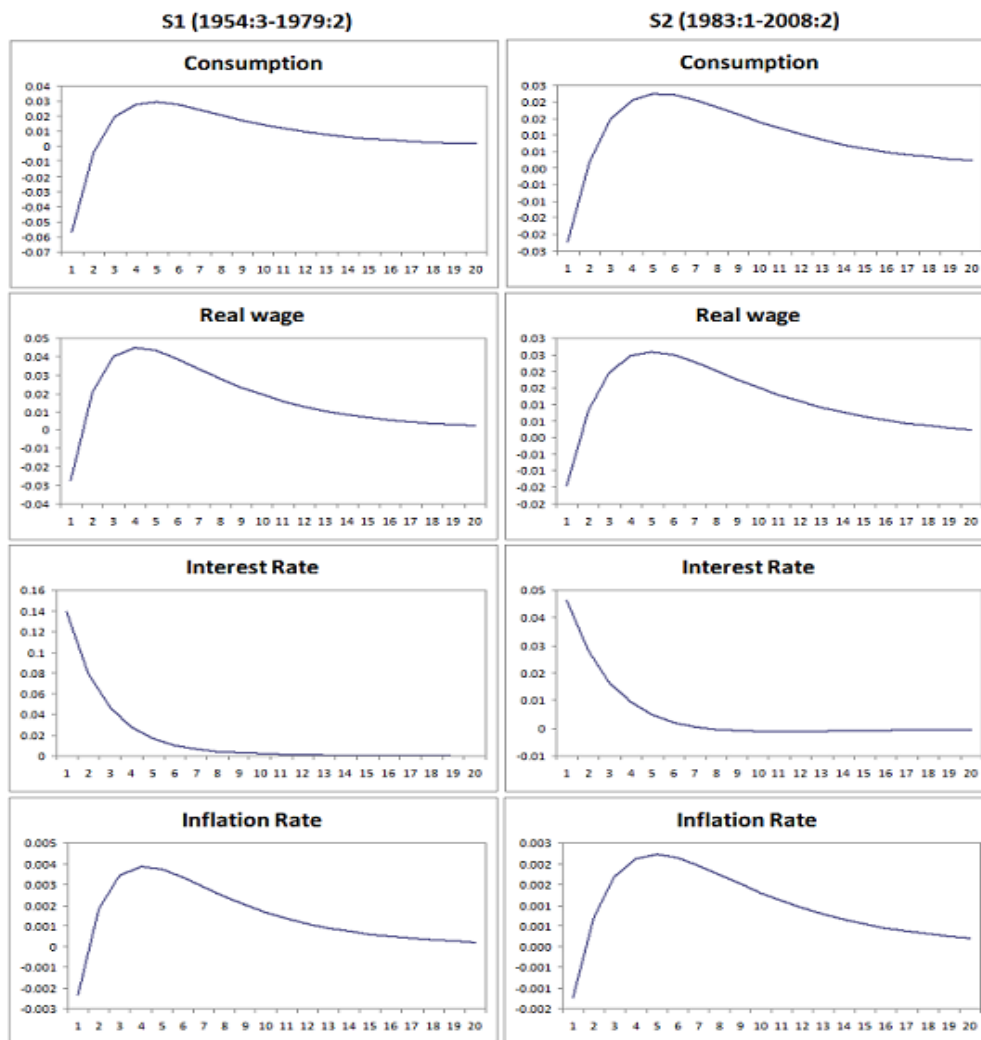
It is interesting to distinguish the effects of an increase in non-military spending with respect to a rise of military expenditure. The response of private consumption can be seen to display a more positive effect in the case of a rise non-military spending. At the contrary, in the case of an increase in military expenditure, private consumption is negative for first few periods. The same is true for the real wage response to civilian and military shocks. An increase of the resources devolved to non-military sector raises the wage level more than an increase in military spending. Also for the interest rate response, differences in the effects of non-military and military shocks are remarkable. In the former case, nominal interest rate displays a higher increase. Finally, the shock to non-military spending shows a much greater response of inflation rate than the shock to military expenditure.

From these empirical results, it is evident that civilian spending as well as military spending generate crowding-in effect on private consumption for US economy. On the other hand, answering the question whether defence spending provides the same economic stimulation of civilian spending is more complex. Although we obtain a positive impact of defence spending on consumption, wages, interest rate and inflation rate, the dimensions of the effect in both sub samples are quite weak. Even if the estimated dimension of the impact might be surprising for some, these estimates are highly in line with the descriptive evidence of the time series. In conclusion, the long-run policy mechanism at work seems, therefore, to structurally sustain greater



returns from civilian spending.

Figure 4: Military Spending Impulse Responses



### 3.5 CONCLUDING REMARKS

In this paper, we add essentially three contributions to previous literature. First, we assess what is the impact of an increase in total public spending and its components,

i.e. the expenditure devolved for non-military and military sectors. Second, we confirm the emerging evidence that fiscal shocks in the U.S. economy has changed substantially in the post-1980s. Finally, we try to account for these changes by considering a DSGE model estimated with recent Bayesian technique. Estimates of the parameters with Bayesian approach offers an effective tool in order assess the impact of exogenous shocks. Indeed, our new-Keynesian model is able to fit the changes in fiscal policies of U.S. economy very well.

The first finding is that an exogenous increase in total government spending leads to a sustained rise in consumption and the real wage in the period 1954-1979 but has less important effects on these variables after 1982. Moreover, we find a much larger positive effect of non-military spending on the economy with respect to the rise of resources devolved to military sector.

Why does U.S. fiscal policy have less expansionary effects in the more recent period? Starting from Bayesian estimates of the structural parameters, we try to relate the differences in fiscal shocks transmission to important institutional changes in the U.S. economy. Specifically, we propose a New Keynesian DSGE model that features limited asset market participation as a potential institutional explanation for different degrees of fiscal policy effectiveness.

We use the same Bayesian approach Smets and Wouters (2007) combining the likelihood function with prior distributions for the parameters of the model, to form the posterior density function. This posterior is optimised with respect to the model parameters through Monte-Carlo Markov-Chain (MCMC) sampling methods. The attraction of this method is evident. When successful, it provides a full characterisation of the data generating process and allows for proper specification testing. The results suggest that asset market participation increased noticeably in the post-1980s, in line with earlier evidence. A *ceteris paribus* increase in asset market participation to the level estimated for the second sample leads to somewhat weaker consumption, and real wage effects of a fiscal spending shocks, thus explaining part of the decline

in the impact of fiscal shocks. Moreover, our results provide consistent evidence that defence spending has weaker effects on consumption and wages with respect to civilian spending. Thus, the military Keynesianism hypothesis that has many supporters/proponents in the U.S. can be questionable. For both the sub-samples, the dimensions of the effect of defence spending on private consumption, wage, interest rate and inflation rate are really low. On the other hand, economy seems to have greater returns from non-military spending. The policy implication we draw is that switching government priorities in favour of supplying civilian goods and services, rather than financing federal defence spending, should create benefits to the economy.

As future work it would be interesting to analyze the crowding in/out effects of fiscal expenditure on aggregate consumption considering the "within" substitution for the resources devolved to military and non-military sectors. It should be interesting to assess the existence of an indirect and contrasting channel for the effects of specific government components of expenditure on private consumption. This should allow us to explain the reasons of crowding in/out effects in the economy.

### 3.6 APPENDIX A: MAXIMIZATION PROBLEMS OF THE MODEL

#### 3.6.1 ASSET HOLDERS.

This kind of households solves the following intertemporal problem:

$$\max_{\{C_{A,t}, L_{A,t}, B_{A,t+1}\}} E_t \sum_{t=0}^{\infty} \beta^t \frac{(C_{A,t} L_{A,t}^\varphi)^{1-\sigma}}{1-\sigma} \quad (\text{Eq. A1})$$

subject to the budget constraint:

$$R_t^{-1} B_{A,t+1} + P_t C_{A,t} + P_t T_t = B_{A,t} + (1-\tau)(W_t N_{A,t} + P_t D_{A,t}) \quad (\text{Eq. A2})$$

We can write the lagrangian of this problem as:

$$\mathcal{L} = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_{A,t} L_{A,t}^\varphi)^{1-\sigma}}{1-\sigma} \right] + \lambda_t \left[ \begin{array}{c} \frac{1}{R_t} B_{A,t+1} + P_t C_{A,t} + \\ P_t T_t - B_{A,t} - (1-\tau)(W_t(1-L_{A,t}) + P_t D_{A,t}) \end{array} \right] \right\}$$

The first order conditions for  $C_{A,t}$  and  $L_{A,t}$  are:

$$\frac{\partial \mathcal{L}}{\partial C_{A,t}} = 0 \Rightarrow$$

$$\begin{aligned} \beta^t (C_{A,t} L_{A,t}^\varphi)^{-\sigma} L_{A,t}^\varphi + \beta^t \lambda_t P_t &= 0 \Rightarrow \\ \lambda_t &= - \frac{L_{A,t}^\varphi}{(C_{A,t} L_{A,t}^\varphi)^\sigma} \frac{1}{P_t} \end{aligned} \quad (\text{Eq. A3})$$

$$\frac{\partial \mathcal{L}}{\partial L_{A,t}} = 0 \Rightarrow$$

$$\beta^t (C_{A,t} L_{A,t}^\varphi)^{-\sigma} C_{A,t} \varphi L_{A,t}^{\varphi-1} + \beta^t \lambda_t [-(1-\tau)(-W_t)] = 0 \Rightarrow$$

$$\frac{\varphi C_{A,t} L_{A,t}^{\varphi-1}}{\left(C_{A,t} L_{A,t}^{\varphi}\right)^{\sigma}} = -\lambda_t [(1-\tau) W_t] \quad (\text{Eq. A4})$$

putting (Eq. A3) into (Eq. A4) we obtain the labour decision equation:

$$\begin{aligned} \frac{\varphi C_{A,t} L_{A,t}^{\varphi-1}}{\left(C_{A,t} L_{A,t}^{\varphi}\right)^{\sigma}} &= \frac{L_{A,t}^{\varphi}}{\left(C_{A,t} L_{A,t}^{\varphi}\right)^{\sigma}} \frac{1}{P_t} [(1-\tau) W_t] \Rightarrow \\ \frac{C_{A,t}}{L_{A,t}} &= \frac{(1-\tau) W_t}{\varphi P_t} \end{aligned}$$

The FOC for  $B_{A,t+1}$  is:

$$\frac{\partial \mathcal{L}}{\partial B_{A,t+1}} = 0 \Rightarrow$$

$$\begin{aligned} \lambda_t \beta^t \frac{1}{R_t} + \lambda_{t+1} \beta^{t+1} (-1) &= 0 \Rightarrow \\ \lambda_t \frac{1}{R_t} &= \lambda_{t+1} \beta \end{aligned} \quad (\text{Eq. A5})$$

putting (Eq. A3) into (Eq. A5) we obtain the Euler equation:

$$\begin{aligned} -\frac{L_{A,t+1}^{\varphi}}{\left(C_{A,t} L_{A,t}^{\varphi}\right)^{\sigma}} \frac{1}{P_t} \frac{1}{R_t} &= -\frac{L_{A,t+1}^{\varphi}}{\left(C_{A,t+1} L_{A,t+1}^{\varphi}\right)^{\sigma}} \beta \Rightarrow \\ \frac{1}{R_t} &= \beta \left(\frac{C_{A,t}}{C_{A,t+1}}\right)^{\sigma} \left(\frac{L_{A,t+1}}{L_{A,t}}\right)^{\varphi(1-\sigma)} \frac{P_t}{P_{t+1}} \end{aligned}$$

thus:

$$R_t^{-1} = \beta E_t [\Lambda_{t,t+1}]$$

where:

$$\Lambda_{t,t+s} = \beta^s \left(\frac{C_{A,t}}{C_{A,t+s}}\right)^{\sigma} \left(\frac{L_{A,t+s}}{L_{A,t}}\right)^{\varphi(1-\sigma)} \frac{P_t}{P_{t+s}}$$

this is the stochastic discount factor.

### 3.6.2 NON-ASSET HOLDERS.

Non-asset holders solve the intratemporal problem:

$$\max_{\{C_{N,t}, L_{N,t}\}} \frac{\left(C_{N,t} L_{N,t}^\varphi\right)^{1-\sigma}}{1-\sigma} \quad (\text{Eq. A6})$$

subject to:

$$P_t C_{N,t} = (1-\tau) W_t N_{N,t} - P_t T_t \quad (\text{Eq. A7})$$

The lagrangian associated to this problem is:

$$\mathcal{L} = \frac{\left(C_{N,t} L_{N,t}^\varphi\right)^{1-\sigma}}{1-\sigma} + \lambda_t [P_t C_{N,t} - (1-\tau) W_t (1 - L_{N,t}) - P_t T_t]$$

the first order conditions for  $C_{N,t}$  and  $L_{N,t}$  are:

$$\frac{\partial \mathcal{L}}{\partial C_{N,t}} = 0 \Rightarrow$$

$$\begin{aligned} \left(C_{N,t} L_{N,t}^\varphi\right)^{-\sigma} L_{N,t}^\varphi + \lambda_t P_t &= 0 \Rightarrow \\ \lambda_t &= \frac{L_{N,t}^\varphi}{\left(C_{N,t} L_{N,t}^\varphi\right)^\sigma} \frac{1}{P_t} \end{aligned} \quad (\text{Eq. A8})$$

$$\frac{\partial \mathcal{L}}{\partial L_{N,t}} = 0 \Rightarrow$$

$$\left(C_{N,t} L_{N,t}^\varphi\right)^{-\sigma} \varphi C_{N,t} L_{N,t}^{\varphi-1} + \lambda_t [(1-\tau) W_t (-1)] = 0 \Rightarrow$$

$$\frac{\varphi C_{N,t} L_{N,t}^{\varphi-1}}{\left(C_{N,t} L_{N,t}^\varphi\right)^\sigma} = \lambda_t [(1-\tau) W_t] \quad (\text{Eq. A9})$$

putting (Eq. A8) into (Eq. A9) gives the labour decision equation:

$$\begin{aligned} \frac{\varphi C_{N,t} L_{N,t}^{\varphi-1}}{\left(C_{N,t} L_{N,t}^{\varphi}\right)^{\sigma}} &= -\frac{L_{N,t}^{\varphi}}{\left(C_{N,t} L_{N,t}^{\varphi}\right)^{\sigma}} \frac{1}{P_t} [(1-\tau) W_t] \Rightarrow \\ \frac{C_{N,t}}{L_{N,t}} &= \frac{(1-\tau) W_t}{\varphi P_t} \end{aligned}$$

### 3.6.3 FINAL GOOD FIRMS.

Given the following aggregation technology:

$$Y_t = \left( \int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (\text{Eq. A10})$$

the final good firm maximizes its profits as:

$$\max_{\{Y_t(i)\}} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \quad (\text{Eq. A11})$$

Putting equation (Eq. A10) into (Eq. A11) we have:

$$\max_{Y_t(i)} P_t \left( \int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_t(i) Y_t(i) di$$

thus, we obtain the demand for each intermediate input:

$$\begin{aligned}
P_t \frac{\varepsilon}{\varepsilon - 1} \left( \int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}-1} \frac{\varepsilon-1}{\varepsilon} Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}-1} - P_t(i) &= 0 \Rightarrow \\
P_t \left( \int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}-1} Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}-1} - P_t(i) &= 0 \Rightarrow \\
P_t \left( \int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}-1} Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}-1} &= P_t(i) \Rightarrow \\
Y_t(i)^{-\frac{1}{\varepsilon}} &= \frac{P_t(i)}{P_t} \left( \int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{-\frac{1}{\varepsilon-1}} \Rightarrow \\
Y_t(i) &= \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} \left( \int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \Rightarrow \\
Y_t(i) &= \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t \tag{Eq. A12}
\end{aligned}$$

Putting (Eq. A12) into (Eq. A10) we obtain the price index:

$$\begin{aligned}
Y_t &= \left( \int_0^1 \left( \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} \right)^{\frac{\varepsilon-1}{\varepsilon}} Y_t^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \Rightarrow \\
Y_t^{\frac{\varepsilon-1}{\varepsilon}} &= Y_t^{\frac{\varepsilon-1}{\varepsilon}} \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{1-\varepsilon} di \Rightarrow \\
1 &= P_t^{\frac{1}{1-\varepsilon}} \int_0^1 (P_t(i))^{1-\varepsilon} di \Rightarrow \\
P_t^{1-\varepsilon} &= \int_0^1 (P_t(i))^{1-\varepsilon} di \Rightarrow \\
P_t &= \left[ \int_0^1 (P_t(i))^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}
\end{aligned}$$



### 3.6.4 INTERMEDIATE GOOD FIRMS.

Given the following production function:

$$Y_t(i) = N_t(i) - F,$$

we can write real profits as:

$$O_t(i) \equiv \left[ \frac{P_t(i)}{P_t} \right] Y_t(i) - \left[ \frac{W_t}{P_t} \right] N_t(i)$$

A firm  $i$  sets  $P(i)$  in order to solve the following problem:

$$\begin{aligned} & \max_{\{P_t^*(i)\}} E_t \sum_{s=0}^{\infty} \alpha^s \Lambda_{t,t+s} [P_t^*(i) Y_{t,t+s}(i) - W_{t+s} Y_{t,t+s}(i)] \\ \text{s.t.} \quad & Y_t(i) = \left( \frac{P_t^*(i)}{P_t} \right)^{-\varepsilon} Y_t \end{aligned}$$

that is:

$$\max_{\{P_t^*(i)\}} E_t \sum_{s=0}^{\infty} \alpha^s \Lambda_{t,t+s} \left[ P_t^*(i) \left( \frac{P_t^*(i)}{P_t} \right)^{-\varepsilon} Y_t - W_t \left( \frac{P_t^*(i)}{P_t} \right)^{-\varepsilon} Y_t \right]$$

The FOC is the following:

$$\begin{aligned} (1 - \varepsilon) \frac{(P_t^*(i))^{-\varepsilon}}{P_t^{-\varepsilon}} Y_t + W_t \varepsilon \left( \frac{P_t^*(i)}{P_t} \right)^{-\varepsilon-1} \frac{1}{P_t} Y_t &= 0 \Rightarrow \\ (1 - \varepsilon) \frac{(P_t^*(i))^{-\varepsilon}}{P_t^{-\varepsilon}} Y_t + W_t \varepsilon \frac{(P_t^*(i))^{-\varepsilon-1}}{P_t^{-\varepsilon-1}} \frac{1}{P_t} Y_t &= 0 \Rightarrow \\ (1 - \varepsilon) \frac{(P_t^*(i))^{-\varepsilon}}{P_t^{-\varepsilon}} + W_t \varepsilon \frac{(P_t^*(i))^{-\varepsilon-1}}{P_t^{-\varepsilon}} &= 0 \Rightarrow \\ (1 - \varepsilon) (P_t^*(i))^{-\varepsilon} + \varepsilon W_t (P_t^*(i))^{-\varepsilon-1} &= 0 \Rightarrow \\ (P_t^*(i))^{-\varepsilon} + \frac{\varepsilon}{1 - \varepsilon} W_t (P_t^*(i))^{-\varepsilon-1} &= 0 \Rightarrow \\ (P_t^*(i))^{\varepsilon+1} (P_t^*(i))^{-\varepsilon} + \frac{\varepsilon}{1 - \varepsilon} W_t (P_t^*(i))^{-\varepsilon-1} (P_t^*(i))^{\varepsilon+1} &= 0 \Rightarrow \\ P_t^*(i) + \frac{\varepsilon}{1 - \varepsilon} W_t &= 0 \Rightarrow \end{aligned}$$

thus, we have:

$$E_t \sum_{s=0}^{\infty} \alpha^s \Lambda_{t,t+s} \left[ P_t^*(i) - \frac{\varepsilon}{\varepsilon - 1} W_{t+s} \right] = 0$$

### 3.6.5 FISCAL POLICY

The structural deficit ( $D_{s,t}$ ) can be obtained as follows:

$$\begin{aligned} D_{s,t} &= D_t + \tau(Y_t - Y) \\ &= D_t + \tau Y_t - \tau Y \\ &= G_t - \tau Y_t - T_t + \tau Y_t - \tau Y \\ &= G_t - T_t - \tau Y \end{aligned}$$

### 3.7 APPENDIX B: STEADY STATES AND LOG-LINEARIZED EQUATIONS

#### 3.7.1 STEADY STATES.

Euler equation (Eq. 3), in steady state, gives:

$$\begin{aligned} \frac{1}{R_t} &= \beta \left( \frac{C_{A,t}}{C_{A,t+1}} \right)^\sigma \left( \frac{L_{A,t+1}}{L_{A,t}} \right)^{\varphi(1-\sigma)} \frac{P_t}{P_{t+1}} \Rightarrow \\ \frac{1}{R} &= \beta \left( \frac{C_A}{C_A} \right)^\sigma \left( \frac{L_A}{L_A} \right)^{\varphi(1-\sigma)} \frac{P}{P} \Rightarrow \\ \frac{1}{R} &= \beta \Rightarrow \\ R &= \frac{1}{\beta} \end{aligned}$$

From the dynamics of the price level (Eq. 13):

$$P_t = \left[ \alpha P_{t-1}^{1-\epsilon} + (1-\alpha) (P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$$

we have that in steady state:

$$\begin{aligned} P &= \left[ \alpha P^{1-\epsilon} + (1-\alpha) (P^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \Rightarrow \\ P^{1-\epsilon} &= \alpha P^{1-\epsilon} + (1-\alpha) (P^*)^{1-\epsilon} \Rightarrow \\ P^{1-\epsilon} (1-\alpha) &= (1-\alpha) (P^*)^{1-\epsilon} \Rightarrow \\ P &= P^* \end{aligned}$$

In steady state, from the FOC of the price setting in the intermediate good firm's

problem (Eq. 12), we have for the real wage:

$$\begin{aligned}
 P - \frac{\varepsilon}{\varepsilon - 1}W &= 0 \Rightarrow \\
 P &= \frac{\varepsilon}{\varepsilon - 1}W \Rightarrow \\
 \frac{W}{P} &= \frac{\varepsilon - 1}{\varepsilon}
 \end{aligned}
 \tag{Eq. A13}$$

In steady state, from the production function (Eq. 11):

$$\begin{aligned}
 Y_t(i) &= N_t(i) - F \Rightarrow \\
 Y &= N - F
 \end{aligned}$$

Defining:

$$\begin{aligned}
 \mu &= \frac{1}{\varepsilon - 1} \Rightarrow \\
 1 + \mu &= 1 + \frac{1}{\varepsilon - 1} = \frac{\varepsilon - 1 + 1}{\varepsilon - 1} = \frac{P}{W}
 \end{aligned}$$

we can rewrite (Eq. A13) as:

$$\frac{W}{P} = \frac{1}{1 + \mu} = \frac{N}{N} \frac{1}{1 + \mu} = \frac{Y + F}{N(1 + \mu)} = \frac{Y}{N} \frac{1 + F_Y}{1 + \mu}
 \tag{Eq. A14}$$

Profits in steady state amount to:

$$\begin{aligned}
 O_t(i) &\equiv \left[ \frac{P_t(i)}{P_t} \right] Y_t(i) - \left[ \frac{W_t}{P_t} \right] N_t(i) \Rightarrow \\
 O &\equiv \left[ \frac{P}{P} \right] Y - \left[ \frac{W}{P} \right] N \Rightarrow \\
 O &\equiv Y - \left[ \frac{W}{P} \right] N
 \end{aligned}$$

so that the ratio of profits to output is given by:

$$\begin{aligned}
O &\equiv Y - \left[ \frac{W}{P} \right] N \Rightarrow \\
O &\equiv Y - \left[ \frac{Y}{N} \frac{1 + F_Y}{1 + \mu} \right] N \Rightarrow \\
O_Y &\equiv 1 - \left[ \frac{1 + F_Y}{1 + \mu} \right] \Rightarrow \\
O_Y &\equiv \frac{1 + \mu - 1 - F_Y}{1 + \mu} \Rightarrow \\
O_Y &\equiv \frac{\mu - F_Y}{1 + \mu}
\end{aligned}$$

We assume, in steady state, that:

$$N_N = N_A = N$$

because of preference homogeneity, we need to ensure that steady-state consumption shares also equal across groups. This can be seen comparing the two labour decision equations (Eq. 5) and (Eq. 8) evaluated in steady state:

$$\frac{C_A}{L} = \frac{1 - \tau}{\varphi} \frac{W}{P} = \frac{C_N}{L}$$

implying:

$$C_A = C_N = C$$

We know from (Eq. A14) that:

$$\begin{aligned}
\frac{W}{P} &= \frac{Y}{N} \frac{1 + F_Y}{1 + \mu} \Rightarrow \\
\frac{W}{P} \frac{N}{Y} &= \frac{1 + F_Y}{1 + \mu} \Rightarrow \\
(1 - \tau) \frac{W}{P} \frac{N}{Y} &= (1 - \tau) \frac{1 + F_Y}{1 + \mu} \tag{Eq. A15}
\end{aligned}$$

From equation (Eq. 7) in steady state:

$$\begin{aligned}
P_t C_{N,t} &= (1 - \tau) W_t N_{N,t} - P_t T_t \Rightarrow \\
P C_N &= (1 - \tau) W N_N - P T \Rightarrow \\
C_N &= (1 - \tau) \frac{W}{P} N_N - T \Rightarrow \\
C_N &= (1 - \tau) \frac{Y}{N} \frac{1 + F_Y}{1 + \mu} N_N - T \Rightarrow \\
\frac{C_N}{Y} &= \frac{1}{Y} (1 - \tau) Y \frac{1 + F_Y}{1 + \mu} - T_Y \Rightarrow \\
\frac{C_N}{Y} &= (1 - \tau) \frac{1 + F_Y}{1 + \mu} - T_Y \tag{Eq. A16}
\end{aligned}$$

From expression (Eq. 19) in steady state:

$$\begin{aligned}
D_t &= G_t - \tau Y_t - T_t \Rightarrow \\
0 &= G - \tau Y - T \Rightarrow \\
0 &= G_Y - \tau - T_Y \Rightarrow \\
T_Y &= G_Y - \tau \tag{Eq. A17}
\end{aligned}$$

From equation (Eq. 24) in steady state:

$$\begin{aligned}
C_t &= \lambda C_{N,t} + (1 - \lambda) C_{A,t} \Rightarrow \\
C &= \lambda C_N + (1 - \lambda) C_A \Rightarrow \\
C_A &= \frac{C - \lambda C_N}{1 - \lambda}
\end{aligned}$$

and thus:

$$\frac{C_A}{Y} = \frac{C - \lambda C_N}{(1 - \lambda) Y}$$

since steady state of expression (Eq. 23) gives:

$$\begin{aligned} Y &= C + G \Rightarrow \\ 1 &= \frac{C}{Y} + \frac{G}{Y} \Rightarrow \\ \frac{C}{Y} &= 1 - G_Y \end{aligned}$$

and from (Eq. A16):

$$\frac{C_N}{Y} = (1 - \tau) \frac{1 + F_Y}{1 + \mu} - T_Y$$

we have that:

$$\begin{aligned} \frac{C_A}{Y} &= \frac{1}{1 - \lambda} \left( \frac{C}{Y} - \frac{\lambda C_N}{Y} \right) \Rightarrow \\ \frac{C_A}{Y} &= \frac{1}{1 - \lambda} \left( 1 - G_Y - \lambda(1 - \tau) \frac{1 + F_Y}{1 + \mu} + \lambda T_Y \right) \end{aligned}$$

using (Eq. A17):

$$\begin{aligned} T_Y &= G_Y - \tau \Rightarrow \\ G_Y &= T_Y + \tau \end{aligned}$$

we get:

$$\begin{aligned} \frac{C_A}{Y} &= \frac{1}{1 - \lambda} \left( 1 - T_Y - \tau - \lambda(1 - \tau) \frac{1 + F_Y}{1 + \mu} + \lambda T_Y \right) \Rightarrow \\ \frac{C_A}{Y} &= \frac{1}{1 - \lambda} \left( 1 - \tau - \lambda(1 - \tau) \frac{1 + F_Y}{1 + \mu} - (1 - \lambda) T_Y \right) \Rightarrow \\ \frac{C_A}{Y} &= \frac{1}{1 - \lambda} \left( 1 - \tau - \lambda(1 - \tau) \frac{1 + F_Y}{1 + \mu} \right) - T_Y \Rightarrow \\ \frac{C_A}{Y} &= \frac{1}{1 - \lambda} \left( (1 - \tau) \left( 1 - \lambda \frac{1 + F_Y}{1 + \mu} \right) \right) - T_Y \Rightarrow \\ \frac{C_A}{Y} &= (1 - \tau) \frac{1}{1 - \lambda} \left( 1 - \lambda \frac{1 + F_Y}{1 + \mu} \right) - T_Y \end{aligned} \tag{Eq. A18}$$

We thus achieve equalization of steady-state consumption shares by making assumption on technology. Specifically, we ensure that asset income in steady state is zero. This requires assuming that the fixed cost of production is characterized by:

$$F_Y = \mu$$

Substituting in (Eq. A18) gives:

$$\frac{C_A}{Y} = \frac{C_N}{Y} = 1 - \tau - T_Y = 1 - G_Y$$

We want to find hours in steady state. Given the equalization of hours and consumption between the two groups and normalizing  $P = 1$ , the labour decision equation of non-asset holders (Eq. 8) implies:

$$\begin{aligned} \frac{C_{N,t}}{L_{N,t}} &= \frac{(1-\tau) W_t}{\varphi} \frac{1}{P_t} \Rightarrow \\ \frac{C}{L} &= \frac{(1-\tau)}{\varphi} W \Rightarrow \\ \frac{C}{1-N} &= \frac{(1-\tau)}{\varphi} W \Rightarrow \\ C &= \frac{(1-\tau)}{\varphi} W (1-N) \end{aligned} \tag{Eq. A19}$$

but from equation (Eq. 7) we have that:

$$\begin{aligned} P_t C_{N,t} &= (1-\tau) W_t N_{N,t} - P_t T_t \Rightarrow \\ P C_N &= (1-\tau) W N_N - P T \Rightarrow \\ C_N &= (1-\tau) \frac{W}{P} N_N - T \Rightarrow \\ C &= (1-\tau) W N - T \end{aligned} \tag{Eq. A20}$$



putting equation (Eq. A20) into equation (Eq. A19) we get:

$$(1 - \tau)WN - T = \frac{(1 - \tau)}{\varphi}W(1 - N)$$

dividing by  $Y$  and using (Eq. A15) and the expression for the fixed cost we obtain the following expression for the steady state hours:

$$\begin{aligned} (1 - \tau)WN - T &= \frac{(1 - \tau)}{\varphi}W(1 - N) \Rightarrow \\ (1 - \tau)\frac{WN}{P} - \frac{T}{Y} &= \frac{(1 - \tau)W}{\varphi P}(1 - N)\frac{1}{Y} \Rightarrow \\ (1 - \tau)\frac{1 + F_Y}{1 + \mu} - T_Y &= \frac{(1 - \tau)Y}{\varphi N} \frac{1 + F_Y}{1 + \mu} (1 - N)\frac{1}{Y} \Rightarrow \\ (1 - \tau)\frac{1 + F_Y}{1 + \mu} - T_Y &= \frac{(1 - \tau)}{\varphi} \frac{1 - N}{N} \frac{1 + F_Y}{1 + \mu} \Rightarrow \\ 1 - \frac{1 + \mu}{1 + F_Y} \frac{T_Y}{1 - \tau} &= \frac{1}{\varphi} \frac{1 - N}{N} \Rightarrow \\ 1 - \frac{T_Y}{1 - \tau} &= \frac{1}{\varphi} \frac{1 - N}{N} \Rightarrow \\ \left(1 - \frac{G_Y - \tau}{1 - \tau}\right)\varphi &= \frac{1 - N}{N} \Rightarrow \\ \left(\frac{1 - \tau - G_Y + \tau}{1 - \tau}\right)\varphi &= \frac{1 - N}{N} \Rightarrow \\ \frac{1 - G_Y}{1 - \tau}\varphi &= \frac{1 - N}{N} \Rightarrow \\ \frac{N}{1 - N} &= \frac{1}{\varphi} \frac{1 - \tau}{1 - G_Y} \end{aligned} \tag{Eq. A21}$$

### 3.7.2 LOG-LINEARIZED EQUATIONS

We obtain expression (Eq. 27) from the log-linearizing of the Euler equation (Eq. 3), substituting steady state hours from (Eq. A21) and assuming that:

$$\pi_t = \log\left(\frac{P_t}{P_{t-1}}\right)$$

thus, we have:

$$\begin{aligned}
\frac{1}{R_t} &= \beta \left( \frac{C_{A,t}}{C_{A,t+1}} \right)^\sigma \left( \frac{1-N_{A,t+1}}{1-N_{A,t}} \right)^{\varphi(1-\sigma)} \frac{P_t}{P_{t+1}} \\
\Rightarrow \frac{1}{R} - \frac{1}{R^2} R r_t &= \beta \left( \frac{C_A}{C_A} \right)^\sigma \left( \frac{1-N_A}{1-N_A} \right)^{\varphi(1-\sigma)} \frac{P}{P} \\
&+ \beta \sigma \left( \frac{C_A}{C_A} \right)^{\sigma-1} \frac{1}{C_A} \left( \frac{1-N_A}{1-N_A} \right)^{\varphi(1-\sigma)} \frac{P}{P} c_{A,t} C_A \\
&- \beta \sigma \left( \frac{C_A}{C_A} \right)^{\sigma-1} \frac{C_A}{C_A^2} \left( \frac{1-N_A}{1-N_A} \right)^{\varphi(1-\sigma)} \frac{P}{P} c_{A,t+1} C_A \\
&- \beta \varphi(1-\sigma) \left( \frac{C_A}{C_A} \right)^\sigma \left( \frac{1-N_A}{1-N_A} \right)^{[\varphi(1-\sigma)]-1} \frac{1}{1-N_A} \frac{P}{P} n_{A,t} N_A \\
&+ \beta \varphi(1-\sigma) \left( \frac{C_A}{C_A} \right)^\sigma \left( \frac{1-N_A}{1-N_A} \right)^{[\varphi(1-\sigma)]-1} \frac{1-N_A}{(1-N_A)^2} \frac{P}{P} n_{A,t+1} N_A \\
&+ \beta \left( \frac{C_A}{C_A} \right)^\sigma \left( \frac{1-N_A}{1-N_A} \right)^{\varphi(1-\sigma)} \frac{1}{P} p_t P \\
&- \beta \left( \frac{C_A}{C_A} \right)^\sigma \left( \frac{1-N_A}{1-N_A} \right)^{\varphi(1-\sigma)} \frac{P}{P^2} p_{t+1} P \\
\Rightarrow \beta - \beta r_t &= \beta + \beta \sigma c_{A,t} - \beta \sigma c_{A,t+1} \\
&- \beta \varphi(1-\sigma) \frac{N_A}{1-N_A} n_{A,t} + \beta \varphi(1-\sigma) \frac{N_A}{1-N_A} n_{A,t+1} \\
&+ \beta p_t - \beta p_{t+1} \\
\Rightarrow 1 - r_t &= 1 + \sigma c_{A,t} - \sigma c_{A,t+1} \\
&- \varphi(1-\sigma) \frac{N_A}{1-N_A} n_{A,t} + \varphi(1-\sigma) \frac{N_A}{1-N_A} n_{A,t+1} \\
&+ p_t - p_{t+1}
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \quad & -\log R_t + \log R = \sigma (\log C_{A,t} - \log C_A) \\
& + \sigma (-\log C_{A,t+1} + \log C_{A,t}) \\
& + \varphi (1 - \sigma) \frac{N_A}{1 - N_A} (-\log N_{A,t} + \log N_A) \\
& + \varphi (1 - \sigma) \frac{N_A}{1 - N_A} (\log N_{A,t+1} - \log N_A) \\
& + \log P_t - \log P - \log P_{t+1} + \log P
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \quad & \sigma (\log C_{A,t+1} - \log C_{A,t}) = \sigma (\log C_{A,t} - \log C_A) \\
& + \log R_t - \log R \\
& + \varphi (1 - \sigma) \frac{N_A}{1 - N_A} (-\log N_{A,t} + \log N_A) \\
& + \varphi (1 - \sigma) \frac{N_A}{1 - N_A} (\log N_{A,t+1} - \log N_A) \\
& + \log P_t - \log P_{t+1}
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \quad & (\log C_{A,t+1} - \log C_{A,t}) = (\log C_{A,t} - \log C_A) \\
& + \frac{1}{\sigma} (\log R_t - \log R) \\
& + \varphi \left( \frac{1}{\sigma} - 1 \right) \frac{N_A}{1 - N_A} (-\log N_{A,t} + \log N_A) \\
& + \varphi \left( \frac{1}{\sigma} - 1 \right) \frac{N_A}{1 - N_A} (\log N_{A,t+1} - \log N_A) \\
& - \frac{1}{\sigma} \log \left( \frac{P_{t+1}}{P_t} \right)
\end{aligned}$$

$$\begin{aligned}
\Rightarrow (\log C_{A,t+1} - \log C_{A,t}) &= (\log C_{A,t} - \log C_A) \\
&+ \frac{1}{\sigma} (\log R_t - \log R) \\
&+ \varphi \left( \frac{1}{\sigma} - 1 \right) \left( \frac{1}{\varphi} \frac{1-\tau}{1-G_Y} \right) (-\log N_{A,t} + \log N_A) \\
&+ \varphi \left( \frac{1}{\sigma} - 1 \right) \left( \frac{1}{\varphi} \frac{1-\tau}{1-G_Y} \right) (\log N_{A,t+1} - \log N_A) \\
&- \frac{1}{\sigma} \log \left( \frac{P_{t+1}}{P_t} \right)
\end{aligned}$$

$$\begin{aligned}
\Rightarrow c_{A,t+1} &= c_{A,t} + \frac{1}{\sigma} r_t \\
&- \left( \frac{1}{\sigma} - 1 \right) \left( \frac{1-\tau}{1-G_Y} \right) n_{A,t} \\
&+ \left( \frac{1}{\sigma} - 1 \right) \left( \frac{1-\tau}{1-G_Y} \right) n_{A,t+1} \\
&- \frac{1}{\sigma} \pi_{t+1}
\end{aligned}$$

$$\begin{aligned}
\Rightarrow c_{A,t} &= c_{A,t+1} - \frac{1}{\sigma} r_t + \frac{1}{\sigma} \pi_{t+1} \\
&+ \left( \frac{1}{\sigma} - 1 \right) \left( \frac{1-\tau}{1-G_Y} \right) n_{A,t} \\
&- \left( \frac{1}{\sigma} - 1 \right) \left( \frac{1-\tau}{1-G_Y} \right) n_{A,t+1}
\end{aligned}$$

$$\begin{aligned}
\Rightarrow c_{A,t} &= c_{A,t+1} - \frac{1}{\sigma} (r_t - \pi_{t+1}) \\
&+ \left( \frac{1}{\sigma} - 1 \right) \left( \frac{1-G_Y + T_Y}{1-G_Y} \right) (n_{A,t} - n_{A,t+1})
\end{aligned}$$

$$\Rightarrow c_{A,t} = E_t c_{A,t+1} - \frac{1}{\sigma} (r_t - E_t \pi_{t+1}) + \left( \frac{1}{\sigma} - 1 \right) \left( 1 + \frac{T_Y}{1-G_Y} \right) (n_{A,t} - E_t n_{A,t+1})$$

Relation (Eq. 28), is obtained log-linearizing expression (Eq. 5) and assuming

that:

$$w_t = \log \left( \frac{W_t/P_t}{W/P} \right)$$

thus:

$$\begin{aligned} \frac{C_{A,t}}{L_{A,t}} &= \frac{(1-\tau) W_t}{\varphi P_t} \Rightarrow \\ \frac{C_{A,t}}{1-N_{A,t}} &= \frac{(1-\tau) W_t}{\varphi P_t} \Rightarrow \\ \frac{C_A}{1-N_A} + \frac{1}{1-N_A} c_{A,t} C_A + \frac{C_A}{(1-N_A)^2} n_{A,t} N_A &= \frac{(1-\tau) W}{\varphi P} + \frac{(1-\tau)}{\varphi} \left[ \frac{1}{P} w_t W - \frac{W}{P^2} p_t P \right] \Rightarrow \\ \frac{C_A}{1-N_A} + \frac{C_A}{1-N_A} c_{A,t} + \frac{C_A}{(1-N_A)^2} n_{A,t} N_A &= \frac{(1-\tau) W}{\varphi P} + \frac{(1-\tau) W}{\varphi P} [w_t - p_t] \Rightarrow \\ 1 + c_{A,t} + \frac{N_A}{1-N_A} n_{A,t} &= 1 + w_t - p_t \Rightarrow \\ \log C_{A,t} - \log C_A + \frac{N_A}{1-N_A} (\log N_{A,t} - \log N_A) &= \log W_t - \log W - \log P_t + \log P \Rightarrow \\ \frac{N_A}{1-N_A} (\log N_{A,t} - \log N_A) &= \log \left( \frac{P W_t}{P_t W} \right) - (\log C_{A,t} - \log C_A) \Rightarrow \\ \frac{N}{1-N} n_{A,t} &= w_t - c_{A,t} \end{aligned}$$

Relation (Eq. 29), is obtained log-linearizing expression (Eq. 8):

$$\begin{aligned}
\frac{C_{N,t}}{L_{N,t}} &= \frac{(1-\tau) W_t}{\varphi P_t} \Rightarrow \\
\frac{C_{N,t}}{1-N_{N,t}} &= \frac{(1-\tau) W_t}{\varphi P_t} \Rightarrow \\
\frac{C_N}{1-N_N} + \frac{1}{1-N_N} c_{N,t} C_N + \frac{C_N}{(1-N_N)^2} n_{N,t} N_N &= \frac{(1-\tau) W}{\varphi P} + \frac{(1-\tau)}{\varphi} \left[ \frac{1}{P} w_t W - \frac{W}{P^2} p_t P \right] \Rightarrow \\
\frac{C_N}{1-N_N} + \frac{C_N}{1-N_N} c_{N,t} + \frac{C_N}{(1-N_N)^2} n_{N,t} N_N &= \frac{(1-\tau) W}{\varphi P} + \frac{(1-\tau) W}{\varphi P} [w_t - p_t] \Rightarrow \\
1 + c_{N,t} + \frac{N_N}{1-N_N} n_{N,t} &= 1 + w_t - p_t \Rightarrow \\
\log C_{N,t} - \log C_N + \frac{N_N}{1-N_N} (\log N_{N,t} - \log N_N) &= \log W_t - \log W - \log P_t + \log P \Rightarrow \\
\frac{N_N}{1-N_N} (\log N_{N,t} - \log N_N) &= \log \left( \frac{P}{P_t} \frac{W_t}{W} \right) - (\log C_{A,t} - \log C_A) \Rightarrow \\
\frac{N}{1-N} n_{N,t} &= w_t - c_{N,t}
\end{aligned}$$

The log-linearized budget constraint for non-asset holders (Eq. 30) is obtained log-

linearizing expression (Eq. 7):

$$\begin{aligned}
P_t C_{N,t} &= (1 - \tau) W_t N_{N,t} - P_t T_t \Rightarrow \\
P_t C_{N,t} + C_N p_t P + P c_{N,t} C_N &= (1 - \tau) W N_N + (1 - \tau) N_N w_t W + (1 - \tau) W n_{N,t} N_N \\
&\quad - P T - T p_t P - P t_t T \Rightarrow \\
C_N p_t P + P c_{N,t} C_N &= (1 - \tau) N_N w_t W + (1 - \tau) W n_{N,t} N_N - T p_t P - P t_t T \Rightarrow \\
C_N p_t P + P c_{N,t} C_N &= (1 - \tau) N_N W (w_t + n_{N,t}) - P T (p_t + t_t) \Rightarrow \\
\frac{P}{P} \frac{C_N}{Y} p_t + \frac{P}{P} \frac{C_N}{Y} c_{N,t} &= (1 - \tau) \frac{N_N}{Y} \frac{W}{P} (w_t + n_{N,t}) - \frac{T}{Y} (p_t + t_t) \Rightarrow \\
(1 - G_Y) p_t + (1 - G_Y) c_{N,t} &= (1 - \tau) \frac{N_N}{Y} \frac{W}{P} (w_t + n_{N,t}) - T_Y (p_t + t_t) \Rightarrow \\
(1 - G_Y) p_t + (1 - G_Y) c_{N,t} &= (1 - \tau) \frac{N}{Y} \frac{C}{L} \frac{\varphi}{(1 - \tau)} (w_t + n_{N,t}) - T_Y (p_t + t_t) \Rightarrow \\
(1 - G_Y) p_t + (1 - G_Y) c_{N,t} &= \varphi \frac{N}{1 - N} \frac{C}{Y} (w_t + n_{N,t}) - T_Y (p_t + t_t) \Rightarrow \\
(1 - G_Y) p_t + (1 - G_Y) c_{N,t} &= \varphi (1 - G_Y) \frac{1}{\varphi} \frac{1 - \tau}{1 - G_Y} (w_t + n_{N,t}) - T_Y (p_t + t_t) \Rightarrow \\
(1 - G_Y) p_t + (1 - G_Y) c_{N,t} &= (1 - \tau) (w_t + n_{N,t}) - T_Y p_t - T_Y t_t \Rightarrow \\
(1 - G_Y + T_Y) p_t + (1 - G_Y) c_{N,t} &= (1 - \tau) (w_t + n_{N,t}) - T_Y t_t \Rightarrow \\
(1 - \tau) p_t + (1 - G_Y) c_{N,t} &= (1 - \tau) (w_t + n_{N,t}) - T_Y t_t \Rightarrow \\
(1 - G_Y) c_{N,t} &= (1 - \tau) (w_t + n_{N,t}) - T_Y t_t - (1 - \tau) p_t
\end{aligned}$$

$$\begin{aligned}
\Rightarrow (1 - G_Y) (\log C_{N,t} - \log C_N) &= (1 - \tau) (\log W_t - \log W) \\
&\quad + (1 - \tau) (\log N_{N,t} - \log N_N) - T_Y (\log T_t - \log T) - (1 - \tau) (\log P_t - \log P)
\end{aligned}$$

$$\begin{aligned}
\Rightarrow (1 - G_Y) (\log C_{N,t} - \log C_N) &= (1 - \tau) \log \left( \frac{P}{P_t} \frac{W_t}{W} \right) \\
&\quad + (1 - \tau) (\log N_{N,t} - \log N_N) - T_Y (\log T_t - \log T)
\end{aligned}$$

$$\Rightarrow (1 - G_Y) c_{N,t} = (1 - \tau) (w_t + n_{N,t}) - T_Y t_t$$

From expressions (Eq. 29) and (Eq. 30) we obtain equation (Eq. 31):

$$\begin{aligned} & \begin{cases} \frac{N}{1-N} n_{N,t} = w_t - c_{N,t} \\ (1 - G_Y) c_{N,t} = (1 - \tau) (w_t + n_{N,t}) - T_Y t_t \end{cases} \\ \Rightarrow & \begin{cases} \frac{1}{\varphi} \frac{1-\tau}{1-G_Y} n_{N,t} = w_t - c_{N,t} \\ c_{N,t} = \frac{1}{(1-G_Y)} [(1 - \tau) (w_t + n_{N,t}) - T_Y t_t] \end{cases} \\ \Rightarrow & \begin{cases} n_{N,t} = \varphi \frac{1-G_Y}{1-\tau} (w_t - c_{N,t}) \\ c_{N,t} = \frac{1}{(1-G_Y)} [(1 - \tau) (w_t + n_{N,t}) - T_Y t_t] \end{cases} \\ \Rightarrow & n_{N,t} = \varphi \frac{1 - G_Y}{1 - \tau} \left[ w_t - \frac{1}{(1 - G_Y)} ((1 - \tau) (w_t + n_{N,t}) - T_Y t_t) \right] \\ \Rightarrow & n_{N,t} = \varphi \left[ \frac{1 - G_Y}{1 - \tau} w_t - w_t - n_{N,t} + \frac{T_Y t_t}{1 - \tau} \right] \\ \Rightarrow & \varphi n_{N,t} + n_{N,t} = \varphi \left[ \frac{1 - G_Y}{1 - \tau} w_t - w_t + \frac{T_Y t_t}{1 - \tau} \right] \\ \Rightarrow & n_{N,t} = \frac{\varphi}{1 + \varphi} \left[ \frac{1 - G_Y}{1 - \tau} w_t - w_t + \frac{T_Y t_t}{1 - \tau} \right] \\ \Rightarrow & n_{N,t} = \frac{\varphi}{1 + \varphi} \left[ \frac{w_t - G_Y w_t - w_t + \tau w_t + T_Y t_t}{1 - \tau} \right] \\ \Rightarrow & n_{N,t} = \frac{\varphi}{1 + \varphi} \left[ \frac{w_t (-G_Y + \tau) + T_Y t_t}{1 - \tau} \right] \\ \Rightarrow & n_{N,t} = \frac{\varphi}{1 + \varphi} \left[ \frac{-w_t T_Y + T_Y t_t}{1 - \tau} \right] \\ \Rightarrow & n_{N,t} = \frac{\varphi}{1 + \varphi} \left[ \frac{-T_Y}{1 - G_Y + T_Y} \right] (w_t - t_t) \end{aligned}$$



Expression (Eq. 32) is obtained log-linearizing equation (Eq. 25) and assuming:

$$N_A = N_N = N$$

thus:

$$\begin{aligned} N_t &= \lambda N_{N,t} + (1 - \lambda) N_{A,t} \Rightarrow \\ N + n_t N &= \lambda N_N + \lambda n_{N,t} N_N + (1 - \lambda) N_A + (1 - \lambda) n_{A,t} N_A \Rightarrow \\ N + n_t N &= \lambda N + \lambda n_{N,t} N + (1 - \lambda) N + (1 - \lambda) n_{A,t} N \Rightarrow \\ 1 + n_t &= \lambda + \lambda n_{N,t} + 1 - \lambda + (1 - \lambda) n_{A,t} \Rightarrow \\ n_t &= \lambda n_{N,t} + (1 - \lambda) n_{A,t} \end{aligned}$$

Expression (Eq. 33) is obtained log-linearizing equation (Eq. 24) and assuming:

$$C_A = C_N = C$$

thus:

$$\begin{aligned} C_t &= \lambda C_{N,t} + (1 - \lambda) C_{A,t} \Rightarrow \\ C + c_t C &= \lambda C_N + \lambda c_{N,t} C_N + (1 - \lambda) C_A + (1 - \lambda) c_{A,t} C_A \Rightarrow \\ C + c_t C &= \lambda C + \lambda c_{N,t} C + (1 - \lambda) C + (1 - \lambda) c_{A,t} C \Rightarrow \\ 1 + c_t &= \lambda + \lambda c_{N,t} + 1 - \lambda + (1 - \lambda) c_{A,t} \Rightarrow \\ c_t &= \lambda c_{N,t} + (1 - \lambda) c_{A,t} \end{aligned}$$

The log-linearized aggregate production function (Eq. 34) is obtained from ex-

pression (Eq. 11):

$$\begin{aligned}
Y_t(i) &= N_t(i) - F \Rightarrow \\
Y + y_t Y &= N + n_t N - F \Rightarrow \\
Y + y_t Y &= Y + F + n_t(Y + F) - F \Rightarrow \\
y_t Y &= n_t(Y + F) \Rightarrow \\
y_t &= (1 + F_Y) n_t
\end{aligned}$$

The new-Keynesian Phillips curve (Eq. 35) is obtained combining the equation of the price setting problem (Eq. 12):

$$E_t \sum_{s=0}^{\infty} \alpha^s \Lambda_{t,t+s} \left[ P_t^*(i) - \frac{\varepsilon}{\varepsilon - 1} W_t \right] = 0$$

together with the dynamics of the price:

$$P_t = \left[ \alpha (P_{t-1})^{1-\varepsilon} + (1 - \alpha) (P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

From the dynamics of the price level:

$$P_t = \left[ \alpha P_{t-1}^{1-\varepsilon} + (1 - \alpha) (P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

we have that in steady state:

$$\begin{aligned}
P &= \left[ \alpha P^{1-\varepsilon} + (1 - \alpha) (P^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \Rightarrow \\
P^{1-\varepsilon} &= \alpha P^{1-\varepsilon} + (1 - \alpha) (P^*)^{1-\varepsilon} \Rightarrow \\
P^{1-\varepsilon} (1 - \alpha) &= (1 - \alpha) (P^*)^{1-\varepsilon} \Rightarrow \\
P &= P^*
\end{aligned}$$

Again from the dynamics of the price level without subscript  $i$ :

$$P_t^{1-\epsilon} = \alpha P_{t-1}^{1-\epsilon} + (1-\alpha)(P_t^*)^{1-\epsilon}$$

dividing by  $P_t^{1-\epsilon}$  we obtain:

$$\frac{P_t^{1-\epsilon}}{P_t^{1-\epsilon}} = \alpha \frac{P_{t-1}^{1-\epsilon}}{P_t^{1-\epsilon}} + (1-\alpha) \frac{(P_t^*)^{1-\epsilon}}{P_t^{1-\epsilon}}$$

defining:

$$V_t = \frac{P_t^*}{P_t}$$

and :  $V = 1$

we have:

$$1 = \alpha \left( \frac{P_{t-1}}{P_t} \right)^{1-\epsilon} + (1-\alpha) V_t^{1-\epsilon}$$

log-linearizing this expression:

$$\begin{aligned} (1-\epsilon) \alpha \left( \frac{P}{P} \right)^{-\epsilon} \frac{1}{P} p_{t-1} P - (1-\epsilon) \alpha \left( \frac{P}{P} \right)^{-\epsilon} \frac{P}{P^2} p_t P + (1-\epsilon)(1-\alpha) V^{-\epsilon} v_t V &= 0 \Rightarrow \\ \alpha p_{t-1} - \alpha p_t + (1-\alpha) v_t &= 0 \Rightarrow \\ -\alpha (p_t - p_{t-1}) + (1-\alpha) v_t &= 0 \Rightarrow \\ (1-\alpha) v_t &= \alpha \pi_t \Rightarrow \end{aligned}$$

thus:

$$v_t = \frac{\alpha}{1-\alpha} \pi_t \tag{Eq. A22}$$

Moreover, from expression (Eq. 12), ignoring subscript  $i$  for simplicity:

$$E_t \sum_{s=0}^{\infty} \alpha^s \Lambda_{t,t+s} \left[ P_t^* - \frac{\epsilon}{\epsilon-1} W_{t+s} \right] = 0$$

in steady state we have:

$$\begin{aligned}
P - \frac{\varepsilon}{\varepsilon - 1}W &= 0 \Rightarrow \\
P &= \frac{\varepsilon}{\varepsilon - 1}W \Rightarrow \\
\frac{W}{P} &= \frac{\varepsilon - 1}{\varepsilon}
\end{aligned}$$

Again from (Eq. 12):

$$\begin{aligned}
E_t \sum_{s=0}^{\infty} \alpha^s \Lambda_{t,t+s} \left[ P_t^* - \frac{\varepsilon}{\varepsilon - 1} W_{t+s} \right] &= 0 \Rightarrow \\
E_t \sum_{s=0}^{\infty} \alpha^s \beta^s \left( \frac{C_{A,t}}{C_{A,t+s}} \right)^\sigma \left( \frac{L_{A,t+s}}{L_{A,s}} \right)^{\varphi(1-\sigma)} \frac{P_t}{P_{t+s}} \left[ P_t^* - \frac{\varepsilon}{\varepsilon - 1} W_{t+s} \right] &= 0 \Rightarrow \\
E_t \sum_{s=0}^{\infty} \alpha^s \beta^s \left( \frac{C_{A,t}}{C_{A,t+s}} \right)^\sigma \left( \frac{L_{A,t+s}}{L_{A,s}} \right)^{\varphi(1-\sigma)} \left[ \frac{P_t}{P_{t+s}} P_t^* - \frac{W_{t+s}}{P_{t+s}} P_t \frac{\varepsilon}{\varepsilon - 1} \right] &= 0
\end{aligned}$$

multiplying both sides for  $\frac{1}{P_t^*}$  gives:

$$\begin{aligned}
E_t \sum_{s=0}^{\infty} \alpha^s \beta^s \left( \frac{C_{A,t}}{C_{A,t+s}} \right)^\sigma \left( \frac{L_{A,t+s}}{L_{A,s}} \right)^{\varphi(1-\sigma)} \frac{1}{P_t^*} \left[ \frac{P_t}{P_{t+s}} P_t^* - \frac{W_{t+s}}{P_{t+s}} P_t \frac{\varepsilon}{\varepsilon - 1} \right] &= 0 \Rightarrow \\
E_t \sum_{s=0}^{\infty} \alpha^s \beta^s \left( \frac{C_{A,t}}{C_{A,t+s}} \right)^\sigma \left( \frac{L_{A,t+s}}{L_{A,s}} \right)^{\varphi(1-\sigma)} \left[ \frac{P_t}{P_{t+s}} \frac{P_t^*}{P_t^*} - \frac{1}{P_{t+s}} \frac{P_t}{P_t^*} \frac{\varepsilon}{\varepsilon - 1} W_{t+s} \right] &= 0 \Rightarrow \\
E_t \sum_{s=0}^{\infty} \alpha^s \beta^s \left( \frac{C_{A,t}}{C_{A,t+s}} \right)^\sigma \left( \frac{L_{A,t+s}}{L_{A,s}} \right)^{\varphi(1-\sigma)} \left[ \frac{P_t}{P_{t+s}} - MC_{t+s} \frac{1}{V_t} \frac{\varepsilon}{\varepsilon - 1} \right] &= 0 \text{ (Eq. A23)}
\end{aligned}$$

where we define:

$$MC_{t+s} = \frac{W_{t+s}}{P_{t+s}} \quad \text{(Eq. A24)}$$

in steady state:

$$MC_{t+s} = \frac{W}{P} = \frac{\varepsilon - 1}{\varepsilon}$$

log-linearizing this expression:

$$\begin{aligned}
& E_t \sum_{s=0}^{\infty} \alpha^s \beta^s \sigma \left( \frac{C_A}{C_A} \right)^{\sigma-1} \frac{1}{C_A} \left( \frac{L_A}{L_A} \right)^{\varphi(1-\sigma)} \left[ \frac{P}{P} - \frac{\varepsilon}{\varepsilon-1} MC \frac{1}{V} \right] c_{A,t} C_A \\
& - E_t \sum_{s=0}^{\infty} \alpha^s \beta^s \sigma \left( \frac{C_A}{C_A} \right)^{\sigma-1} \frac{C_A}{C_A^2} \left( \frac{L_A}{L_A} \right)^{\varphi(1-\sigma)} \left[ \frac{P}{P} - \frac{\varepsilon}{\varepsilon-1} MC \frac{1}{V} \right] c_{A,t+1} C_A \\
& + E_t \sum_{s=0}^{\infty} \alpha^s \beta^s \varphi(1-\sigma) \left( \frac{C_A}{C_A} \right)^{\sigma} \left( \frac{L_A}{L_A} \right)^{\varphi(1-\sigma)-1} \frac{1}{L_A} \left[ \frac{P}{P} - \frac{\varepsilon}{\varepsilon-1} MC \frac{1}{V} \right] l_{A,t+1} L_A \\
& - E_t \sum_{s=0}^{\infty} \alpha^s \beta^s \varphi(1-\sigma) \left( \frac{C_A}{C_A} \right)^{\sigma} \left( \frac{L_A}{L_A} \right)^{\varphi(1-\sigma)-1} \frac{L_A}{L_A^2} \left[ \frac{P}{P} - \frac{\varepsilon}{\varepsilon-1} MC \frac{1}{V} \right] l_{A,t} L_A \\
& + E_t \sum_{s=0}^{\infty} \alpha^s \beta^s \left( \frac{C_A}{C_A} \right)^{\sigma} \left( \frac{L_A}{L_A} \right)^{\varphi(1-\sigma)} \left[ \frac{1}{P} \right] p_t P \\
& - E_t \sum_{s=0}^{\infty} \alpha^s \beta^s \left( \frac{C_A}{C_A} \right)^{\sigma} \left( \frac{L_A}{L_A} \right)^{\varphi(1-\sigma)} \left[ \frac{\varepsilon}{\varepsilon-1} \frac{P}{P^2} \frac{1}{V} \right] p_{t+s} P \\
& - E_t \sum_{s=0}^{\infty} \alpha^s \beta^s \left( \frac{C_A}{C_A} \right)^{\sigma} \left( \frac{L_A}{L_A} \right)^{\varphi(1-\sigma)} \left[ \frac{1}{V} \frac{\varepsilon}{\varepsilon-1} \right] m c_{t+s} MC \\
& + E_t \sum_{s=0}^{\infty} \alpha^s \beta^s \left( \frac{C_A}{C_A} \right)^{\sigma} \left( \frac{L_A}{L_A} \right)^{\varphi(1-\sigma)} \left[ \frac{\varepsilon}{\varepsilon-1} \frac{W}{P} \frac{1}{V^2} \right] v_t V \\
& = 0
\end{aligned}$$

for simplicity, we eliminate expectation operator:

$$\begin{aligned}
\sum_{s=0}^{\infty} \alpha^s \beta^s p_t - \sum_{s=0}^{\infty} \alpha^s \beta^s p_{t+s} - \sum_{s=0}^{\infty} \alpha^s \beta^s m c_{t+s} + \sum_{s=0}^{\infty} \alpha^s \beta^s v_t &= 0 \Rightarrow \\
\sum_{s=0}^{\infty} \alpha^s \beta^s v_t &= \sum_{s=0}^{\infty} \alpha^s \beta^s p_{t+s} - \sum_{s=0}^{\infty} \alpha^s \beta^s p_t + \sum_{s=0}^{\infty} \alpha^s \beta^s m c_{t+s} \Rightarrow \\
v_t \sum_{s=0}^{\infty} \alpha^s \beta^s &= -p_t \sum_{s=0}^{\infty} \alpha^s \beta^s + \sum_{s=0}^{\infty} \alpha^s \beta^s (p_{t+s} + m c_{t+s})
\end{aligned}$$

since:

$$0 < \alpha^s \beta^s < 1 \Rightarrow$$

$$\sum_{s=0}^{\infty} \alpha^s \beta^s = \frac{1}{1 - \alpha\beta}$$

Thus, we can rewrite:

$$v_t \frac{1}{1 - \alpha\beta} = -p_t \frac{1}{1 - \alpha\beta} + \beta \sum_{s=0}^{\infty} \alpha^s \beta^s (p_{t+s} + mc_{t+s}) \Rightarrow$$

$$v_t \frac{1}{1 - \alpha\beta} + p_t \frac{1}{1 - \alpha\beta} = \beta \sum_{s=0}^{\infty} \alpha^s \beta^s (p_{t+s} + mc_{t+s}) \Rightarrow$$

$$v_t + p_t = (1 - \alpha\beta) \sum_{s=0}^{\infty} \alpha^s \beta^s (p_{t+s} + mc_{t+s}) \quad (\text{Eq. A25})$$

Last expression can be rewritten as:

$$v_t + p_t = (1 - \alpha\beta) (p_t + w_t) + \alpha\beta (p_{t+1} + v_{t+1}) \quad (\text{Eq. A26})$$

Now we demonstrate that expressions (Eq. A25) and (Eq. A26) are equivalent.

Indeed, taking equation (Eq. A26) at  $t - 1$ :

$$v_{t-1} + p_{t-1} = (1 - \alpha\beta) (p_{t-1} + w_{t-1}) - \alpha\beta (p_t + v_t) \Rightarrow$$

$$-(v_{t-1} + p_{t-1}) + \alpha\beta (p_t + v_t) = -(1 - \alpha\beta) (p_{t-1} + w_{t-1}) \Rightarrow$$

$$(p_t + v_t) - \frac{1}{\alpha\beta} (v_{t-1} + p_{t-1}) = -\frac{(1 - \alpha\beta)}{\alpha\beta} (p_{t-1} + w_{t-1})$$

defining:

$$\begin{aligned}l_t &= p_t + v_t \\u_{t-1} &= p_{t-1} + w_{t-1} \\ \zeta &= \frac{1}{\alpha\beta} \\ \xi &= -\frac{(1-\alpha\beta)}{\alpha\beta}\end{aligned}$$

we have:

$$\begin{aligned}l_t - \zeta l_{t-1} &= \xi u_{t-1} \Rightarrow \\ l_{t-1} &= \zeta^{-1} l_t - \xi \zeta^{-1} u_{t-1}\end{aligned}$$

lagging one period:

$$\begin{aligned}l_t &= \zeta^{-1} l_{t+1} - \xi \zeta^{-1} u_t \Rightarrow \\ (1 - \zeta^{-1} L^{-1}) l_t &= -\xi \zeta^{-1} u_t \\ \text{where } : \quad \zeta^{-1} &< 1 \\ \text{and } : \quad L &\text{ is the lag operator}\end{aligned}$$

thus:

$$\begin{aligned}l_t &= -\xi \zeta^{-1} \frac{1}{(1 - \zeta^{-1} L^{-1})} u_t \Rightarrow \\ l_t &= -\xi \frac{1}{\zeta} \sum_{s=0}^{\infty} \zeta^{-s} u_{t+s}\end{aligned}$$

substituting we obtain expression (Eq. A25):

$$\begin{aligned}
p_t + v_t &= -\left(-\frac{(1-\alpha\beta)}{\alpha\beta}\right) (\alpha\beta) \sum_{s=0}^{\infty} \alpha^s \beta^s (p_{t+s} + mc_{t+s}) \Rightarrow \\
p_t + v_t &= (1-\alpha\beta) \sum_{s=0}^{\infty} \alpha^s \beta^s (p_{t+s} + mc_{t+s})
\end{aligned}$$

Thus, taking equation (Eq. A26):

$$\begin{aligned}
v_t + p_t &= (1-\alpha\beta) (p_t + w_t) + \alpha\beta (p_{t+1} + v_{t+1}) \Rightarrow \\
v_t + p_t &= p_t + mc_t - \alpha\beta p_t - \alpha\beta mc_t + \alpha\beta (p_{t+1} + v_{t+1}) \Rightarrow \\
v_t &= mc_t (1-\alpha\beta) + \alpha\beta (p_{t+1} - p_t + v_{t+1}) \Rightarrow \\
v_t &= mc_t (1-\alpha\beta) + \alpha\beta (\pi_{t+1} + v_{t+1}) \tag{Eq. A27}
\end{aligned}$$

putting expression (Eq. A22) into (Eq. A27) gives:

$$\begin{aligned}
\frac{\alpha}{1-\alpha} \pi_t &= mc_t (1-\alpha\beta) + \alpha\beta \left( \pi_{t+1} + \frac{\alpha}{1-\alpha} \pi_{t+1} \right) \Rightarrow \\
\frac{\alpha}{1-\alpha} \pi_t &= mc_t (1-\alpha\beta) + \alpha\beta \left( 1 + \frac{\alpha}{1-\alpha} \right) \pi_{t+1} \Rightarrow \\
\frac{\alpha}{1-\alpha} \pi_t &= mc_t (1-\alpha\beta) + \beta \frac{\alpha}{1-\alpha} \pi_{t+1} \Rightarrow \\
\pi_t &= \beta E_t \pi_{t+1} + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha} mc_t \tag{Eq. A28}
\end{aligned}$$

Moreover, from (Eq. A24):

$$\begin{aligned}
MC_{t+s} &= \frac{W_{t+s}}{P_{t+s}} \\
mc_{t+s} MC &= \frac{1}{P} \log(W_{t+s} - W) W - \frac{W}{P^2} \log(P_{t+s} - P) P \Rightarrow \\
mc_{t+s} &= \log W_{t+s} - \log W - \log P_{t+s} + \log P \Rightarrow \\
mc_{t+s} &= \log \left( \frac{W_{t+s}/P_{t+s}}{W/P} \right) \Rightarrow \\
mc_{t+s} &= w_t \tag{Eq. A29}
\end{aligned}$$



thus, putting expression (Eq. A29) into (Eq. A28) we obtain the new-Keynesian Phillips curve (Eq. 35):

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \alpha\beta)(1 - \alpha)}{\alpha} w_t$$

Linearization of the government budget constraint (Eq. 14) around a steady state with zero debt and a balanced primary budget yields expression (Eq. 36):

$$\frac{1}{R_t} B_{t+1} = B_t + P_t [G_t - \tau Y_t - T_t]$$

dividing by  $P_t$ :

$$\begin{aligned} R_t^{-1} B_{t+1} \frac{1}{P_t Y_t} &= B_t \frac{1}{P_t Y_t} + \frac{P_t}{P_t} \left[ \frac{G_t}{Y_t} - \tau \frac{Y_t}{Y_t} - \frac{T_t}{Y_t} \right] \Rightarrow \\ R_t^{-1} \frac{B_{t+1}}{P_t Y_t} &= \frac{B_t}{P_t Y_t} + \frac{G_t}{Y_t} - \tau - \frac{T_t}{Y_t} \end{aligned}$$

Linearizing this expression:

$$\begin{aligned} &-R^{-2} \frac{B}{PY} (R_t - R) + R^{-1} \frac{1}{PY} (B_{t+1} - B) \\ &-R^{-1} \frac{B}{P^2 Y} (P_t - P) - R^{-1} \frac{B}{PY^2} (Y_t - Y) \\ = &\frac{1}{PY} (B_t - B) - \frac{B}{P^2 Y} (P_t - P) - \frac{B}{PY^2} (Y_t - Y) \\ &+ \frac{1}{Y} g_t G - \frac{G}{Y^2} y_t Y - \frac{1}{Y} t_t T + \frac{T}{Y^2} y_t Y \end{aligned}$$

Assuming steady state with zero debt:

$$R^{-1} \frac{B_{t+1}}{PY} = \frac{B_t}{PY} + G_Y g_t - T_Y t_t + y_t (T_Y - G_Y) \quad (\text{Eq. A30})$$

Assuming a balanced primary budget:

$$P_{t-1} = P_t = P$$

Using expression (Eq. A17) and defining:

$$\hat{b}_t = \frac{B_t}{P_{t-1}Y_{t-1}}$$

Finally, adding all these conditions, equation (Eq. A30) gives the government budget constraint linearized (Eq. 36):

$$\beta\hat{b}_{t+1} = \hat{b}_t + G_Y g_t - T_T t_t - \tau y_t$$

Expression (Eq. 37) is obtained log-linearizing expression (Eq. 16) and dividing by  $Y_t$ :

$$\begin{aligned} G_t &= NM_t + M_t \Rightarrow \\ \frac{G_t}{Y_t} &= \frac{NM_t}{Y_t} + \frac{M_t}{Y_t} \Rightarrow \\ \frac{1}{Y} g_t G - \frac{G}{Y^2} y_t Y &= \frac{1}{Y} n m_t N M - \frac{N M}{Y^2} y_t Y + \frac{1}{Y} m_t M - \frac{M}{Y^2} y_t Y \Rightarrow \\ g_t G_Y - G y_t &= n m_t N M_Y - N M y_t + m_t M_Y - M y_t \Rightarrow \\ g_t G_Y &= n m_t N M_Y + m_t M_Y + y_t (G - N M - M) \Rightarrow \\ g_t G_Y &= n m_t N M_Y + m_t M_Y \end{aligned}$$

The log-linearized structural primary deficit (Eq. 38) is obtained by expression (Eq. 20):

$$D_{s,t} = G_t - T_t - \tau Y$$

Assuming that:

$$\hat{d}_{s,t} = \frac{Y}{Y}$$

We have that:

$$\begin{aligned}
D_{s,t} &= G_t - T_t - \tau Y \Rightarrow \\
D_s + d_{s,t}D_s &= G + g_tG - T - t_tT \Rightarrow \\
d_{s,t}D_s &= g_tG - t_tT \Rightarrow \\
d_{s,t}\frac{1}{Y}D_s &= g_t\frac{1}{Y}G - t_t\frac{1}{Y}T \Rightarrow \\
\hat{d}_{s,t} &= G_Y g_t - T_Y t_t
\end{aligned}$$

Log-linearized good market clearing condition (Eq. 41) is obtained from expression (Eq. 23):

$$\begin{aligned}
Y_t &= C_t + G_t \Rightarrow \\
Y + y_tY &= C + c_tC + G + g_tG \Rightarrow \\
y_tY &= c_tC + g_tG \Rightarrow \\
y_t &= c_tC_Y + g_tG_Y \Rightarrow \\
y_t &= g_tG_Y + c_t(1 - G_Y)
\end{aligned}$$

## References

- [1] Bilbiie, F., & Straub, R., (2006). "Asset Market Participation, Monetary Policy Rules, and the Great Inflation", *IMF Working Papers 06/200*, International Monetary Fund.
- [2] Bilbiie, F., O., Meier, A., & Gernot J., M. (2009). "What accounts for the changes in the U.S. fiscal policy transmission?", *Journal of Money, Credit and Banking*, 40(7).
- [3] Blanchard, O., & Perotti, R. (2002). "An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output", *Quarterly Journal of Economics*, 117(4), 1329-1368.
- [4] Bohm, H. (1998). "The Behavior of U.S. Public Debt and Deficits", *Quarterly Journal of Economics*, 113, 949–63.
- [5] Boivin, J., & Giannoni, M., (2006). "Has Monetary Policy Become More Effective?", *The Review of Economics and Statistics*, MIT Press, vol. 88(3), pages 445-462,
- [6] Calvo, G., A. (1983). "Staggered Prices in a Utility-Maximizing Framework", *Journal of Monetary Economics*, 13, 383–98.
- [7] Canova, F., & Ferroni, F. (2010). "The Dynamics of US Inflation: Can Monetary Policy explain the Changes?", *Working Paper*.
- [8] Castelnuovo, E. (2010). "Monetary Policy Shocks, Cholesky Identification, and DSGE models: An Empirical Investigation for the U.S", *mimeo*.
- [9] Christiano, L., Eichenbaum, M., & Evans, C. (2001). "Nominal rigidities and the dynamic effects of a shock to monetary policy", May 2001.
- [10] Clarida, R., Galí, J., & Gertler, M. (1998). "Monetary policy rules in practice: Some international evidence", *European Economic Review*, 42, 6, 1033-67.
- [11] Fatás, A. & Mihov, I. (2001). "The Effects of Fiscal Policy on Consumption and Employment: Theory and Evidence", *INSEAD, mimeo*.

- [12] Feldstein, M. (2008). "Defense Spending Would Be Great Stimulus". *The Wall Street Journal*, December 24, 2008.
- [13] Fernandez-Villaverde, J., & Rubio-Ramirez, J. (2001), "Comparing dynamic equilibrium models to data", *mimeo*.
- [14] Galí, J., & Gertler, M. (1999). "Inflation Dynamics: A Structural Econometric Analysis", *Journal of Monetary Economics*, 44, 195–222.
- [15] Galí, J., & Perotti, R. (2003). "Fiscal Policy and Monetary Integration in Europe", *Economic Policy*, 37, 534-72.
- [16] Galí, J., López-Salido, D., & Vallés, J. (2007). "Understanding the Effects of Government Spending on Consumption", *Journal of the European Economic Association*, 5, 227-70.
- [17] Geweke, J. (1998). "Using simulation methods for Bayesian econometric models: inference, development and communication", *mimeo*, *University of Minnesota and Federal Reserve Bank of Minneapolis*.
- [18] Geweke, J. (1999). "Computational experiments and reality", *University of Minnesota and Federal Reserve Bank of Minneapolis*, June 1999.
- [19] Ireland, P., N. (1999). "A method for taking models to the data", *Boston College and NBER*, 1999.
- [20] Ireland, P., N. (1999). "Interest rates, inflation, and Federal Reserve policy since 1980", *Boston College*, 1999.
- [21] King, R., Plosser, C., & Rebelo, S., (1988). "Production, growth and business cycles : I. The basic neoclassical model", *Journal of Monetary Economics*, Elsevier, vol. 21(2-3), pages 195-232.
- [22] Kydland, F., & Prescott, E. (1982). "Time to build and aggregate fluctuations", *Econometrica*, 50, 1345-70.

- [23] Kydland, F., & Prescott, E. (1996). "The computational experiment: an econometric Tool", *Journal of Economic Perspectives*, 10(1), 68-86.
- [24] Linnemann, L., & Schabert, A. (2003). "Fiscal Policy and the New Neoclassical Synthesis", *Journal of Money, Credit and Banking*, 35, 911-29.
- [25] Linnemann, L., & Schabert, A. (2006). "Productive Government Expenditure In Monetary Business Cycle Models", *Scottish Journal of Political Economy*, 53(1), 28-46.
- [26] Landon-Lane, J. (1998). "Bayesian comparison of dynamic macroeconomic models", *mimeo*.
- [27] Mankiw, N., G. (2000). "The Savers-Spenders Theory of Fiscal Policy", *American Economic Review*, 90(2), 120-125.
- [28] Mishkin, F., S. (1991). "Financial Innovation and Current Trends in U.S. Financial Markets", *NBER Reprints 1662*.
- [29] Mihov, I. (2003). "Discussion of: Understanding the Effects of Government Spending on Consumption, by Jordi Galí, J. David López-Salido, and Javier Vallés". *Mimeo, INSEAD*.
- [30] Monacelli, T., & Perotti, R. (2006). "Fiscal Policy, the Trade Balance, and the Real Exchange Rate: Implications for International Risk Sharing", *manuscript, IGIER, Università Bocconi*.
- [31] Otrok, C. (2001). "On measuring the welfare costs of business cycles", *Journal of Monetary Economics*, 47, 61-92.
- [32] Perotti, R. (2005). "Estimating the effects of fiscal policy in OECD countries", *Proceedings, Federal Reserve Bank of San Francisco*.
- [33] Rabanal, P., & Rubio-Ramirez, J. (2001). "Nominal versus real wage rigidities: a Bayesian approach", *mimeo*.
- [34] Rotemberg, J., & Woodford, M. (1998). "An optimization-based econometric framework for the evaluation of monetary policy: expanded version", *NBER Technical Working Paper 233*.

- [35] Sargent, T., J. (1989). "Two models of measurements and the Investment Accelerator", *Journal of Political Economy*, 97(2), 251-287.
- [36] Schorfheide, F., (2000). "Loss function-based evaluation of DSGE models", *Journal of Applied Econometrics*, 15(6), 645-670.
- [37] Smets, F. & Wouters, R. (2003). " An estimated DSGE model for the Euro area", *Journal of the European Economic Association*, 1, 1123-1175.
- [38] Smets, F. & Wouters, R. (2007). "Shocks and Frictions in US Business Cycles: a Bayesian DSGE approach", *American Economic Review*, 97(3), 586-606.
- [39] Taylor, J. (1993). "Discretion versus policy rules in practice", *Carnegie-Rochester Conference Series on Public Policy*, 39, 195-214.
- [40] Yun, T. (1996). "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles", *Journal of Monetary Economics*, 37, 345-70.

## 4 *THE ROLE OF GOVERNMENT SPENDING COMPONENTS: A RE-EXAMINATION OF THE EFFECTS ON PRIVATE CONSUMPTION*

### 4.1 *INTRODUCTION*

A classic question in macroeconomics concerns the size of the government spending multiplier. There is a large empirical literature that analyses the dimension of government spending multiplier. Authors such as Barro (1981) argue that the multiplier is around 0.8 while authors such as Ramey (2008) estimate the multiplier to be closer to 1.2<sup>17</sup>. There is also a large literature that uses general-equilibrium models to study the size of the government-spending multiplier. In standard new-Keynesian models the government-spending multiplier can be somewhat above or below one depending on the exact specification of agent's preferences (see Gali, López-Salido and Vallés 2007, and Monacelli and Perotti 2008). In frictionless Real Business Cycle models this multiplier is typically less than one (see e.g. Aiyagari, Christiano, and Eichenbaum 1992, Baxter and King 1993, Burnside, Eichenbaum and Fisher 2004, Ramey and Shapiro 1998, and Ramey 2008).

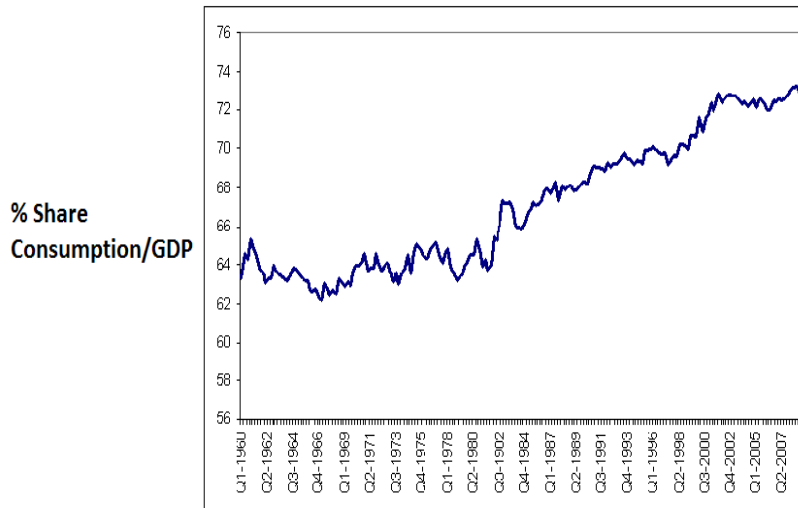
From this debate comes out a very basic question: which fiscal policy component is associated with the highest multiplier? For this reason, the present paper investigates how different categories of public expenditure affect private consumption. The relevance of the issue concerns private consumption as major weight among aggregate demand components, as showed by Figure 1. This in turn is the reason why consumption's response to economic stimulus plans is the key determinant of output multipliers.

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<sup>17</sup>For recent contributions to the VAR-based empirical literature on the size of the government spending multiplier see Fisher and Peters (2009) and Ilzetzki, Mendoza, and Vegh (2009).



Figure 1: Private consumption as % of GDP



In the present paper, we perform a structural VAR analysis on the US economy. As it is well known, there are two alternative approaches to the identification of fiscal policy shocks in the VAR literature. On one side, Blanchard and Perotti (2002) have perhaps the most careful and comprehensive approach to estimate fiscal shocks using VARs. To identify shocks, they first incorporate institutional information on taxes, transfers, and spending to set parameters, and then estimate the VAR. They analyse the contemporaneous relationship between taxes and GDP and they find that government spending does not respond to GDP or taxes contemporaneously. Thus, their identification of government spending shocks is identical to a Choleski decomposition in which government spending is ordered as the most exogenous compared to the other variables. When they augment the system to include consumption, they find that consumption rises in response to a positive government spending shock. Galí et al. (2007) use this basic identification method in their study which focuses only on government spending shocks and not taxes. They estimate a VAR with additional variables of interest, such as real wages, and order government spending as first.

On the other side, Ramey and Shapiro (1998) suggest that defence spending is a major part of the variation in government spending around trend. For this reason many analyses of government spending, including early contributions by Barro (1981) and Hall (1986), focus on military spending when studying the macroeconomic effects of government spending. Ramey and Shapiro (1998) use a narrative approach to identify shocks to government spending. Because of their concern that many shocks identified from a VAR are simply anticipated changes in government spending, they focus only on episodes where Business Week suddenly began to forecast large rises in defence spending induced by major political events that were unrelated to the state of the U.S. economy. In her paper, Ramey (2008) states that a concern with the VAR identification scheme is that some of what it classifies as “shocks” to government spending may well be anticipated. Thus, it is essential to identify when news becomes available about a major change in the present discounted value of government spending. For this reason she shows that the war dates (the Korean war military build-up, the Vietnam war build-up, the Reagan fiscal expansion, the US reaction of 2001 terrorist attacks), as well as professional forecasts, predict the VAR government spending shocks. She also shows how in each war episode, the VAR shocks are positive several quarters after Business Week or the Office of Management and Budget started forecasting increases in defence spending. Finally she shows that delaying the timing of the Ramey-Shapiro dates produces the Keynesian results.

In this paper we take a strategy more in line with a new-Keynesian approach. We justify it in several ways, imposing alternative types of structural restrictions: they can be sign restrictions on the impulse response functions (Uhlig 2005, Mountford and Uhlig 2005, Canova and Pappa 2007, Enders et al. 2008), external and institutional information exploiting the quarterly nature of data and fiscal policy decision lags (Blanchard and Perotti 2002, Perotti 2005, Muller 2008, Monacelli and Perotti 2008), or restrictions on contemporaneous relations among variables and error terms in the structural form (Marcellino 2006, Beetsma et al. 2006, Beetsma 2008, Benetrix

and Lane 2009). Using quarterly data from 1960 to 2008 and in line with some recent studies (Beetsma et al. 2006, Beetsma 2008, Giordano et al. 2007, Cavallo 2005 and 2007) we do not focus on public finance aggregates but rather on budget deficit's single components. Our disaggregation is mainly on the expenditure side, as we are primarily concerned with the aggregate consumption effects of different public expenditure categories. Here, we distinguish among military and non-military expenditures.

Our results show that the only component resulting in a positive and significant response of private consumption and gross domestic product is non-military expenditure. On the other hand, military spending seems to have a negative and significant effect. Regarding the magnitude of those effects, the impact on private consumption of a civilian spending shock equal to 1% is in absolute terms higher than the one (of the same magnitude) to the military component: after eight quarters, while shocks to the former lead to a +1.91% impact on GDP, shocks to the latter cause a -0.21% cumulative response. A consequence of our analysis is that using total government expenditure (by aggregating the two components above) does not seem to be a reasonable simplification: in fact, when these two components of government expenditure enter the VAR in a unique aggregate measure, the result is a lower impact on private consumption.

In the second part of our work we turn to the development of a simple dynamic general equilibrium model that can potentially account for that evidence. Our framework encompasses many ingredients of recent dynamic optimizing sticky price models, though we modify the latter by allowing for the presence of rule-of-thumb behaviour by some households<sup>18</sup>. Following Campbell and Mankiw (1989), we assume that rule-of-thumb consumers do not borrow or save; instead, they are assumed to consume their current income fully. In our model, rule-of-thumb consumers coexist with conventional infinite-horizon Ricardian consumers. As described subsequently,

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<sup>18</sup>See, for example, Rotemberg and Woodford (1999), Clarida, Gali, and Gertler (1999), or Woodford (2003) for a description of the standard new Keynesian model.

our model predicts responses of aggregate consumption and other variables that are in line with the existing evidence, given plausible calibrations of the fraction of rule-of-thumb consumers, the degree of price stickiness, and the extent of deficit financing.

The present paper can be relevant because we provide empirical and theoretical evidence that considering the indistinct aggregate of government expenditure can be very misleading. The identification of non-military as the only government expenditure category which is effective in stimulating private consumption leads to two remarks: (i) the complementarity/substitutability issue cannot be discussed independently from a sufficient disaggregation of government expenditure (ii) the rule-of-thumb-consumers approach can be justified on the existence of a precise portion of public expenditure that stimulates a fraction of consumers, specifically those who are the beneficial of non-military expenditure, and who consume out of it.

The remainder of this paper is organized as follows. Section 4.2 presents our empirical evidence, showing the data and discussing the identification procedure. Section 4.3 contains the estimation results with particular regard to the reaction of private consumption to different kinds of government expenditure shocks. Section 4.4 lays out the model and its different blocks. Section 4.5 contains the model calibration. Section 4.6 examines the equilibrium response to the different government spending shocks, focusing on the response of consumption and its consistency with the existing evidence. Section 4.7 concludes and discusses policy implications.

## **4.2 SPECIFICATION AND IDENTIFICATION OF THE MODEL**

The strategy we adopt in this paper is based on two basic assumptions. The first is related to the nature of fiscal policy shocks. Indeed, unlike monetary policy measures, changes to government spending and taxes are typically decided and publicized well in advance of their implementation. As a consequence, the estimated innovations of a VAR are such only with respect to the information set of the econometrician, but

not of the private sector, and the interest rate response will pick up the effects of the anticipated components of fiscal policy (the “Sims conjecture”, 1988). However, there are many reasons why fiscal decisions announced in advance might not be taken at face value by the public. The yearly budget is often largely a political document, which is discounted by the private sector as such; any decision to change taxes or spending in the future can be modified before the planned implementation time arrives; and “...changes in expenditure policy typically have involved not simply changes in program rules, but rather changes in future spending targets, with the ultimate details left to be worked out later and the feasibility of eventually meeting the targets uncertain” (Auerbach (2000), p. 16).

The second assumption concerns the variables used in order to estimate the model. With respect to the narrative approach we introduce the distinction of military and non-military expenditures and government budget deficit. Indeed, we believe that the effects of public spending on the economy cannot be correctly analyzed without accounting for both the interaction of military/non-military expenditures and the financing mechanism of the spending. First, complementarity/substitution effects between defence and non-defence expenditures are essential in order to identify the final effect on consumption and GDP. Second, the way in which government decides to finance public expenditure, by tax increases or issuing debt, plays an important role on the agent decisions about consumption.

Thus, as benchmark specification of our model we adopt a structural VAR, whose reduced form is defined by the following dynamic equation:

$$Y_t = c + A(L)Y_{t-1} + U_t \tag{Eq. 1}$$

where  $Y_t$  indicates the vector of variables specified below,  $A(L)$  represents an autoregressive lag polynomial,  $c$  is a constant term and  $U_t$  denotes the vector of reduced-form innovations.

We focus on United States and use data from 1960Q1 to 2008Q4. According to Giordano et al. (2007) the availability of quarterly fiscal variables represents the main constraint for the analysis of fiscal policy with VAR models. The end of the sample corresponds the acceleration of the financial crises began with the bankruptcy of Lehman Brothers in September 2008, which triggered unconventional policy moves by the Federal Reserve Bank (Brunner-meier 2009, and Castelnuovo 2010).

In equation (Eq. 1) we assume a five-variables vector  $Y_t = [C_t, N_t, DINC_t, BD_t, GTOT_t]$  composed by private consumption ( $C_t$ ), hours worked ( $N_t$ ), disposable income ( $DINC_t$ ), the government budget deficit ( $BD_t$ ) and the total government spending ( $GTOT_t$ ). The variables are all integrated of order 1. The source for almost all of the variables that we used is the OECD Economic Outlook No. 88. The specification includes: the log of real private final consumption expenditure per capita  $C$ , a measure of hours worked  $N$  was obtained by multiplying the hours worked per employee in the total economy by total employment and expressing it in log per capita terms, disposable income  $DINC$  corresponds to real personal disposable income (obtained from the FRED-II database), the log of real total government spending per capita  $GTOT$  (obtained from the Bureau of Economic Analysis, National Economic Accounts), a measure of budget deficit  $BD$  that corresponds to gross government fixed capital formation ( $IG$ ) minus net government saving ( $SAVG$ ). The resulting variable, expressed in nominal terms, was normalized by the lagged trend nominal GDP. All the other real variables are deflated by the GDP deflator. The variables expressed in per capita terms are divided by working-age population.

As identification strategy, we adopt a Cholesky factorization so to recover the vector of structural shocks  $\varepsilon_t$  (and its variance  $\Omega$ ) from the reduced-form error  $U_t$  in

(Eq. 1), according to the following scheme:

$$\begin{bmatrix} \varepsilon_t^C \\ \varepsilon_t^N \\ \varepsilon_t^{DINC} \\ \varepsilon_t^{BD} \\ \varepsilon_t^{GTOT} \end{bmatrix} = \begin{pmatrix} 1 & \alpha_N^C & \alpha_{DINC}^C & \alpha_{BD}^C & \alpha_{GTOT}^C \\ 0 & 1 & \alpha_{DINC}^N & \alpha_{BD}^N & \alpha_{GTOT}^N \\ 0 & 0 & 1 & \alpha_{BD}^{DINC} & \alpha_{GTOT}^{DINC} \\ 0 & 0 & 0 & 1 & \alpha_{GTOT}^{BD} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} u_t^C \\ u_t^N \\ u_t^{DINC} \\ u_t^{BD} \\ u_t^{GTOT} \end{bmatrix} \quad (\text{Eq. 2})$$

The Cholesky ordering as in (Eq. 2) corresponds to assume the following set of conditions. Consumption is the most endogenous variable and it is therefore affected by all contemporaneous values of all the variables in the VAR; this is natural, as the present study is primarily concerned with the analysis of macroeconomic effects on private consumption. Hours worked are allowed to depend on disposable income. Indeed, agents choose their optimal quantity of labour supply depending on changes in their net income. Total public spending is modeled as the most exogenous variable. The interaction between government expenditure and taxation rate influences budget deficit and disposable income. If the government spending increase is financed by tax raises budget deficit is positive and private disposable income decreases. Contrary, if the public expenditure rise is not followed by a corresponding increase in taxation rate budget deficit is negative and private disposable income increases.

Since we are primarily concerned with private consumption effects of different public spending categories we impose a disaggregation on the expenditure side, distinguishing among civilian and military expenditures. Thus, the vector of variables  $Y_t$  in equation (Eq. 1) can be expressed as  $Y_t = [C_t, N_t, DINC_t, BD_t, NM_t, M_t]$  where  $NM_t$  and  $M_t$  are the public civilian spending and government spending for military sector, respectively. We collect data for both variables from the Bureau of Economic Analysis, National Economic Accounts. Military spending,  $M$ , corresponds to national defence data, whereas civilian spending,  $NM$ , is obtained by the difference

between government consumption expenditures and gross investment data and national defence data. Both the variables are deflated by respective deflators and are expressed in log per capita terms. The Cholesky factorization we adopt in this case is given by:

$$\begin{bmatrix} \varepsilon_t^C \\ \varepsilon_t^N \\ \varepsilon_t^{DINC} \\ \varepsilon_t^{BD} \\ \varepsilon_t^{NM} \\ \varepsilon_t^M \end{bmatrix} = \begin{pmatrix} 1 & \alpha_N^C & \alpha_{DINC}^C & \alpha_{BD}^C & \alpha_{NM}^C & \alpha_M^C \\ 0 & 1 & \alpha_{DINC}^N & \alpha_{BD}^N & \alpha_{NM}^N & \alpha_M^N \\ 0 & 0 & 1 & \alpha_{BD}^{DINC} & \alpha_{NM}^{DINC} & \alpha_M^{DINC} \\ 0 & 0 & 0 & 1 & \alpha_{NM}^{BD} & \alpha_M^{BD} \\ 0 & 0 & 0 & 0 & 1 & \alpha_M^M \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} u_t^C \\ u_t^N \\ u_t^{DINC} \\ u_t^{BD} \\ u_t^{NM} \\ u_t^M \end{bmatrix} \quad (\text{Eq. 3})$$

In the Cholesky ordering of relation (Eq. 3) we assume that following conditions hold. According to the previous reasoning private consumption, hours worked, disposable income and budget deficit are allowed to depend on fiscal variables that are modeled as follows. Military expenditure is assumed to be more rigid than civilian spending (following Ramey, 2008). First, it is well known that defence spending accounts for almost all of the volatility of total government spending (including non-defence share). Second, large rises in defence spending are induced by major political events that are unrelated to the state of the U.S. economy.

In order to test total government, civilian and military expenditures multipliers on output we include as additional variable the log of real GDP per capita (obtained from the Bureau of Economic Analysis, National Economic Accounts). We would like to remark in advance that the two schemes presented above could be arguable (as it is often the case of Cholesky ordering). However we believe that the data frequency grants us a sufficient degree of flexibility in the choice.



### 4.3 ESTIMATION RESULTS

We use two different specifications for the VAR reduced form equation (Eq. 1). In the case of the aggregated government spending, we also include a linear time trend because we observed that at least four of five variables contain a deterministic trend. Moreover, as suggested by the descriptive analysis, we include a shift-dummy for a break point related to the monetary policy change in October 1978 just before the beginning of the Volcker chairmanship. This choice conforms to the evidence on two phenomena that are relevant, namely the financial liberalization occurring in the early 1980s and the general changes in business-cycle dynamics. Whereas, in the case of disaggregation of public spending into military and civilian components we add a trend that starts in the second quarter of 1973 (following Ramey, 1998) and a shift-dummy for a break point related to the oil price shock of 1974 (Perron, 1989).

We estimate both specifications using least squares. The number of lags is set to two according to the Akaike Information Criterion and the absence of serial correlation in the residuals, positively checked with a Lagrange Multiplier test. Moreover, we failed to reject the hypothesis of normality of residuals with the Jarque-Bera statistics and we checked the stability condition of the VAR, finding that all eigenvalues comfortably lie inside the unit circle. We also tested for the presence of cointegrating relationships among the variables, finding mixed evidence according to the rank and the maximum eigenvalue tests. Due to that, and given that our a priori did not include a meaningful long-run relationship among the variables, we decided not to impose any cointegrating restriction and, thus, estimate the VAR with the variables in levels (Sims et al. 1990, Giordano et al. 2007).

Now we turn to discuss the results of the two specifications. Figure 2 shows the effects of aggregate government spending on all the five variables of the first specification. In order to derive the 16th and 84th percentiles of the impulse response distribution in the graphs, we perform Monte Carlo simulations and assume normal-

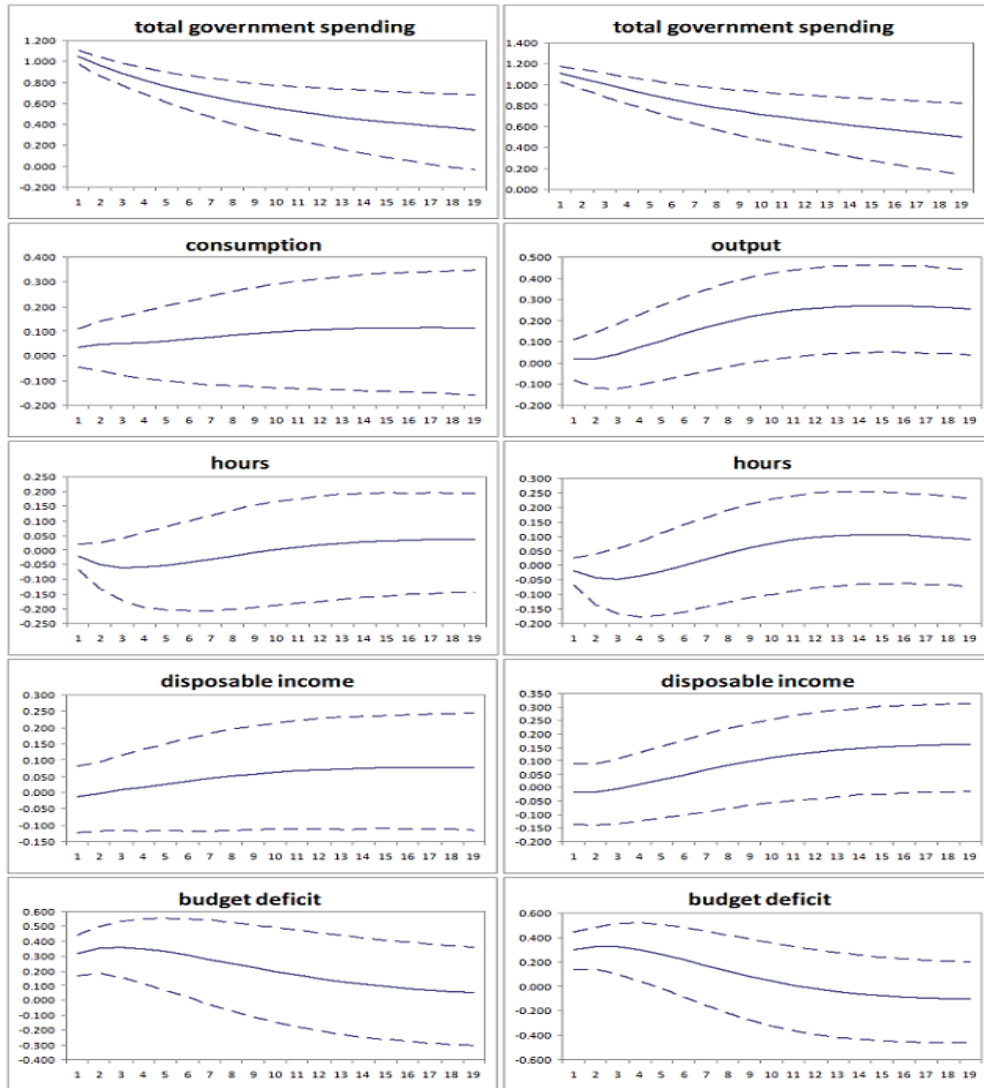
ity in the parameter distribution. Based on that information, we construct  $t$ -tests based on 1000 different responses generated by simulations, and check whether the point estimates of the mean impulse responses are statistically different from zero. The responses of all the six variables are expressed by multiplying the response from the VAR (which is expressed in logs) by the sample average share of total public spending in GDP (as in Monacelli and Perotti, 2006).

Figure 2 shows the effects of aggregate spending shock when consumption is ordered as first in the Cholesky factorization (left column) and when GDP is the most endogenous variable (right column). Our results are in line with them shown by Galí et al. (2007). In the first row we note that total government spending raises significantly for both consumption and GDP cases, although it displays greater persistence in the latter case. Second row shows the responses of consumption and GDP. Output rises persistently to the shock, as predicted by the theory and, most interestingly, consumption is also shown to rise on impact and to remain persistently above zero. The responses are significant in both cases, but only in GDP responses does the increase stay significant for an extended period. With respect to our labour variable, the response of hours worked is reported on the third row. Here a significant increase can be observed only on the GDP case where hours rise persistently in response to the fiscal shock, although with some delay relative to government spending itself<sup>19</sup>. The fourth row depicts the response of disposable income. Although the point estimates for the first few periods look rather similar, the response is significantly positive in GDP for about two years, but not so in consumption case. The last set of panel pertain the government budget deficit. Here the differences across the two cases are most remarkable: in the case of GDP the budget deficit rises significantly on impact, and afterwards decreases becoming negative after two years, whereas in the consumption case budget deficit remains always positive.

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<sup>19</sup>Fatás and Mihov (2001) also uncover a significant rise in the real wage in response to a spending shock, using compensation per hour in the non-farm business sector as a measure of the real wage.

Figure 2: Response of all variables to a shock of GTOT

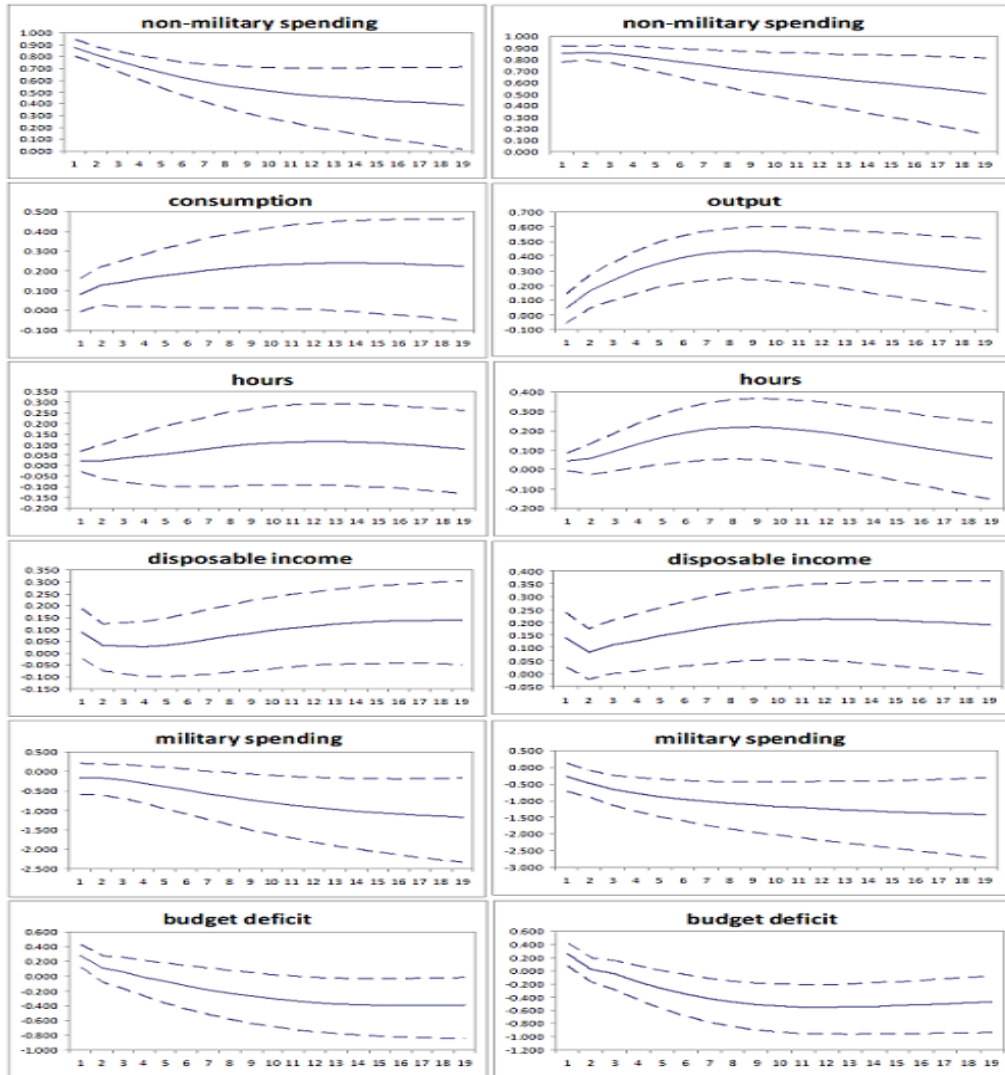


Note: Estimated impulse responses to aggregate government spending shock in the SVAR. The horizontal axis represents quarters after the shock.

Confidence intervals correspond to  $\pm 1$  standard deviations of empirical distributions, based on 1,000 Monte Carlo replications.

In Figure 3 we show the impulse response functions to a civilian spending shock of all the six variables in the second specification. The responses of all the six variables are expressed by multiplying the response from the VAR (which is expressed in logs) by the sample average share of civilian spending in GDP. As before we distinguish on one hand the consumption effects (left column) and on the other hand the output effects (right column) of a civilian spending shock. The first row shows that civilian spending increases significantly for both consumption and GDP cases, even if in the latter case the effect is more persistent. From second row we note that shock in civilian spending leads to an increase in private consumption, as it is assumed by the credit constrained approach (Galí et al., 2007). Non-military shock has also a positive effect on output. Both responses are statistically significant at conventional levels. Moreover, both shocks are very persistent, even though the effects are perceived after two and four quarters in case of, respectively, consumption and output. The former reaches the peak after 13 quarters, the latter after 7 quarters. The third row displays the pattern of hours worked responses. Although in both cases there is an increase to a shock in civilian spending, only for GDP case the response is significant. In the fourth row is shown the response of disposable income that increases in both cases, but also in this case the response is significant only in GDP case. More interestingly, fifth row depicts a significant negative response of military spending due to a shock in civilian spending. This substitution effect is evidently significant for both cases of consumption and GDP. Finally, from the sixth row we note that, for both cases, budget deficit decreases and remains significantly negative after a civilian spending shock. From an economic point of view, this means that non-defence expenditure is financed by an increase in the taxation rate.

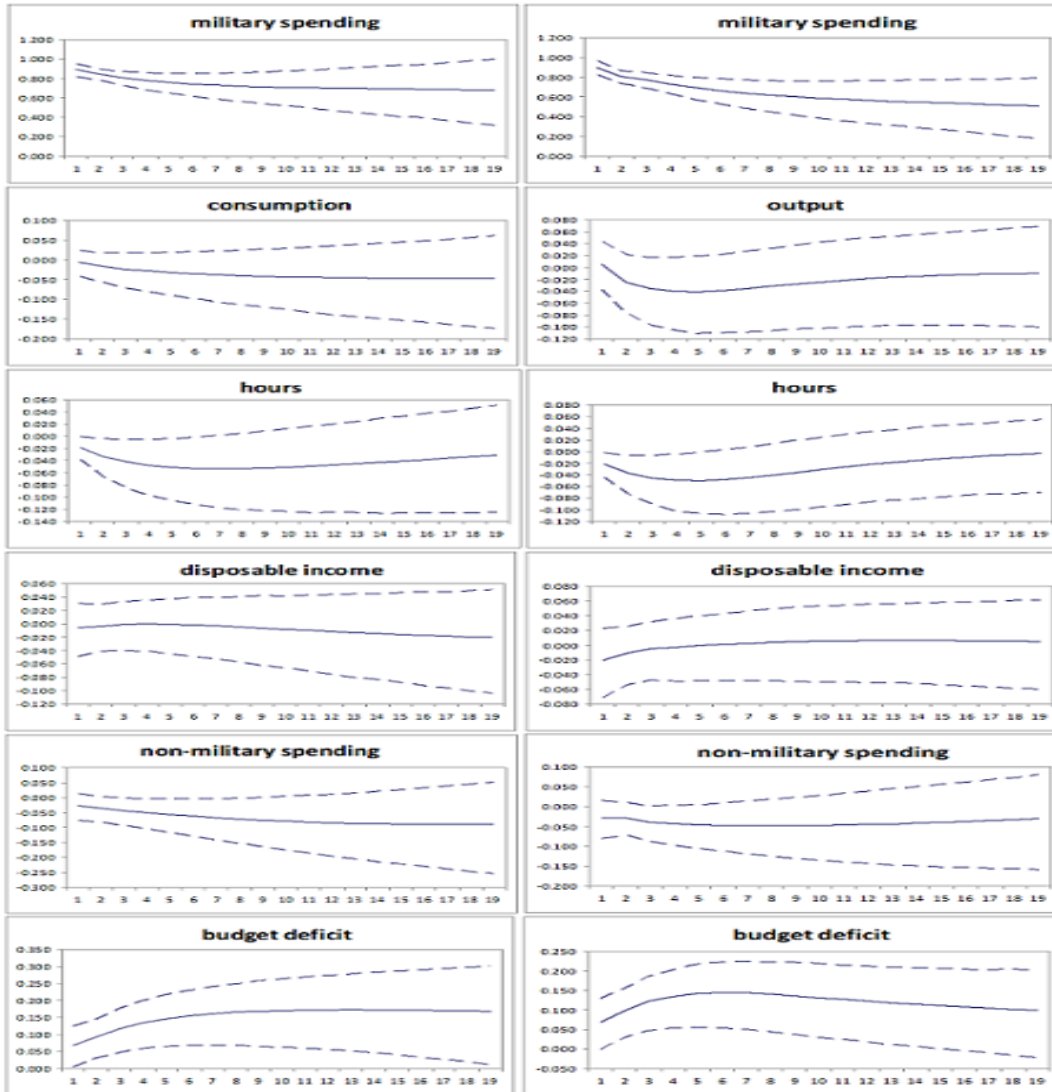
Figure 3: Response of all variables to a shock of NM



Note: Estimated impulse responses to non-military spending shock in the SVAR. The horizontal axis represents quarters after the shock.

Confidence intervals correspond  $\pm 1$  standard deviations of empirical distributions, based on 1,000 Monte Carlo replications.

Figure 4: Response of all variables to a shock of  $M$



Note: Estimated impulse responses to military spending shock in the SVAR. The horizontal axis represents quarters after the shock.

Confidence intervals correspond to  $\pm 1$  standard deviations of empirical distributions, based on 1,000 Monte Carlo replications.

Figure 4 displays the impulse response functions of all six variables of the second specification to an increase military spending. The responses of all the six variables are expressed by multiplying the response from the VAR (which is expressed in logs) by the sample average share of military spending in GDP. Also in Figure 4 we depict on the left column consumption effects and on right column GDP effects of a military spending shock. First row depicts a significant increase of defence expenditure that is more persistent in the consumption case. In the second row are shown the impulse responses of consumption and output. We note that a shock in military spending depresses private consumption, as predicted by neoclassical models. Also output has a negative response to an increase in defence expenditure. Although both responses have similar patterns they are not significant. The response of the hours worked is reported on the third row. Also in this case there is a negative but not significant effect. Fourth row depicts the impulse responses of disposable income that decrease in both case but are not significant. In the fifth row is reported the significant negative effect of civilian spending when there is an increase in military expenditure. This substitution effect across the two sectors is evident both in consumption and GDP effects. Finally, sixth row pertains to the response of budget deficit. We note a significant positive response of deficit to a military spending shock. This result confirms the assumption that defence sector is financed by government debt.

Table 1: Estimated effects of total government, non-military and military shocks

Quarters	Output			Consumption		
	4	8	12	4	8	12
Total government spending	0.09	0.57	1.47	0.01	0.04	0.12
Non-military spending	0.44	1.91	3.62	0.36	1.09	1.99
Military spending	-0.05	-0.21	-0.31	-0.05	-0.18	-0.35

Note: Estimated Fiscal Multipliers were obtained from the two different Cholesky factorizations.

Output corresponds to the results of 5-variable SVAR described in the text.

Consumption SVAR estimates are based on 6-variable SVAR including civilian and military spending.

Our point estimates in Figure 2-4 imply that total government, civilian and military spending multipliers on output,  $dY_{t+k}/dG_t$ ,  $dY_{t+k}/dNM_t$ ,  $dY_{t+k}/dM_t$ , are equal to +0.09, +0.44 and -0.05, respectively, on fourth quarter ( $k = 4$ ). After eight quarters ( $k = 8$ ) they are +0.57, +1.91 and -0.21, respectively. Finally, after twelve quarters ( $k = 12$ ) these multipliers correspond to +1.47, 3.62 and -0.31, respectively. Such estimated multipliers are of a magnitude similar to the ones reported by Blanchard and Perotti (2002). They are also roughly consistent with the range of estimated short-run expenditure multipliers generated by a variety of macroeconomic models<sup>20</sup>. Most important for our purposes is the observation that multipliers on consumption are positive for total and civilian expenditures whereas is negative for military expenditure, in the ranges (+0.01,+0.12), (+0.36,+1.99) and (-0.05,-0.35), respectively (see Table 1).

#### 4.4 A BASELINE MODEL

We follow the seminal model of Galí et al. (2007). In particular, it is a new-Keynesian model that encompasses two types of households. Each household maximises a util-

<sup>20</sup> Among the others, see Hemming, Kell, and Mahfouz (2002).



ity function with two arguments: consumption goods and labour, over an infinite life horizon. Moreover, we assume a perfectly competitive firm producing a final good, and a continuum of intermediate firms that produce differentiated goods and set prices, according to sticky prices à la Calvo (1983). In this economy, government is in charge of the fiscal policy that includes the several components of government spending. Finally, a Central Bank drives monetary policy fixing the nominal interest rate by a Taylor rule (1993). In the next sections we describe the several maximization problems that we derive in detail in Appendix A.

#### 4.4.1 HOUSEHOLDS

The economy is populated by a continuum of households, indexed by  $i \in [0, 1]$ , that live for an infinite life horizon. These households are divided into Ricardian and non-Ricardian ones. Ricardian households represent a share  $1 - \lambda$  and are able to trade securities and accumulate physical capital that rent out to firms. Non-Ricardian households consist of a share  $\lambda$  and do not have access to capital markets, thus they just consume their current labour income. In order to justify this kind of distinction we refer to the paper of Campbell and Mankiw (1989) in which they provide evidence of the existence of Ricardian households in the U.S. economy.

**RICARDIAN HOUSEHOLDS.** This kind of households have a life-time utility function,  $U(\cdot)$ , given by:

$$U(C_t^o, N_t^o) = E_0 \sum_{t=o}^{\infty} \beta^t \left[ \log C_t^o - \frac{(N_t^o)^{1+\varphi}}{1+\varphi} \right] \quad (\text{Eq. 4})$$

Here  $E_0$  is the conditional expectation operator, and  $C_t^o$  and  $N_t^o$  denote time- $t$  consumption and hours worked, respectively. The discount factor is  $\beta \in (0, 1)$ , and the elasticity of wages with respect to hours worked is  $\varphi \geq 0$ .

Ricardian households face the following budget constraint:

$$P_t (C_t^o + I_t^o) + R_t^{-1} B_{t+1}^o = W_t P_t N_t^o + R_t^k P_t K_t^o + B_t^o + D_t^o - P_t T_t^o \quad (\text{Eq. 5})$$

and capital accumulation equation:

$$K_{t+1}^o = (1 - \delta) K_t^o + \phi \left( \frac{I_t^o}{K_t^o} \right) K_t^o \quad (\text{Eq. 6})$$

We denote with  $T_t^o$  the lump sum taxes (or transfers, if negative) paid by these consumers to government, while  $D_t^o$  are dividends from ownership of firms. The variable  $B_{t+1}^o$  denotes the quantity of one-period bonds purchased by these households at time  $t$ . Also,  $P_t$  denotes the price level and  $W_t$  denotes the real wage rate.  $R_t$  denotes the one-period nominal rate of interest that pays off in period  $t$ .  $K_t^o$  represents the capital holding and  $R_t^k$  its real rental cost. Finally  $I_t^o$  indicates investment expenditures in real terms. Capital adjustment costs are introduced through the term  $\phi \left( \frac{I_t^o}{K_t^o} \right) K_t^o$ , which determines the change in the capital stock induced by investment spending. We assume:

$$\begin{aligned} \phi' \left( \frac{I_t^o}{K_t^o} \right) &> 0 \\ \phi'' \left( \frac{I_t^o}{K_t^o} \right) &\leq 0 \end{aligned}$$

with:

$$\begin{aligned} \phi'(\delta) &= 1 \\ \phi(\delta) &= \delta \end{aligned}$$

The subsequent maximization problem gives the following first order conditions:

$$1 = R_t E_t \left[ \Lambda_{t,t+1} \frac{P_t}{P_{t+1}} \right] \quad (\text{Eq. 7})$$

where the stochastic discount factor for real  $k$ -period ahead payoffs characterizing Ricardian households, who own the firms, is given by:

$$\Lambda_{t,t+k} = \beta^k \left( \frac{C_{t+k}^o}{C_t^o} \right)^{-1} \quad (\text{Eq. 8})$$

and:

$$Q_t = E_t \left( \Lambda_{t,t+1} \left\{ R_{t+1}^k + Q_{t+1} \left[ \begin{array}{c} (1-\delta) \\ -\phi' \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \frac{I_{t+1}^o}{K_{t+1}^o} + \phi \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \end{array} \right] \right\} \right) \quad (\text{Eq. 9})$$

where the real shadow value of capital, the so-called Tobin's  $Q$ , is given by:

$$Q_t = \frac{1}{\phi' \left( \frac{I_t^o}{K_t^o} \right)} \quad (\text{Eq. 10})$$

We note that, under our assumption on  $\phi$ , the elasticity of the investment-capital ratio with respect to  $Q$  is given by:

$$-\frac{1}{\phi''(\delta) \delta} \equiv \eta$$

We don't insert in this list of optimality conditions the labour decision equation because we assume that wage is determined in a non-competitive labour market structure (we analyze in details this aspect below).

**NON-RICARDIAN HOUSEHOLDS.** Typical households of this type have a life-time utility function,  $U(\cdot)$ , given by:

$$U(C_t^r, N_t^r) = \log C_t^r - \frac{(N_t^r)^{1+\varphi}}{1+\varphi} \quad (\text{Eq. 11})$$

and face the following budget constraint:

$$P_t C_t^r = W_t P_t N_t^r - P_t T_t^r \quad (\text{Eq. 12})$$

Expression (Eq. 12), implies that non-Ricardian households consume their disposable income:

$$C_t^r = W_t N_t^r - T_t^r \quad (\text{Eq. 13})$$

We note that taxes paid by non-Ricardian ( $T_t^r$ ) and Ricardian ( $T_t^o$ ) households can differ. As before, we omit the labour decision equation in the FOCs for this kind of households since we assume non-competitive labour market structure<sup>21</sup> (see the following discussion).

**THE WAGE SCHEDULE.** Labour market structure is such that there is an economy-wide union setting wages in a centralized manner. Hence, hours worked are not chosen optimally by households but are determined by firms given the wage set by the union (see Appendix C for details). We assume that wages are determined according to the following generalized schedule:

$$W_t = H(C_t, N_t) \quad (\text{Eq. 14})$$

where  $H$  is an increasing function on both its arguments  $C_t$  and  $N_t$ , which guarantees the convex marginal disutility of labour and wealth effects. Once that wage is set by the union, firms decide the quantity of labour to hire allocating labour demand uniformly across households  $N_t^r = N_t^o$  (see Appendix C). We also assume that wage mark-up is such that the following inequalities hold all times:

$$H(C_t, N_t) > C_t^j N_t^j, \quad \text{for } j = o, r$$

Last relation implies that Ricardian and non-Ricardian households will to meet firms' labour demand at the prevailing wage. Since the latter is assumed to remain above

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<sup>21</sup>Assuming a non-competitive labour market structure we allow for the hours worked and consumption of non-Ricardian households to comove positively with real wages. Under a perfectly competitive labour market this condition no longer applies.

the marginal rate of substitution all times, households choose optimally to supply as much labour as it is demanded by the firms<sup>22</sup>.

#### 4.4.2 FIRMS

The final good,  $Y_t$ , is produced by competitive firms using the technology:

$$Y_t = \left( \int_0^1 X_t(j)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} dj \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$$

where the constant elasticity of substitution is  $\varepsilon_p > 1$ , and  $X_t(j)$ ,  $j \in [0, 1]$ , denotes the intermediate good  $j$ .

Profit maximization implies the following first order condition for  $X_t(j)$ :

$$X_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon_p} Y_t$$

where  $P_t(j)$  denotes the price of intermediate good  $j$  and  $P_t$  is the price of the homogeneous final good. Perfect competition in the final goods market implies that the latter can be written as:

$$P_t = \left[ \int_0^1 P_t(j)^{1 - \varepsilon_p} dj \right]^{\frac{1}{1 - \varepsilon_p}}$$

The intermediate good,  $X_t(j)$ , is produced by a monopolistically competitive firms using the following Cobb-Douglas technology:

$$Y_t(j) = K_t(j)^\alpha N_t(j)^{1 - \alpha} \tag{Eq. 15}$$

where  $N_t(j)$  and  $K_t(j)$  denote, respectively, employment and capital used by the  $j^{\text{th}}$

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<sup>22</sup>It is useful to remark that consistency with balanced-growth requires that  $H$  can be written as  $C_t h(N_t)$ .

intermediate firm, while  $\alpha$  is the capital elasticity. Cost minimization implies:

$$\frac{K_t(j)}{N_t(j)} = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k}$$

The firms' marginal costs are given by:

$$\begin{aligned} MC_t &= \Psi (W_t)^{1-\alpha} (R_t^k)^\alpha \\ \text{where } : \quad \Psi &= \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} \end{aligned}$$

The intermediate firm is subject to Calvo style price-setting frictions and can optimize its price with probability  $1-\theta$ , while with probability  $\theta$  it keeps its price unchanged. Thus, the maximization problem can be expressed as:

$$\max_{\{P_t^*\}} E_t \sum_{k=0}^{\infty} \theta^k \left\{ \Lambda_{t,t+k} Y_{t,t+k}(j) \left( \frac{P_t^*}{P_{t+k}} - MC_{t+k} \right) \right\}$$

(we note that  $\Lambda_{t,t+k}$  is the stochastic discount factor already defined in the household maximization program) given the following demand constraints:

$$Y_{t+k}(j) = X_{t+k}(j) = \left( \frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon_p} Y_{t+k}$$

where  $P_t^*$  denotes the optimal price chosen by the intermediate firm resetting prices at time  $t$ .

The first order condition of this problem is given by:

$$E_t \sum_{k=0}^{\infty} \theta^k \left\{ \Lambda_{t,t+k} Y_{t+k}(j) \left( \frac{P_t^*}{P_{t+k}} - \mu^p MC_{t+k} \right) \right\} = 0 \quad (\text{Eq. 16})$$

where the mark-up is given by:

$$\mu_p \equiv \frac{\varepsilon_p}{\varepsilon_p - 1}$$

Finally, the law of motion of aggregate price is given by:

$$P_t = \left[ \theta P_{t-1}^{1-\varepsilon_p} + (1-\theta) (P_t^*)^{1-\varepsilon_p} \right]^{\frac{1}{1-\varepsilon_p}} \quad (\text{Eq. 17})$$

#### 4.4.3 *MONETARY POLICY*

We assume that monetary authority sets the nominal interest rate,  $r_t \equiv R_t - 1$ , following the rule:

$$r_t = r + \phi_\pi \pi_t \quad (\text{Eq. 18})$$

where  $r$  is the steady state level of interest rate and  $\pi_t$  denotes the time- $t$  rate of inflation. According to equation (Eq. 18), the central bank follows a particular case of the standard Taylor rule. Indeed, we assume that the coefficients related to the long run responses of the monetary authority to deviation of inflation target and output from its steady state value are equal to zero. Finally, from expression (Eq. 18), we note that the so-called Taylor principle is satisfied if the coefficient related to the long run response of interest rate to inflation,  $\phi_\pi$ , is bigger than one.

#### 4.4.4 *FISCAL POLICY*

Since the aim of the present paper is to analyse the effects of different components of government spending on the economy we distinguish two different fiscal policies. First, we focus on aggregate government expenditure and successively we split total spending into civilian and military ones.

***AGGREGATE GOVERNMENT SPENDING.*** The government purchases final goods, issues bonds and raises lump-sum taxes to finance spending. Thus, its budget con-

straint is given by:

$$P_t T_t + R_t^{-1} B_{t+1} = B_t + P_t G_t \quad (\text{Eq. 19})$$

$$\text{where} \quad : \quad T_t \equiv \lambda T_t^r + (1 - \lambda) T_t^o$$

The financing of government expenditure is determined by the steady state deviations of deficit and total spending. Thus, the fiscal policy rule takes the following form (see Galí et al., 2007):

$$t_t = \phi_b b_t + \phi_g g_t \quad (\text{Eq. 20})$$

$$\text{where} \quad : \quad g_t \equiv \frac{G_t - G}{Y}$$

$$\text{and} \quad : \quad t_t \equiv \frac{T_t - T}{Y}$$

$$\text{and} \quad : \quad b_t \equiv \frac{(B_t/P_{t-1}) - (B/P)}{Y}$$

where  $\phi_b$  and  $\phi_g$  are the parameters capturing the response of taxes to budget deficit and total government spending respectively. Total government expenditure (in deviations from steady state, and normalized by steady state output) evolves exogenously according to a second order autoregressive process. We assume an AR(2) process in line with our VAR based estimates of the dynamic responses of total government spending:

$$g_t = \rho_1^g g_{t-1} + \rho_2^g g_{t-2} + \epsilon_t^g \quad (\text{Eq. 21})$$

$$\text{where} \quad : \quad 0 < \rho_1^g < 1$$

$$0 < \rho_2^g < 1$$

$$\epsilon_t^g \sim N(0, \sigma_\epsilon^2)$$

where  $\rho_1^g$  and  $\rho_2^g$  are the persistence parameters, whereas  $\epsilon_t^g$  is an i.i.d. shock.



**NON-MILITARY AND MILITARY EXPENDITURES.** Focusing on the disaggregation of total public spending we can express the government budget constraint as:

$$P_t T_t + R_t^{-1} B_{t+1} = B_t + P_t G_t \quad (\text{Eq. 22})$$

$$\text{where : } T_t \equiv \lambda T_t^r + (1 - \lambda) T_t^o$$

$$\text{and : } G_t = N M_t + M_t$$

Expression (Eq. 22) encompasses the sum of civilian and military components according to the additive principle. Government expenditure is financed by the following fiscal policy:

$$t_t = \phi_b b_t + \phi_{nm} nm_t + \phi_m m_t \quad (\text{Eq. 23})$$

$$\text{where : } nm_t \equiv \frac{N M_t - N M}{Y}$$

$$\text{and : } m_t \equiv \frac{M_t - M}{Y}$$

where  $\phi_{nm}$  and  $\phi_m$  are the parameters indicating the response of taxes to non-military and military spending, respectively. Moreover,  $nm_t$  and  $m_t$  are the resources devolved to non-military and military sectors expressed as deviations from their respective steady states, and normalized by steady state output. We assume that they are exogenous autoregressive processes of order two driving the economy:

$$nm_t = \rho_1^{nm} nm_{t-1} + \rho_2^{nm} nm_{t-2} + \epsilon_t^{nm} \quad (\text{Eq. 24})$$

$$\text{where : } 0 < \rho_1^{nm} < 1$$

$$0 < \rho_2^{nm} < 1$$

$$\epsilon_t^{nm} \sim N(0, \sigma_\epsilon^2)$$

and:

$$m_t = \rho_1^m m_{t-1} + \rho_2^m m_{t-2} + \epsilon_t^m \quad (\text{Eq. 25})$$

$$\text{where } : 0 < \rho_1^m < 1$$

$$0 < \rho_2^m < 1$$

$$\epsilon_t^m \sim N(0, \sigma_\epsilon^2)$$

where  $\rho_1^{nm}$ ,  $\rho_1^m$  and  $\rho_2^{nm}$ ,  $\rho_2^m$  are the persistence parameters, whereas  $\epsilon_t^{nm}$ ,  $\epsilon_t^m$  are i.i.d. shock of civilian and military expenditures. We would like to remark that, again, we assume AR(2) processes in line with our VAR based estimates of the dynamic responses of civilian and military expenditures.

#### 4.4.5 AGGREGATION AND MARKET EQUILIBRIUM

The sum of the Ricardian and non-Ricardian consumption shares gives aggregate consumption:

$$C_t \equiv \lambda C_t^r + (1 - \lambda) C_t^o \quad (\text{Eq. 26})$$

Similarly, adding labor supplied by Ricardian and non-Ricardian households gives total hours:

$$N_t \equiv \lambda N_t^r + (1 - \lambda) N_t^o \quad (\text{Eq. 27})$$

By our assumption, all capital stock is held by Ricardian households:

$$K_t \equiv (1 - \lambda) K_t^o$$

and aggregate investment is given by:

$$I_t \equiv (1 - \lambda) I_t^o$$

A dynamic stochastic general equilibrium is a set of values for prices and quantities such that the representative household's and firm's optimality conditions and the market clearing conditions are satisfied. In this case, the clearing of factor markets implies:

$$\begin{aligned} N_t &= \int_0^1 N_t(j) dj \\ K_t &= \int_0^1 K_t(j) dj \\ Y_t(j) &= X_t(j) \text{ for all } j \end{aligned}$$

Final good market is in equilibrium if production equals demand by total household consumption, aggregate private investment and total government spending:

$$Y_t = C_t + I_t + G_t \tag{Eq. 28}$$

In the case of disaggregated government components, condition (Eq. 28) can be expressed as:

$$Y_t = C_t + I_t + NM_t + M_t \tag{Eq. 29}$$

#### 4.4.6 *LINEARIZED EQUILIBRIUM CONDITIONS*

In this section we show the system in the log-linear form where the variables with small letters are log-deviations from the steady state and the variables without time subscripts are at steady state (all the derivations are shown in Appendix B).

**HOUSEHOLDS.** The  $Q$  equation states that the current value of capital stock depends positively on its expected future value and expected rental rate but negatively

on the ex-ante interest rate:

$$q_t = \beta E_t \{q_{t+1}\} + [1 - \beta (1 - \delta)] E_t \{r_{t+1}^k\} - (r_t - E_t \{\pi_{t+1}\}) \quad (\text{Eq. 30})$$

The relationship between Tobin's  $Q$  equation and investment is given by:

$$i_t - k_t = \eta q_t \quad (\text{Eq. 31})$$

The capital accumulation equation is standard:

$$k_{t+1} = \delta i_t + (1 - \delta) k_t \quad (\text{Eq. 32})$$

The Euler equation for Ricardian households controls the dynamics of consumption and real balances, where current consumption and real balances depend on their expected values and the ex-ante interest rate:

$$c_t^o = E_t \{c_{t+1}^o\} - (r_t - E_t \{\pi_{t+1}\}) \quad (\text{Eq. 33})$$

The non-Ricardian consumption equation is given by:

$$c_t^r = \left( \frac{WN^r}{C^r} \right) (w_t + n_t^r) - \left( \frac{Y}{C^r} \right) t_t^r \quad (\text{Eq. 34})$$

where :  $t_t^r = \frac{T_t^r - T^r}{Y}$

We assume that in steady state consumption across different household types is equal:  $C^r = C^o = C$  and this implies that  $N^r = N^o = N$ , as well. The assumption of equal consumption levels in steady state is guaranteed by an appropriate choice of  $T^r$  and  $T^o$  (see Appendix C).

Thus, the aggregate consumption and labour expressions are given by:

$$c_t = \lambda c_t^r + (1 - \lambda) c_t^o \quad (\text{Eq. 35})$$

$$n_t = \lambda n_t^r + (1 - \lambda) n_t^o \quad (\text{Eq. 36})$$

In the case of non-competitive labour market structure, a log-linearized expression for the generalized wage schedule  $W_t = H(C_t, N_t)$  is given by (see Appendix C):

$$w_t = c_t + \varphi n_t \quad (\text{Eq. 37})$$

The intertemporal equilibrium condition for aggregate consumption is given by:

$$c_t = E_t \{c_{t+1}\} - \sigma (r_t - E_t \{\pi_{t+1}\}) - \Theta_n E_t \{\Delta n_{t+1}\} + \Theta_t E_t \{\Delta t_{t+1}^r\} \quad (\text{Eq. 38})$$

We can derive expression (Eq. 38) in the non-competitive case, assuming that  $n_t^r = n_t^o = n_t$  and combining expressions (Eq. 37), (Eq. 33), (Eq. 34), (Eq. 35), (Eq. 36). The coefficients correspond to (for details see Appendix B):

$$\sigma = \gamma_c \Phi \mu^p (1 - \lambda)$$

$$\Theta_n = \lambda \Phi (1 - \alpha) (1 + \varphi)$$

$$\Theta_t = \lambda \Phi \mu^p$$

$$\text{where : } \Phi = (\gamma_c \mu^p - \lambda (1 - \alpha))^{-1}$$

$$\text{and : } \gamma_c \equiv \frac{C}{Y}$$

We note that steady state consumption-output ratio,  $\gamma_c$ , does not depend on  $\lambda$  (as shown in Appendix B).

**FIRMS.** The inflation equation is a standard new-Keynesian Phillips curve stating

that current inflation depends on the expected future inflation and on the deviation of the average mark-up from its steady state level:

$$\pi_t = \beta \{\pi_{t+1}\} - \lambda_p \hat{\mu}_t^p \quad (\text{Eq. 39})$$

where :  $\lambda_p = (1 - \beta\theta)(1 - \theta) \frac{1}{\theta}$

knowing that:

$$\hat{\mu}_t^p = (y_t - n_t) - w_t \quad (\text{Eq. 40})$$

or, equivalently:

$$\hat{\mu}_t^p = (y_t - k_t) - r_t^k \quad (\text{Eq. 41})$$

Final output is produced using a Cobb-Douglas function where we assume that the total factor productivity is equal to one:

$$y_t = (1 - \alpha) n_t + \alpha k_t \quad (\text{Eq. 42})$$

## **FISCAL POLICY**

**AGGREGATE GOVERNMENT SPENDING.** The linearized government budget constraint around a steady state with zero debt and a balanced primary budget is given by:

$$b_{t+1} = \frac{1}{\beta} (b_t + g_t - t_t) \quad (\text{Eq. 43})$$

**NON-MILITARY AND MILITARY EXPENDITURES.** In the case of disaggregation of public expenditure components the linearized budget constraint (Eq. 22) is given by:

$$b_{t+1} = \frac{1}{\beta} (b_t + nm_t + m_t - t_t) \quad (\text{Eq. 44})$$

**MARKET CLEARING.** The log-linearized market equilibrium condition for total government spending can be expressed as follows:

$$y_t = \gamma_c c_t + \gamma_i i_t + g_t \tag{Eq. 45}$$

where :  $\gamma_c = \frac{C}{Y}$

$$\gamma_i = \frac{I}{Y}$$

where  $\gamma_i$  denotes the steady state investment-output ratio. Considering civilian and military expenditures, log-linearization of expression (Eq. 29) is given by:

$$y_t = \gamma_c c_t + \gamma_i i_t + nm_t + m_t \tag{Eq. 46}$$

We solve numerically the system of equations (Eq. 30)-(Eq. 46) including also the log-linearized Taylor rule (Eq. 18), the different fiscal policy rules and shocks.

#### 4.5 MODEL CALIBRATION

We assume a quarterly calibration where the discount factor,  $\beta$ , is set equal to 0.99, which implies an annual steady state real interest rate of 4%. The depreciation rate,  $\delta$ , is set equal to 0.025 per quarter, which implies an annual depreciation on capital equal to 10 percent. We set  $\alpha$  equal to 0.30, which roughly implies a steady state share of labour income in total output of 70%. In addition, we fix the parameter capturing the mark-up,  $\mu^P$  equal to 0.2. The fraction of non-Ricardian households,  $\lambda$ , is set equal to 1/2, a value that is assumed by Galí et al. (2007) and is within the range of estimated values in the literature (see Mankiw, 2000). The probability of firms that keep their prices unchanged,  $\theta$ , is fixed to 0.75 (see Bilbiie et al., 2009). Whereas, the value for the elasticity of wages with respect to hours worked,  $\varphi$ , is set equal to 0.2. This value is line with Rotemberg and Woodford's (1997, 1999) calibrations. According to follow King and Watson (1996), the elasticity of investment with respect

to  $q$ ,  $\eta$ , is fixed to 1. We follow Galí et al. (2007) in the setting of the parameter capturing the response of the monetary authority to inflation,  $\phi_\pi$ , equal to 1.5. This value satisfies the so-called Taylor principle.

Focusing on the parameters describing the different fiscal policy rules we distinguish between the cases of total government spending and the disaggregation of civilian and military components. In the first case (equation Eq. 20), we set the parameters capturing the response of taxes to budget deficit and total government spending,  $\phi_b = 0.13$  and  $\phi_g = 0.10$ , respectively. Both values are within the range of estimated values in Galí et al. (2007).

In order to calibrate the parameters of relation (Eq. 23) we follow the patterns of the IRFs based on the VAR estimates. Moreover, from Galí et al. (2007), we know that a positive comovement of consumption and output in response to government spending shocks requires a sufficiently high response of taxes to debt, and a sufficiently low response of taxes to current government spending. Thus, in the case of non-military spending shock, we increase  $\phi_b$  to 0.25 and fix the parameter capturing the response of taxes to civilian and military expenditures,  $\phi_{nm}$ ,  $\phi_m$ , respectively equal to 0.01 and 0.89. Whereas, in the case of military spending shock we set  $\phi_b = 0.01$  and  $\phi_{nm} = 0.5$ ,  $\phi_m = 0.9$ .

The persistence parameters of the AR(2) shocks are calibrated using the VAR based estimates of the dynamic responses of government spending. In particular, we set  $\rho_1^g$ ,  $\rho_1^{nm}$  and  $\rho_1^m$  equal to 0.8 and  $\rho_2^g$ ,  $\rho_2^{nm}$  and  $\rho_2^m$  equal to 0.1. These values reflect the highly persistent responses of total government, civilian and military expenditures to their own shock.

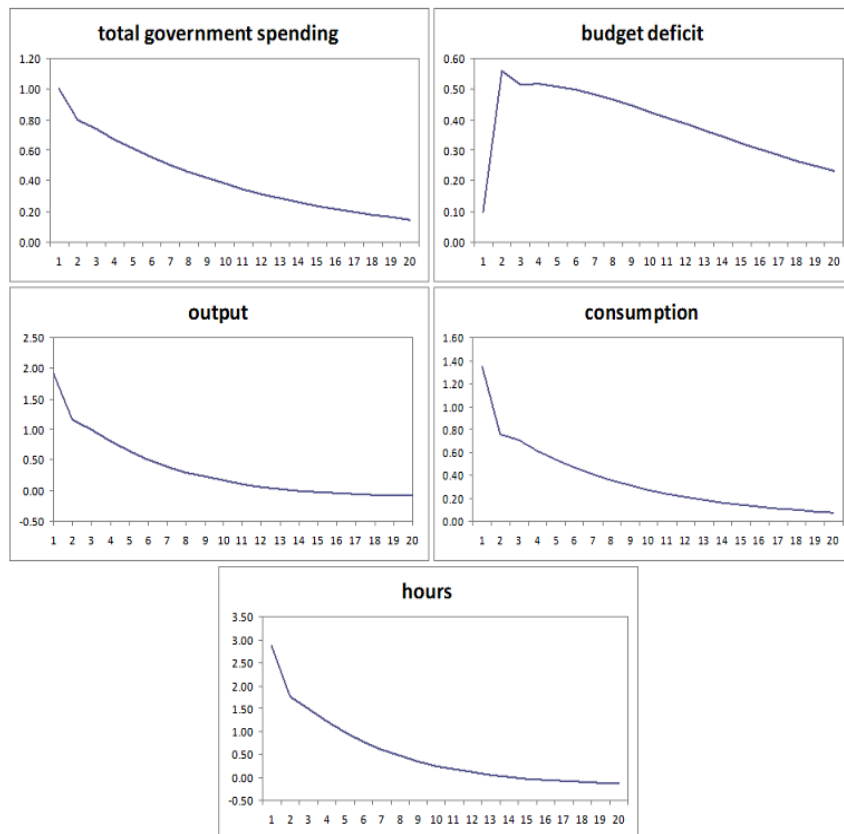
Finally, we set  $\gamma_g = 0.2$ ,  $\gamma_{nm} = 0.13$  and  $\gamma_m = 0.07$  which roughly correspond respectively to the average share of total government, civilian and military expenditures in GDP for the period 1960-2008.



#### 4.6 THE EFFECTS OF GOVERNMENT SPENDING SHOCKS

Figure 5 displays the dynamic responses of some key variables in our model to a positive total government spending shock.

Figure 5: The dynamic effects of a total government spending shock

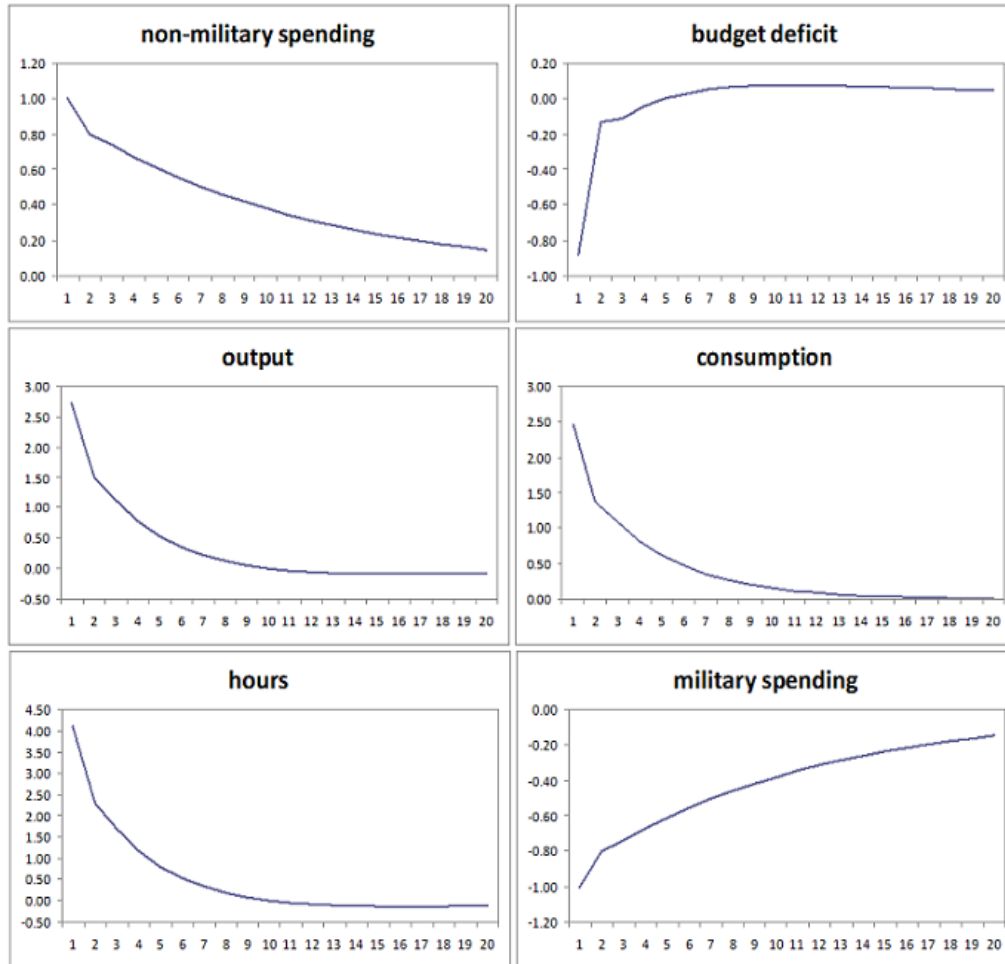


Note: Aggregate government spending shock equal to 1%.

The graphs in the first row of Figure 5 display the pattern of spending and budget deficit in response to the shock considered. We notice that the pattern of both variables is close to the one found by Galí et al. (2007). Both the responses of output and consumption (graphs in the second row) are positive in a way consistent with much of the recent evidence. In particular, the interaction between non-Ricardian households (whose consumption equals their labour income) and sticky prices (modelled as in the recent new-Keynesian literature) makes it possible to generate an increase in consumption in response to a persistent expansion in total government spending. Furthermore, in our model, and in contrast with the neoclassical model, the increase in aggregate hours coexists with an increase in real wages (graphs in third row).

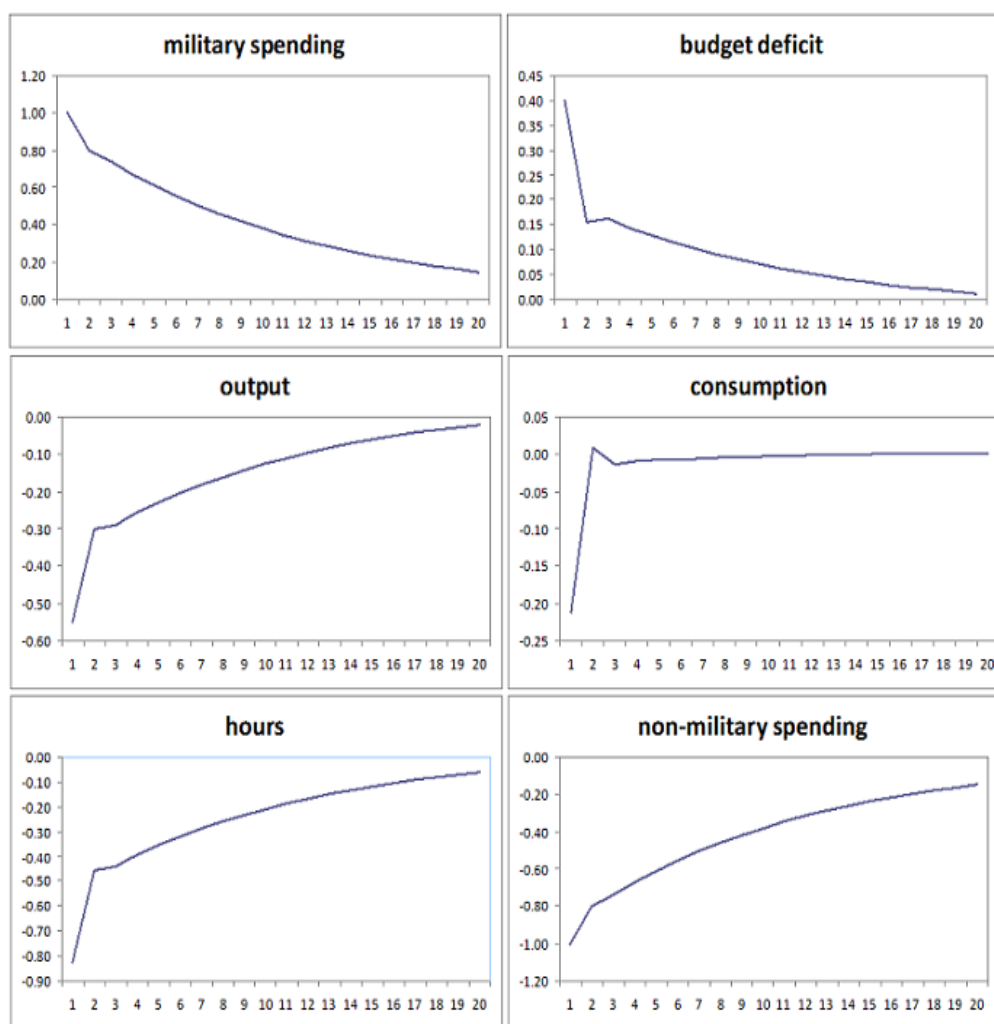
The introduction of non-Ricardian households offsets the negative wealth effects generated by the higher levels of taxes needed to finance the fiscal expansion, while making consumption more sensitive to current labour income (net of current taxes). Sticky prices make it possible for real wages to increase, even if the marginal product of labour goes down, since the price mark-up may decline sufficiently to more than offset the latter effect. The increase in the real wage, together with that of employment, raises current labour income and hence stimulates the consumption of non-Ricardian households. In this way we are able to obtain a crowding-in effect of aggregate consumption in response to total government spending shock.

Figure 6: The dynamic effects of a non-military spending shock



Note: Non-military military spending shock equal to 1%.

Figure 7: The dynamic effects of a military spending shock



Note: Military spending shock equal to 1%.

Now we turn to discuss the simulation results of total public spending disaggregation into civilian and military components. Figure 6 shows the impulse response functions of the variables to a positive civilian spending shock. The graphs in the first row display respectively the pattern of civilian spending and budget deficit in response to the shock considered. First, we note that in order to finance civilian spending, budget deficit remains negative for all the period considered, consistently with its estimated impulse response (see Figure 3). As regards the responses of output and consumption, graphs of second row in Figure 6 show that all are positive but higher with respect to the case of total government spending case and in line with our empirical evidence.

The graphs in the last row display the responses of hours worked and military spending to a shock in civilian sector. The former shows a positive pattern for all the period considered. The latter clearly indicates that non-military and military expenditures are clearly substitute. This means that the increase of non-defence resources play a negative role in defence purchases decreasing it. This substitution effect of non-military/military expenditure is particularly relevant and underlines the importance of disaggregating public spending components. Indeed, splitting public spending components, and assuming a unique shock on civilian side, the effect on output, consumption and hours worked is amplified.

Finally, we analyze the military spending shock displaying the relative IRFs concerning the several variables (Figure 7). The graphs in the first row display the pattern of military expenditure and budget deficit in response to the shock considered. As we can see, contrary to civilian spending case, budget deficit response is positive when there is an increase in military spending. This is in line with the relative estimated impulse response (see Figure 4).

Interestingly, consequently to a military shock the responses of output and consumption are negative. In particular, private consumption (right graph in the second row) decreases because hours worked (left graph in the third row) and wages comove

negatively. This effect, in turn, implies a negative response of output (left graph on the second row). Also, these results are confirmed by our estimated impulse response functions (see Figure 4).

The substitution effect between military and civilian expenditures, clearly, appears in Figure 7 (right graph of the last row). An increase to the resources devolved to military sectors diminishes the ones devolved for the non-defence component.

#### **4.7 CONCLUSIONS**

This paper carried out an analysis on US economy from 1960 to 2008 with the objective of verifying and quantifying the effects of different broad categories of government expenditure on private consumption, so to contribute to the empirical literature which has reported mixed evidence so far. Our findings obtained using structural VAR method and reproduced by a DSGE model simulation can be summarized as follows. GDP and private consumption seems to respond: i) positively to total government purchases of goods and services; ii) positively to civilian spending; iii) negatively to military expenditure. While i) and ii) strengthens the new-Keynesian theoretical approach, known as the “credit-constrained” agents, iii) seems to confirm the standard neoclassical wealth effect. Moreover, the way of financing of different public spending components is extremely relevant. In particular, military spending is financed by an increase in government budget deficit, contrary to the civilian spending case. We also found a clear substitution effect between the resources devolved for the defence and non-defence sectors.

Quantitative estimates of the responses’ magnitude in our benchmark specification lead to an important policy implication: shocks to civilian spending have a cumulative impact on GDP after three years - via private consumption - that is ten times higher than military spending, with opposite signs. This suggests that any expansionary effect of non-defense expenditure might be potentially offset by a

parallel increase in pure military expenditure, with a negative effect on aggregate demand even though a overall increase in aggregate government expenditure has occurred. This result shows that trying to measure the fiscal multiplier on private consumption by considering the whole government expenditure aggregate - and not its decomposition according to features and goals - can indeed be misleading.

While we believe that this analysis can represent a useful contribution to a more effective management of fiscal policy tools on the expenditure side, the general validity of the findings is certainly limited by the closed-economy one-country investigation. We believe that an analysis on United Kingdom economy would permit the use of easily-available annual data, allowing an interesting comparison and more complete answer to our original question. This should probably be the most rationale next step.

## 4.8 APPENDIX A: MAXIMIZATION PROBLEMS OF THE MODEL

### 4.8.1 RICARDIAN HOUSEHOLDS.

A typical household of this type maximizes:

$$\max E_t \sum_{t=0}^{\infty} \beta^t U(C_t^o, N_t^o) \quad (\text{Eq. A1})$$

$$\text{with : } 0 < \beta < 1$$

$$\text{where : } U(C_t^o, N_t^o) = \log C_t^o - \frac{(N_t^o)^{1+\varphi}}{1+\varphi} \quad (\text{Eq. A2})$$

$$\text{with : } \varphi \geq 0$$

subject to the sequence budget constraints:

$$P_t(C_t^o + I_t^o) + R_t^{-1}B_{t+1}^o = W_tP_tN_t^o + R_t^kP_tK_t^o + B_t^o + D_t^o - P_tT_t^o \quad (\text{Eq. A3})$$

and the capital accumulation equation:

$$K_{t+1}^o = (1 - \delta)K_t^o + \phi\left(\frac{I_t^o}{K_t^o}\right)K_t^o \quad (\text{Eq. A4})$$

$$\text{where : } \phi'\left(\frac{I_t^o}{K_t^o}\right) > 0$$

$$\text{and : } \phi''\left(\frac{I_t^o}{K_t^o}\right) \leq 0$$

$$\text{and : } \phi'(\delta) = 1$$

$$\text{finally : } \phi(\delta) = \delta$$

The lagrangian of this problem is:

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \left[ \log C_t^o - \frac{(N_t^o)^{1+\varphi}}{1+\varphi} \right] + \lambda_t \left[ \begin{array}{l} P_t(C_t^o + I_t^o) + R_t^{-1}B_{t+1}^o \\ -W_tP_tN_t^o - R_t^kP_tK_t^o - B_t^o - D_t^o + P_tT_t^o \end{array} \right] \right. \\ \left. + \tau_t \left[ (1 - \delta)K_t^o + \phi\left(\frac{I_t^o}{K_t^o}\right)K_t^o - K_{t+1}^o \right] \right\}$$



The first order conditions for  $C_t^o$  and  $L_t^o$  are:

$$\frac{\partial \mathcal{L}}{\partial C_t^o} = 0 \Rightarrow$$

$$\begin{aligned} \beta^t \frac{1}{C_t^o} + \beta^t \lambda_t P_t &= 0 \Rightarrow \\ \lambda_t &= -\frac{1}{C_t^o} \frac{1}{P_t} \end{aligned} \quad (\text{Eq. A5})$$

$$\frac{\partial \mathcal{L}}{\partial N_t^o} = 0 \Rightarrow$$

$$\begin{aligned} -\beta^t (N_t^o)^\varphi - \beta^t \lambda_t W_t P_t &= 0 \Rightarrow \\ (N_t^o)^\varphi &= -\lambda_t W_t P_t \end{aligned} \quad (\text{Eq. A6})$$

We show in Appendix C the details of the non competitive labour market structure, i.e. under the case of a non-competitive labour market condition.

The FOC for  $B_{t+1}^o$  is:

$$\frac{\partial \mathcal{L}}{\partial B_{t+1}^o} = 0 \Rightarrow$$

$$\begin{aligned} -\beta^t \tau_t - \beta^{t+1} \lambda_{t+1} R_{t+1} &= 0 \Rightarrow \\ \lambda_t \frac{1}{R_t} &= \lambda_{t+1} \beta \end{aligned} \quad (\text{Eq. A8})$$

putting (Eq. A5) into (Eq. A8) we obtain the Euler equation:

$$\begin{aligned} -\frac{1}{C_t^o} \frac{1}{P_t} \frac{1}{R_t} &= \beta - \frac{1}{C_{t+1}^o} \frac{1}{P_{t+1}} \Rightarrow \\ \frac{1}{R_t} &= \beta \frac{C_t^o}{C_{t+1}^o} \frac{P_t}{P_{t+1}} \end{aligned}$$

thus:

$$1 = R_t E_t \left[ \Lambda_{t,t+1} \frac{P_t}{P_{t+1}} \right] \quad (\text{Eq. A9})$$

where the stochastic discount factor is:

$$\Lambda_{t,t+k} = \beta^k \left( \frac{C_{t+k}^o}{C_t^o} \right)^{-1} \quad (\text{Eq. A10})$$

The FOC for  $K_{t+1}^o$  is:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial K_{t+1}^o} = 0 \Rightarrow \\ -\beta^t \tau_t - \beta^{t+1} \lambda_{t+1} R_{t+1}^k P_{t+1} + \beta^{t+1} \tau_{t+1} \left[ \begin{array}{c} (1-\delta) \\ -\phi' \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \frac{I_{t+1}^o}{(K_{t+1}^o)^2} K_{t+1}^o \\ +\phi \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \end{array} \right] = 0 \end{aligned} \quad (\text{Eq. A11})$$

The FOC for  $I_{t+1}^o$  is:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial I_{t+1}^o} = 0 \Rightarrow \\ \beta^t \lambda_t P_t + \beta^t \tau_t \left[ \phi' \left( \frac{I_t^o}{K_t^o} \right) \frac{1}{K_t^o} K_t^o \right] = 0 \Rightarrow \\ \lambda_t P_t + \tau_t \left[ \phi' \left( \frac{I_t^o}{K_t^o} \right) \right] = 0 \Rightarrow \\ \tau_t = -\frac{1}{\phi' \left( \frac{I_t^o}{K_t^o} \right)} (\lambda_t P_t) \end{aligned} \quad (\text{Eq. A12})$$

putting (Eq. A12) into (Eq. A11) we obtain the Tobin's  $Q$  equation:

$$\begin{aligned} -\tau_t - \beta \lambda_{t+1} R_{t+1}^k P_{t+1} + \beta \tau_{t+1} \left[ (1-\delta) - \phi' \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \frac{I_{t+1}^o}{K_{t+1}^o} + \phi \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \right] = 0 \Rightarrow \\ \frac{1}{\phi' \left( \frac{I_t^o}{K_t^o} \right)} (\lambda_t P_t) - \beta \lambda_{t+1} R_{t+1}^k P_{t+1} - \beta \frac{1}{\phi' \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right)} (\lambda_{t+1} P_{t+1}) \left[ (1-\delta) - \phi' \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \frac{I_{t+1}^o}{K_{t+1}^o} + \phi \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \right] = 0 \Rightarrow \end{aligned}$$

$$\begin{aligned}
Q_t (\lambda_t P_t) - \beta \lambda_{t+1} R_{t+1}^k P_{t+1} - \beta Q_{t+1} (\lambda_{t+1} P_{t+1}) \left[ (1 - \delta) - \phi' \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \frac{I_{t+1}^o}{K_{t+1}^o} + \phi \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \right] &= 0 \Rightarrow \\
Q_t (\lambda_t P_t) = \beta \lambda_{t+1} R_{t+1}^k P_{t+1} + \beta Q_{t+1} (\lambda_{t+1} P_{t+1}) \left[ (1 - \delta) - \phi' \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \frac{I_{t+1}^o}{K_{t+1}^o} + \phi \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \right] &\Rightarrow \\
Q_t (\lambda_t P_t) = (\beta \lambda_{t+1} P_{t+1}) \left\{ R_{t+1}^k + Q_{t+1} \left[ (1 - \delta) - \phi' \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \frac{I_{t+1}^o}{K_{t+1}^o} + \phi \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \right] \right\} &\Rightarrow \\
Q_t (\lambda_t P_t) = (\beta \lambda_{t+1} P_{t+1}) \left\{ R_{t+1}^k + Q_{t+1} \left[ (1 - \delta) - \phi' \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \frac{I_{t+1}^o}{K_{t+1}^o} + \phi \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \right] \right\} &\Rightarrow \\
Q_t \left( -\frac{1}{C_t^o} \frac{1}{P_t} P_t \right) = \beta \left( -\frac{1}{C_{t+1}^o} \frac{1}{P_{t+1}} P_{t+1} \right) \left\{ R_{t+1}^k + Q_{t+1} \left[ (1 - \delta) - \phi' \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \frac{I_{t+1}^o}{K_{t+1}^o} + \phi \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \right] \right\} &\Rightarrow \\
Q_t = \beta \frac{C_t^o}{C_{t+1}^o} \left\{ R_{t+1}^k + Q_{t+1} \left[ (1 - \delta) - \phi' \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \frac{I_{t+1}^o}{K_{t+1}^o} + \phi \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \right] \right\} &\Rightarrow \\
Q_t = E_t \left( \Lambda_{t,t+1} \left\{ R_{t+1}^k + Q_{t+1} \left[ \begin{array}{c} (1 - \delta) \\ -\phi' \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \frac{I_{t+1}^o}{K_{t+1}^o} \\ + \phi \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \end{array} \right] \right\} \right) & \text{(Eq. A13)}
\end{aligned}$$

where:

$$Q_t = \frac{1}{\phi' \left( \frac{I_t^o}{K_t^o} \right)} \quad \text{(Eq. A14)}$$

#### 4.8.2 NON-RICARDIAN HOUSEHOLDS.

A typical household of this type seeks to maximize:

$$\max U (C_t^r, N_t^r) \quad \text{(Eq. A15)}$$

$$\text{where : } U (C_t^r, N_t^r) = \log C_t^r - \frac{(N_t^r)^{1+\varphi}}{1+\varphi} \quad \text{(Eq. A16)}$$

subject to the budget constraint:

$$P_t C_t^r = W_t P_t N_t^r - P_t T_t^r \quad \text{(Eq. A17)}$$

The lagrangian associated to this problem is:

$$\mathcal{L} = \left[ \log C_t^r - \frac{(N_t^r)^{1+\varphi}}{1+\varphi} \right] + \lambda_t [P_t C_t^r - W_t P_t N_t^r + P_t T_t^r] \quad (\text{Eq. A18})$$

the first order conditions for  $C_t^r$  and  $N_t^r$  are:

$$\frac{\partial \mathcal{L}}{\partial C_t^r} = 0 \Rightarrow$$

$$\begin{aligned} \frac{1}{C_t^r} + \lambda_t P_t &= 0 \Rightarrow \\ \lambda_t &= -\frac{1}{C_t^r} \frac{1}{P_t} \end{aligned} \quad (\text{Eq. A19})$$

$$\frac{\partial \mathcal{L}}{\partial N_t^r} = 0 \Rightarrow$$

$$-(N_t^r)^\varphi - \lambda_t W_t P_t = 0 \Rightarrow$$

$$\lambda_t W_t P_t = -(N_t^r)^\varphi \quad (\text{Eq. A20})$$

Alternatively, when the wage is set by a union, hours are determined by firms' labour demand. Again we refer the reader to the subsequent discussion.

### 4.8.3 FINAL GOOD FIRMS.

Taking the following constant returns technology:

$$Y_t = \left( \int_0^1 X_t(j)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} dj \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \quad (\text{Eq. A21})$$

Profit maximization, taking as given the final goods price  $P_t$  and the prices for the intermediate goods  $P_t(j)$ , all  $j \in [0, 1]$ , yields the set of demand schedules:

$$\max_{X_t(j)} P_t Y_t - \int_0^1 P_t(j) X_t(j) dj \quad (\text{Eq. A22})$$

Putting equation (Eq. A21) into (Eq. A22):

$$\max_{X_t(j)} P_t \left( \int_0^1 X_t(j)^{\frac{\varepsilon_p-1}{\varepsilon_p}} dj \right)^{\frac{\varepsilon_p}{\varepsilon_p-1}} - \int_0^1 P_t(j) X_t(j) dj$$

thus, we obtain the downward sloping demand for each intermediate input:

$$\begin{aligned} P_t \frac{\varepsilon_p}{\varepsilon_p-1} \left( \int_0^1 X_t(j)^{\frac{\varepsilon_p-1}{\varepsilon_p}} dj \right)^{\frac{\varepsilon_p}{\varepsilon_p-1}-1} \frac{\varepsilon_p-1}{\varepsilon_p} X_t(j)^{\frac{\varepsilon_p-1}{\varepsilon_p}-1} - P_t(j) &= 0 \Rightarrow \\ P_t \left( \int_0^1 X_t(j)^{\frac{\varepsilon_p-1}{\varepsilon_p}} dj \right)^{\frac{\varepsilon_p}{\varepsilon_p-1}-1} X_t(j)^{\frac{\varepsilon_p-1}{\varepsilon_p}-1} - P_t(j) &= 0 \Rightarrow \\ P_t \left( \int_0^1 X_t(j)^{\frac{\varepsilon_p-1}{\varepsilon_p}} dj \right)^{\frac{\varepsilon_p}{\varepsilon_p-1}-1} X_t(j)^{\frac{\varepsilon_p-1}{\varepsilon_p}-1} &= P_t(j) \Rightarrow \\ X_t(j)^{-\frac{1}{\varepsilon_p}} &= \frac{P_t(j)}{P_t} \left( \int_0^1 X_t(j)^{\frac{\varepsilon_p-1}{\varepsilon_p}} dj \right)^{-\frac{1}{\varepsilon_p-1}} \Rightarrow \\ X_t(j) &= \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon_p} \left( \int_0^1 X_t(j)^{\frac{\varepsilon_p-1}{\varepsilon_p}} dj \right)^{\frac{\varepsilon}{\varepsilon_p-1}} \Rightarrow \\ X_t(j) &= \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon_p} Y_t \quad (\text{Eq. A23}) \end{aligned}$$

Putting (Eq. A23) into (Eq. A21) we obtain the price index:

$$\begin{aligned}
Y_t &= \left( \int_0^1 \left( \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon_p} \right)^{\frac{\varepsilon_p-1}{\varepsilon_p}} Y_t^{\frac{\varepsilon_p-1}{\varepsilon_p}} dj \right)^{\frac{\varepsilon_p}{\varepsilon_p-1}} \Rightarrow \\
Y_t^{\frac{\varepsilon_p-1}{\varepsilon_p}} &= Y_t^{\frac{\varepsilon_p-1}{\varepsilon_p}} \int_0^1 \left( \frac{P_t(j)}{P_t} \right)^{1-\varepsilon_p} dj \Rightarrow \\
1 &= \frac{1}{P_t^{1-\varepsilon_p}} \int_0^1 (P_t(j))^{1-\varepsilon_p} dj \Rightarrow \\
P_t^{1-\varepsilon_p} &= \int_0^1 (P_t(j))^{1-\varepsilon_p} dj \Rightarrow \\
P_t &= \left[ \int_0^1 P_t(j)^{1-\varepsilon_p} dj \right]^{\frac{1}{1-\varepsilon_p}} \tag{Eq. A24}
\end{aligned}$$

#### 4.8.4 INTERMEDIATE GOOD FIRMS.

Taking the production function for a typical intermediate good firm:

$$Y_t(j) = K_t(j)^\alpha N_t(j)^{1-\alpha} \tag{Eq. A25}$$

The maximization of real profits is thus given by:

$$\max_{K_t(j), N_t(j)} O_t(j) = \frac{P_t(j)}{P_t} Y_t(j) - R_t^k K_t(j) - W_t N_t(j) + \lambda_t(j) \left[ K_t(j)^\alpha N_t(j)^{1-\alpha} - Y_t(j) \right]$$

The FOCs are:

$$\frac{\partial O_t(j)}{\partial N_t(j)} = 0 \Rightarrow$$

$$\begin{aligned}
-W_t + (1 - \alpha) \lambda_t(j) K_t(j)^\alpha N_t(j)^{-\alpha} &= 0 \Rightarrow \\
(1 - \alpha) \lambda_t(j) \left( \frac{K_t(j)}{N_t(j)} \right)^\alpha &= W_t
\end{aligned} \tag{Eq. A26}$$

and:

$$\begin{aligned}
\frac{\partial O_t(j)}{\partial K_t(j)} = 0 \Rightarrow \\
-R_t^k + \alpha \lambda_t(j) K_t(j)^{\alpha-1} N_t(j)^{1-\alpha} &= 0 \Rightarrow \\
\alpha \lambda_t(j) \left( \frac{K_t(j)}{N_t(j)} \right)^{\alpha-1} &= R_t^k
\end{aligned} \tag{Eq. A27}$$

from (Eq. A26) and (Eq. A27):

$$\begin{aligned}
\frac{W_t}{R_t^k} &= (1 - \alpha) \lambda_t(j) \left( \frac{K_t(j)}{N_t(j)} \right)^\alpha \frac{1}{\alpha} \frac{1}{\lambda_t(j)} \left( \frac{K_t(j)}{N_t(j)} \right)^{1-\alpha} \Rightarrow \\
\frac{W_t}{R_t^k} &= \frac{1 - \alpha}{\alpha} \left( \frac{K_t(j)}{N_t(j)} \right) \Rightarrow \\
\frac{K_t(j)}{N_t(j)} &= \frac{\alpha}{1 - \alpha} \frac{W_t}{R_t^k}
\end{aligned} \tag{Eq. A28}$$

Real marginal cost is common to all firms and given as follows. From (Eq. A26) and (Eq. A27):

$$\begin{aligned}
COST(j) &= \lambda_t(j) (1 - \alpha) \left( \frac{K_t(j)}{N_t(j)} \right)^\alpha N_t(j) + \lambda_t(j) \alpha \left( \frac{K_t(j)}{N_t(j)} \right)^{\alpha-1} K_t(j) \\
&= \lambda_t(j) W_t N_t(j) + \lambda_t(j) R_t^k K_t(j) \\
&= \lambda_t(j) Y_t(j)
\end{aligned}$$

and:

$$MC(j) = \frac{\partial COST(j)}{\partial Y_t(j)} = \frac{\partial \lambda_t(j) Y_t(j)}{\partial Y_t(j)} = \lambda_t(j)$$

where:

$$W_t = (1 - \alpha) \lambda_t(j) \left( \frac{K_t(j)}{N_t(j)} \right)^\alpha$$

and:

$$\begin{aligned} R_t^k &= \alpha \lambda_t(j) \left( \frac{K_t(j)}{N_t(j)} \right)^{\alpha-1} \Rightarrow \\ (R_t^k)^{\frac{\alpha}{\alpha-1}} &= (\alpha \lambda_t(j))^{\frac{\alpha}{\alpha-1}} \left( \frac{K_t(j)}{N_t(j)} \right)^\alpha \end{aligned}$$

thus:

$$\begin{aligned} \frac{W_t}{(R_t^k)^{\frac{\alpha}{\alpha-1}}} &= \frac{(1-\alpha) \lambda_t(j) \left( \frac{K_t(j)}{N_t(j)} \right)^\alpha}{(\alpha \lambda_t(j))^{\frac{\alpha}{\alpha-1}} \left( \frac{K_t(j)}{N_t(j)} \right)^\alpha} \Rightarrow \\ \frac{W_t}{(R_t^k)^{\frac{\alpha}{\alpha-1}}} &= \lambda_t(j)^{1-\frac{\alpha}{\alpha-1}} \frac{1-\alpha}{\alpha^{\frac{\alpha}{\alpha-1}}} \Rightarrow \\ \frac{W_t}{(R_t^k)^{\frac{\alpha}{\alpha-1}}} &= \lambda_t(j)^{-\frac{1}{\alpha-1}} \frac{1-\alpha}{\alpha^{\frac{\alpha}{\alpha-1}}} \Rightarrow \\ \lambda_t(j)^{-\frac{1}{\alpha-1}} &= \frac{W_t}{(R_t^k)^{\frac{\alpha}{\alpha-1}}} \frac{\alpha^{\frac{\alpha}{\alpha-1}}}{1-\alpha} \Rightarrow \\ \lambda_t(j) &= \frac{(W_t)^{-(\alpha-1)}}{(R_t^k)^{-\alpha}} \frac{\alpha^{-\alpha}}{(1-\alpha)^{-(\alpha-1)}} \Rightarrow \\ \lambda_t(j) &= (W_t)^{1-\alpha} (R_t^k)^\alpha \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} \end{aligned}$$

finally:

$$MC_t = \Psi (W_t)^{1-\alpha} (R_t^k)^\alpha \quad (\text{Eq. A29})$$

$$\text{where } : \quad \Psi = \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)}$$

#### 4.8.5 PRICE SETTING

A firm resetting its price in period  $t$  solves:

$$\max_{P_t^*} E_t \sum_{k=0}^{\infty} \theta^k \left\{ \Lambda_{t,t+k} Y_{t,t+k}(j) \left( \frac{P_t^*}{P_{t+k}} - MC_{t+k} \right) \right\} \quad (\text{Eq. A30})$$



subject to the sequence of demand constraints:

$$Y_{t+k}(j) = X_{t+k}(j) = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon_p} Y_{t+k} \quad (\text{Eq. A31})$$

Thus, we can rewrite the problem as:

$$\begin{aligned} & \max_{P_t^*} E_t \sum_{k=0}^{\infty} \theta^k \left\{ \Lambda_{t,t+k} \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon_p} Y_{t+k} \left(\frac{P_t^*}{P_{t+k}} - MC_{t+k}\right) \right\} \\ \Rightarrow & \max_{P_t^*} E_t \sum_{k=0}^{\infty} \theta^k \left\{ \Lambda_{t,t+k} Y_{t+k} \left( \left(\frac{P_t^*}{P_{t+k}}\right)^{1-\varepsilon_p} - \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon_p} MC_{t+k} \right) \right\} \end{aligned}$$

The FOC is the following:

$$\begin{aligned} (1 - \varepsilon_p) \frac{(P_t^*)^{-\varepsilon_p}}{(P_{t+k})^{-\varepsilon_p}} \frac{1}{P_{t+k}} + \varepsilon_p \frac{(P_t^*)^{-\varepsilon_p - 1}}{(P_{t+k})^{-\varepsilon_p - 1}} \frac{1}{P_{t+k}} MC_{t+k} &= 0 \Rightarrow \\ (1 - \varepsilon_p) + \varepsilon_p \frac{(P_t^*)^{-1}}{(P_{t+k})^{-1}} MC_{t+k} &= 0 \Rightarrow \\ P_t^* (1 - \varepsilon_p) + \varepsilon_p \frac{(P_t^*) (P_t^*)^{-1}}{(P_{t+k})^{-1}} MC_{t+k} &= 0 \Rightarrow \\ P_t^* (1 - \varepsilon_p) + \varepsilon_p P_{t+k} MC_{t+k} &= 0 \Rightarrow \\ P_t^* + \frac{\varepsilon_p}{1 - \varepsilon_p} P_{t+k} MC_{t+k} &= 0 \Rightarrow \\ \frac{P_t^*}{P_{t+k}} - \frac{\varepsilon_p}{\varepsilon_p - 1} MC_{t+k} &= 0 \end{aligned}$$

thus, we have:

$$\begin{aligned} E_t \sum_{k=0}^{\infty} \theta^k \left\{ \Lambda_{t,t+k} Y_{t+k}(j) \left(\frac{P_t^*}{P_{t+k}} - \mu^p MC_{t+k}\right) \right\} &= 0 \quad (\text{Eq. A32}) \\ \text{where } \mu^p &\equiv \frac{\varepsilon_p}{\varepsilon_p - 1} \end{aligned}$$

## 4.9 APPENDIX B: LOG-LINEARIZED EQUILIBRIUM CONDITIONS

### 4.9.1 STEADY STATE ANALYSIS

In this short section we show that the steady state ratio of aggregate consumption to total output does not depend upon the fraction of rule-of-thumb consumers. In doing so, we just notice that the market clearing condition for final goods implies.

#### AGGREGATE GOVERNMENT SPENDING.

$$Y_t = C_t + I_t + G_t \Rightarrow$$

$$C_t = Y_t - I_t - G_t$$

in steady state:

$$C = Y - I - G$$

dividing by  $Y$ :

$$\frac{C}{Y} = \frac{Y}{Y} - \frac{I}{Y} - \frac{G}{Y}$$

where:

$$\frac{I}{Y} = \frac{I}{K} \frac{K}{Y}$$

knowing that:

$$\frac{I}{K} = \delta$$

we can rewrite:

$$\frac{I}{Y} = \delta \frac{K}{Y}$$

rearranging:

$$\begin{aligned}\frac{I}{Y} &= \delta \frac{\alpha}{\alpha} \frac{1}{\frac{Y}{K}} \Rightarrow \\ \frac{I}{Y} &= \frac{\delta \alpha}{\alpha \frac{Y}{K}}\end{aligned}$$

thus:

$$\frac{C}{Y} = \frac{Y}{Y} - \frac{I}{Y} - \frac{G}{Y}$$

since:

$$\begin{aligned}\gamma_c &= \frac{C}{Y} \\ \gamma_g &= \frac{G}{Y}\end{aligned}$$

we can rewrite:

$$\gamma_c = 1 - \frac{\delta \alpha}{\alpha \frac{Y}{K}} - \gamma_g$$

Moreover, starting from the production function:

$$Y_t(j) = K_t(j)^\alpha N_t(j)^{1-\alpha}$$

$$\begin{aligned}\frac{\partial Y_t(j)}{\partial K_t(j)} &= R_t^k \Rightarrow \\ \alpha K_t(j)^\alpha N_t(j)^{1-\alpha} \frac{1}{K_t(j)} \lambda_t(j) &= R_t^k \Rightarrow \\ \alpha \frac{Y_t(j)}{K_t(j)} \lambda_t(j) &= R_t^k\end{aligned}$$

in steady state:

$$\alpha \frac{Y}{K} \lambda_t(j) = R^k$$

from relation (Eq. 16):

$$E_t \sum_{k=0}^{\infty} \theta^k \left\{ \Lambda_{t,t+k} Y_{t+k}(j) \left( \frac{P_t^*}{P_{t+k}} - \mu_p MC_{t+k} \right) \right\} = 0$$

in steady state:

$$\begin{aligned} 1 - \mu_p MC &= 0 \Rightarrow \\ MC &= \frac{1}{\mu_p} = \lambda_t(j) \end{aligned}$$

thus:

$$\begin{aligned} \alpha \frac{Y}{K} \frac{1}{\mu_p} &= R^k \Rightarrow \\ R^k &= \frac{\alpha Y}{\mu_p K} \end{aligned}$$

Knowing that:

$$\begin{aligned} Q &= \beta \frac{C}{C} \left\{ R^k + Q \left[ (1 - \delta) - \phi' \left( \frac{I}{K} \right) \frac{I}{K} + \phi \left( \frac{I}{K} \right) \right] \right\} \Rightarrow \\ 1 &= \beta \left\{ R^k + \left[ (1 - \delta) - \phi'(\delta) \delta + \phi(\delta) \right] \right\} \Rightarrow \\ 1 &= \beta \left\{ R^k + [(1 - \delta) - \delta + \delta] \right\} \Rightarrow \\ 1 &= \beta \left\{ R^k + (1 - \delta) \right\} \Rightarrow \\ \frac{1}{\beta} &= R^k + 1 - \delta \Rightarrow \end{aligned}$$

$$R^k = \frac{1}{\beta} - 1 + \delta$$

we can equate:

$$R^k = \frac{1}{\beta} - 1 + \delta = \frac{\alpha Y}{\mu_p K} = R_t^k$$

solving for:

$$\begin{aligned}\frac{1}{\beta} - 1 + \delta &= \frac{\alpha Y}{\mu_p K} \Rightarrow \\ \alpha \frac{Y}{K} &= \mu_p \left( \frac{1}{\beta} - 1 + \delta \right)\end{aligned}$$

finally:

$$\gamma_c = 1 - \frac{\delta \alpha}{\alpha \frac{Y}{K}} - \gamma_g = (1 - \gamma_g) - \frac{\delta \alpha}{\alpha \frac{Y}{K}} = (1 - \gamma_g) - \frac{\delta \alpha}{\mu_p \left( \frac{1}{\beta} - 1 + \delta \right)}$$

***NON-MILITARY AND MILITARY EXPENDITURES.*** In the case of total government spending disaggregation, we have:

$$\begin{aligned}Y_t &= C_t + I_t + NM_t + M_t \Rightarrow \\ C_t &= Y_t - I_t - NM_t - M_t\end{aligned}$$

in steady state:

$$C = Y - I - NM - M$$

dividing by  $Y$ :

$$\frac{C}{Y} = \frac{Y}{Y} - \frac{I}{Y} - \frac{NM}{Y} - \frac{M}{Y}$$

since:

$$\begin{aligned}\gamma_{nm} &= \frac{NM}{Y} \\ \gamma_m &= \frac{M}{Y}\end{aligned}$$

Finally, applying the same computations as in total government spending case, we obtain:

$$\gamma_c = 1 - \frac{\delta\alpha}{\alpha\frac{Y}{K}} - \gamma_{nm} - \gamma_m = (1 - \gamma_{nm} - \gamma_m) - \frac{\delta\alpha}{\alpha\frac{Y}{K}} = (1 - \gamma_{nm} - \gamma_m) - \frac{\delta\alpha}{\mu_p\left(\frac{1}{\beta} - 1 + \delta\right)}$$

#### 4.9.2 LOG-LINEARIZED EQUATIONS

In the present section we derive the log-linear versions of the key optimality and market-clearing conditions used in our analysis of the model's equilibrium dynamics. Some of these conditions hold exactly, whereas others represent first-order approximations around a zero-inflation steady state. Henceforth, and unless otherwise noted, lower-case letters denote log-deviations with respect to the corresponding steady state values:

$$x_t \equiv \log \frac{X_t}{\bar{X}} \tag{Eq. A33}$$

**HOUSEHOLDS.** Equation (Eq. 30) is obtained log-linearizing expression (Eq. 9):

$$\begin{aligned} Q_t &= E_t \left( \Lambda_{t,t+1} \left\{ R_{t+1}^k + Q_{t+1} \left[ (1 - \delta) - \phi' \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \frac{I_{t+1}^o}{K_{t+1}^o} + \phi \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \right] \right\} \right) \Rightarrow \\ Q_t &= \beta \frac{C_t^o}{C_{t+1}^o} \left\{ R_{t+1}^k + Q_{t+1} \left[ (1 - \delta) - \phi' \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \frac{I_{t+1}^o}{K_{t+1}^o} + \phi \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \right] \right\} \end{aligned}$$

we know that in steady state:

$$\begin{aligned}
 \frac{I}{K} &= \delta \\
 \Rightarrow \phi(\delta) &= \delta \\
 \Rightarrow \phi'(\delta) &= 1 \\
 \text{and } : Q &= \frac{1}{\phi'(\frac{I}{K})} \\
 \Rightarrow Q &= \frac{1}{\phi'(\delta)} \\
 \Rightarrow Q &= 1 \\
 \text{and } : -\frac{1}{\phi''(\frac{I}{K})\frac{I}{K}} &\equiv \eta \\
 \Rightarrow -\frac{1}{\phi''(\delta)\delta} &\equiv \eta
 \end{aligned}$$

thus:

$$\begin{aligned}
 Q &= \beta \frac{C}{C} \left\{ R^k + Q \left[ (1-\delta) - \phi' \left( \frac{I}{K} \right) \frac{I}{K} + \phi \left( \frac{I}{K} \right) \right] \right\} \Rightarrow \\
 1 &= \beta \left\{ R^k + \left[ (1-\delta) - \phi'(\delta)\delta + \phi(\delta) \right] \right\} \Rightarrow \\
 1 &= \beta \left\{ R^k + [(1-\delta) - \delta + \delta] \right\} \Rightarrow \\
 1 &= \beta \left\{ R^k + (1-\delta) \right\} \Rightarrow \\
 \frac{1}{\beta} &= R^k + 1 - \delta \Rightarrow
 \end{aligned}$$

$$R^k = \frac{1}{\beta} - 1 + \delta$$

we obtain:

$$\begin{aligned}
Q_t &= \beta \frac{C_t^o}{C_{t+1}^o} \left\{ R_{t+1}^k + Q_{t+1} \left[ (1 - \delta) - \phi' \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \frac{I_{t+1}^o}{K_{t+1}^o} + \phi \left( \frac{I_{t+1}^o}{K_{t+1}^o} \right) \right] \right\} \\
\Rightarrow q_t Q &= \beta \left( \frac{1}{C} c_t^o C - \frac{C}{C^2} c_{t+1}^o C \right) \left\{ R^k + Q \left[ (1 - \delta) - \phi' \left( \frac{I}{K} \right) \frac{I}{K} + \phi \left( \frac{I}{K} \right) \right] \right\} \\
&+ \beta \frac{C}{C} \{ r_{t+1}^k R^k \} \\
&+ \beta \frac{C}{C} \left\{ q_{t+1} \left[ (1 - \delta) - \phi' \left( \frac{I}{K} \right) \frac{I}{K} + \phi \left( \frac{I}{K} \right) \right] Q \right\} \\
&+ \beta \frac{C}{C} \left\{ Q \left[ -\phi'' \left( \frac{I}{K} \right) \frac{1}{K} \frac{I}{K} i_{t+1} I \right] \right\} \\
&+ \beta \frac{C}{C} \left\{ Q \left[ -\phi' \left( \frac{I}{K} \right) \frac{1}{K} i_{t+1} I \right] \right\} \\
&+ \beta \frac{C}{C} \left\{ Q \left[ \phi'' \left( \frac{I}{K} \right) \frac{I}{K^2} \frac{I}{K} k_{t+1} K \right] \right\} \\
&+ \beta \frac{C}{C} \left\{ Q \left[ \phi' \left( \frac{I}{K} \right) \frac{I}{K^2} k_{t+1} K \right] \right\} \\
&+ \beta \frac{C}{C} \left\{ Q \left[ \phi' \left( \frac{I}{K} \right) \frac{1}{K} i_{t+1} I \right] \right\} \\
&+ \beta \frac{C}{C} \left\{ Q \left[ -\phi' \left( \frac{I}{K} \right) \frac{I}{K^2} k_{t+1} K \right] \right\}
\end{aligned}$$



$$\begin{aligned}
&\Rightarrow q_t = \beta (c_t^o - c_{t+1}^o) \left\{ \frac{1}{\beta} - 1 + \delta + \left[ (1 - \delta) - \phi'(\delta) \delta + \phi(\delta) \right] \right\} \\
&\quad + \beta r_{t+1}^k \left( \frac{1}{\beta} - 1 + \delta \right) \\
&\quad + \beta q_{t+1} \left[ (1 - \delta) - \phi'(\delta) \delta + \phi(\delta) \right] \\
&\quad - \beta \left[ \phi''(\delta) \delta \delta i_{t+1} \right] - \beta \left[ \phi'(\delta) \delta i_{t+1} \right] + \beta \left[ \phi''(\delta) \delta \delta k_{t+1} \right] \\
&\quad + \beta \left[ \phi'(\delta) \delta k_{t+1} \right] + \beta \left[ \phi'(\delta) \delta i_{t+1} \right] - \beta \left[ \phi'(\delta) \delta k_{t+1} \right] \\
&\Rightarrow q_t = \beta (c_t^o - c_{t+1}^o) \left\{ \frac{1}{\beta} - 1 + \delta + [(1 - \delta) - \delta + \delta] \right\} \\
&\quad + \beta r_{t+1}^k \left( \frac{1}{\beta} - 1 + \delta \right) + \beta q_{t+1} [(1 - \delta) - \delta + \delta] \\
&\quad - \beta \left[ \phi''(\delta) \delta \delta i_{t+1} \right] + \beta \left[ \phi''(\delta) \delta \delta k_{t+1} \right] \\
&\Rightarrow q_t = \beta (c_t^o - c_{t+1}^o) \frac{1}{\beta} + \beta r_{t+1}^k \left( \frac{1}{\beta} - 1 + \delta \right) + \beta q_{t+1} (1 - \delta) - \beta \phi''(\delta) \delta \delta (i_{t+1} - k_{t+1})
\end{aligned}$$

using the log-linearized relationship between Tobin's  $Q$  and investment:

$$i_t - k_t = \eta q_t$$

and knowing that:

$$-\frac{1}{\phi''(\delta) \delta} = \eta \Rightarrow \phi''(\delta) \delta = -\frac{1}{\eta}$$

we have that:

$$\begin{aligned}
q_t &= \beta (c_t^o - c_{t+1}^o) \frac{1}{\beta} + \beta r_{t+1}^k \left( \frac{1}{\beta} - 1 + \delta \right) + \beta q_{t+1} - \beta \delta q_{t+1} + \beta \frac{1}{\eta} \delta (\eta q_{t+1}) \\
&\Rightarrow q_t = \beta (c_t^o - c_{t+1}^o) \frac{1}{\beta} + \beta r_{t+1}^k \left( \frac{1}{\beta} - 1 + \delta \right) + \beta q_{t+1} - \delta \beta q_{t+1} + \beta \delta q_{t+1} \\
&\Rightarrow q_t = \beta (c_t^o - c_{t+1}^o) \frac{1}{\beta} + \beta r_{t+1}^k \left( \frac{1}{\beta} - 1 + \delta \right) + \beta q_{t+1} \\
&\Rightarrow q_t = (c_t^o - c_{t+1}^o) + \beta r_{t+1}^k \left( \frac{1}{\beta} - 1 + \delta \right) + \beta q_{t+1}
\end{aligned}$$

using the log-linearized Euler equation:

$$\begin{aligned} c_t^o &= E_t \{c_{t+1}^o\} - (r_t - E_t \{\pi_{t+1}\}) \\ &\Rightarrow c_t^o - E_t \{c_{t+1}^o\} = -(r_t - E_t \{\pi_{t+1}\}) \end{aligned}$$

finally, we obtain expression (Eq. 30):

$$\begin{aligned} q_t &= -(r_t - E_t \{\pi_{t+1}\}) + \beta r_{t+1}^k \left( \frac{1}{\beta} - 1 + \delta \right) + \beta q_{t+1} \Rightarrow \\ q_t &= \beta E_t \{q_{t+1}\} + \beta \left( \frac{1}{\beta} - 1 + \delta \right) E_t \{r_{t+1}^k\} - (r_t - E_t \{\pi_{t+1}\}) \Rightarrow \\ q_t &= \beta E_t \{q_{t+1}\} + [1 - \beta(1 - \delta)] E_t \{r_{t+1}^k\} - (r_t - E_t \{\pi_{t+1}\}) \end{aligned}$$

Equation (Eq. 31) is obtained log-linearizing (Eq. 10):

$$\begin{aligned} Q_t &= \frac{1}{\phi' \left( \frac{I_t^o}{K_t^o} \right)} \Rightarrow \\ Q_t &= \phi' \left( \frac{I_t}{K_t} \right)^{-1} \Rightarrow \\ q_t Q &= (-1) \phi' \left( \frac{I}{K} \right)^{-2} \left( \phi'' \left( \frac{I}{K} \right) \frac{1}{K} i_t I - \phi'' \left( \frac{I}{K} \right) \frac{I}{K^2} k_t K \right) \Rightarrow \\ q_t Q &= (-1) \phi' (\delta)^{-2} \left( \phi'' (\delta) \delta i_t - \phi'' (\delta) \delta k_t \right) \Rightarrow \\ q_t &= (-1) \left( \phi'' (\delta) \delta (i_t - k_t) \right) \end{aligned}$$

knowing that:

$$-\frac{1}{\phi'' (\delta) \delta} = \eta \Rightarrow \phi'' (\delta) \delta = -\frac{1}{\eta}$$

we have expression (Eq. 31):

$$\begin{aligned} q_t &= (-1) \left( -\frac{1}{\eta} (i_t - k_t) \right) \Rightarrow \\ q_t \eta &= i_t - k_t \Rightarrow \\ i_t - k_t &= \eta q_t \end{aligned}$$

The log-linearized capital accumulation equation (Eq. 32) is obtained from expression (Eq. 6):

$$\begin{aligned}
K_{t+1}^o &= (1 - \delta) K_t^o + \phi \left( \frac{I_t^o}{K_t^o} \right) K_t^o \Rightarrow \\
K_{t+1} &= (1 - \delta) K_t + \phi \left( \frac{I_t}{K_t} \right) K_t \Rightarrow \\
k_{t+1}K &= (1 - \delta) k_t K + \phi' \left( \frac{I}{K} \right) \frac{1}{K} K i_t I - \phi' \left( \frac{I}{K} \right) \frac{I}{K^2} K k_t K + \phi \left( \frac{I}{K} \right) k_t K \Rightarrow \\
k_{t+1} &= (1 - \delta) k_t + \phi' \left( \frac{I}{K} \right) \frac{I}{K} i_t - \phi' \left( \frac{I}{K} \right) \frac{I}{K} k_t + \phi \left( \frac{I}{K} \right) k_t \Rightarrow \\
k_{t+1} &= (1 - \delta) k_t + \phi'(\delta) \delta i_t - \phi'(\delta) \delta k_t + \phi(\delta) k_t \Rightarrow \\
k_{t+1} &= (1 - \delta) k_t + \delta i_t - \delta k_t + \delta k_t \Rightarrow \\
k_{t+1} &= (1 - \delta) k_t + \delta i_t \Rightarrow \\
k_{t+1} &= \delta i_t + (1 - \delta) k_t
\end{aligned}$$

The log-linearized Euler equation for optimizing households (Eq. 33) is given as follows. From equation (Eq. 7):

$$1 = R_t E_t \left[ \Lambda_{t,t+1} \frac{P_t}{P_{t+1}} \right]$$

and (Eq. 8):

$$\Lambda_{t,t+k} = \beta^k \left( \frac{C_{t+k}^o}{C_t^o} \right)^{-1}$$

thus, we obtain expression (Eq. 33):

$$1 = R_t \beta \frac{C_t^o}{C_{t+1}^o} \frac{P_t}{P_{t+1}} \Rightarrow$$

$$\begin{aligned}
\beta \frac{C}{C} \frac{P}{P} r_t R + R\beta \frac{1}{C} \frac{P}{P} c_t^o C - R\beta \frac{C}{C^2} \frac{P}{P} c_{t+1}^o C + R\beta \frac{C}{C} \frac{1}{P} p_t P - R\beta \frac{C}{C} \frac{P}{P^2} p_{t+1} P &= 0 \Rightarrow \\
\beta r_t R + R\beta c_t^o - R\beta c_{t+1}^o + R\beta p_t - R\beta p_{t+1} &= 0 \Rightarrow \\
r_t + c_t^o - c_{t+1}^o + p_t - p_{t+1} &= 0 \Rightarrow
\end{aligned}$$

$$\begin{aligned}
c_t^o &= c_{t+1}^o - r_t + p_{t+1} - p_t \Rightarrow \\
c_t^o &= c_{t+1}^o - r_t + \pi_{t+1} \Rightarrow \\
c_t^o &= E_t \{c_{t+1}^o\} - (r_t - E_t \{\pi_{t+1}\})
\end{aligned}$$

The log-linearized equation of consumption for non-Ricardian households (Eq. 34) is obtained from equation (Eq. 13):

$$\begin{aligned}
C_t^r &= W_t N_t^r - T_t^r \Rightarrow \\
c_t^r C &= N^r w_t W + W n_t^r N^r - (T_t^r - T^r) \Rightarrow \\
c_t^r C^r &= W N^r (w_t + n_t^r) - (T_t^r - T^r) \Rightarrow \\
c_t^r &= \left( \frac{W N^r}{C^r} \right) (w_t + n_t^r) - \frac{1}{C^r} (T_t^r - T^r) \frac{Y}{Y} \Rightarrow \\
c_t^r &= \left( \frac{W N^r}{C^r} \right) (w_t + n_t^r) - \left( \frac{Y}{C^r} \right) \frac{T_t^r - T^r}{Y} \Rightarrow \\
c_t^r &= \left( \frac{W N^r}{C^r} \right) (w_t + n_t^r) - \left( \frac{Y}{C^r} \right) t_t^r \\
\text{since } : \quad t_t^r &= \frac{T_t^r - T^r}{Y}
\end{aligned}$$

The log-linearized expression for aggregate consumption (Eq. 35) is obtained, assuming  $C^r = C^o = C$ , from equation (Eq. 26):

$$\begin{aligned}
C_t &= \lambda C_t^r + (1 - \lambda) C_t^o \Rightarrow \\
c_t C &= \lambda c_t^r C^r + (1 - \lambda) c_t^o C^o \Rightarrow \\
c_t &= \lambda c_t^r + (1 - \lambda) c_t^o
\end{aligned}$$

Equivalently, the log-linearization equation (Eq. 36), assuming that  $C^r = C^o = C$ , gives expression (Eq. 27):

$$\begin{aligned} N_t &= \lambda N_t^r + (1 - \lambda) N_t^o \Rightarrow \\ n_t N &= \lambda n_t^r N^r + (1 - \lambda) n_t^o N^o \Rightarrow \\ n_t &= \lambda n_t^r + (1 - \lambda) n_t^o \end{aligned}$$

Since we have assumed a non-competitive labour market structure, we can derive the intertemporal equilibrium condition for aggregate consumption (Eq. 38) as follows. We first substitute expression (Eq. 37) into expression (Eq. 34), yielding:

$$\begin{aligned} c_t^r &= \left( \frac{WN}{C} \right) (c_t + \varphi n_t + n_t) - \left( \frac{Y}{C} \right) t_t^r \Rightarrow \\ c_t^r &= \left( \frac{WN}{C} \right) (c_t + (1 + \varphi) n_t) - \left( \frac{Y}{C} \right) t_t^r \Rightarrow \\ c_t^r &= \left( (1 - \alpha) \frac{Y}{C} \frac{1}{\mu^p} \right) (c_t + (1 + \varphi) n_t) - \left( \frac{Y}{C} \right) t_t^r \Rightarrow \\ c_t^r &= \left( \frac{(1 - \alpha)}{\gamma_c \mu^p} \right) (c_t + (1 + \varphi) n_t) - \left( \frac{1}{\gamma_c} \right) t_t^r \Rightarrow \\ c_t^r &= \frac{(1 - \alpha)}{\gamma_c \mu^p} c_t + \frac{(1 - \alpha)(1 + \varphi)}{\gamma_c \mu^p} n_t - \left( \frac{1}{\gamma_c} \right) t_t^r \end{aligned}$$

We proceed to use the operator  $(1 - L^{-1})$  in the previous expression, yielding:

$$\begin{aligned} c_t^r - E_t \{c_{t+1}^r\} &= \frac{(1 - \alpha)}{\gamma_c \mu^p} [c_t - E_t \{c_{t+1}\}] \\ &+ \frac{(1 - \alpha)(1 + \varphi)}{\gamma_c \mu^p} [n_t - E_t \{n_{t+1}\}] - \left( \frac{1}{\gamma_c} \right) [t_t^r - E_t \{t_{t+1}^r\}] \end{aligned} \quad (\text{Eq. A34})$$

We also apply the operator  $(1 - L^{-1})$  to expression (Eq. 35), which yields:

$$c_t - c_{t+1} = \lambda [c_t^r - c_{t+1}^r] + (1 - \lambda) [c_t^o - c_{t+1}^o]$$

Finally, we substitute expressions (Eq. A34) and (Eq. 33) into the previous one, which after rearranging terms yields an Euler equation for aggregate consumption (Eq. 38).

Thus, knowing that:

$$\begin{aligned} c_t^r - E_t \{c_{t+1}^r\} &= \frac{(1-\alpha)}{\gamma_c \mu^p} [c_t - E_t \{c_{t+1}\}] \\ &+ \frac{(1-\alpha)(1+\varphi)}{\gamma_c \mu^p} [n_t - E_t \{n_{t+1}\}] - \left(\frac{1}{\gamma_c}\right) [t_t^r - E_t \{t_{t+1}^r\}] \end{aligned}$$

and:

$$c_t^o = c_{t+1}^o - (r_t - \pi_{t+1})$$

we substitute:

$$\begin{aligned} c_t - c_{t+1} &= \lambda \left[ \frac{(1-\alpha)}{\gamma_c \mu^p} [c_t - c_{t+1}] + \frac{(1-\alpha)(1+\varphi)}{\gamma_c \mu^p} [n_t - n_{t+1}] - \left(\frac{1}{\gamma_c}\right) [t_t^r - t_{t+1}^r] \right] + (1-\lambda) [-(r_t - \pi_{t+1})] \\ \Rightarrow c_t - c_{t+1} - \lambda \frac{(1-\alpha)}{\gamma_c \mu^p} [c_t - c_{t+1}] &= \lambda \left[ \frac{(1-\alpha)(1+\varphi)}{\gamma_c \mu^p} [n_t - n_{t+1}] - \left(\frac{1}{\gamma_c}\right) [t_t^r - t_{t+1}^r] \right] - (1-\lambda) [r_t - \pi_{t+1}] \\ \Rightarrow c_t - \lambda \frac{(1-\alpha)}{\gamma_c \mu^p} c_t - c_{t+1} + \lambda \frac{(1-\alpha)}{\gamma_c \mu^p} c_{t+1} &= \lambda \left[ \frac{(1-\alpha)(1+\varphi)}{\gamma_c \mu^p} [n_t - n_{t+1}] - \left(\frac{1}{\gamma_c}\right) [t_t^r - t_{t+1}^r] \right] - (1-\lambda) [r_t - \pi_{t+1}] \\ \Rightarrow c_t \left[ 1 - \lambda \frac{(1-\alpha)}{\gamma_c \mu^p} \right] &= c_{t+1} \left[ 1 - \lambda \frac{(1-\alpha)}{\gamma_c \mu^p} \right] + \lambda \left[ \frac{(1-\alpha)(1+\varphi)}{\gamma_c \mu^p} [n_t - n_{t+1}] - \left(\frac{1}{\gamma_c}\right) [t_t^r - t_{t+1}^r] \right] - (1-\lambda) [r_t - \pi_{t+1}] \\ \Rightarrow c_t \left[ \frac{\gamma_c \mu^p - \lambda(1-\alpha)}{\gamma_c \mu^p} \right] &= c_{t+1} \left[ \frac{\gamma_c \mu^p - \lambda(1-\alpha)}{\gamma_c \mu^p} \right] + \lambda \left[ \frac{(1-\alpha)(1+\varphi)}{\gamma_c \mu^p} [n_t - n_{t+1}] - \left(\frac{1}{\gamma_c}\right) [t_t^r - t_{t+1}^r] \right] - (1-\lambda) [r_t - \pi_{t+1}] \end{aligned}$$

$$\begin{aligned}
\Rightarrow c_t &= c_{t+1} \left[ \frac{\gamma_c \mu^p}{\gamma_c \mu^p - \lambda(1-\alpha)} \right] \left[ \frac{\gamma_c \mu^p - \lambda(1-\alpha)}{\gamma_c \mu^p} \right] \\
&\quad - \left[ \frac{\gamma_c \mu^p}{\gamma_c \mu^p - \lambda(1-\alpha)} \right] \lambda \left[ -\frac{(1-\alpha)(1+\varphi)}{\gamma_c \mu^p} [n_{t+1} - n_t] + \left( \frac{1}{\gamma_c} \right) [t_{t+1}^r - t_t^r] \right] \\
&\quad - \left[ \frac{\gamma_c \mu^p}{\gamma_c \mu^p - \lambda(1-\alpha)} \right] (1-\lambda) [r_t - \pi_{t+1}] \\
\Rightarrow c_t &= c_{t+1} - \frac{\gamma_c \mu^p (1-\lambda)}{\gamma_c \mu^p - \lambda(1-\alpha)} [r_t - \pi_{t+1}] \\
&\quad - \frac{\lambda(1-\alpha)(1+\varphi)}{\gamma_c \mu^p - \lambda(1-\alpha)} [n_{t+1} - n_t] + \frac{\lambda \mu^p}{\gamma_c \mu^p - \lambda(1-\alpha)} [t_{t+1}^r - t_t^r] \\
\Rightarrow c_t &= c_{t+1} - \frac{\gamma_c \mu^p (1-\lambda)}{\gamma_c \mu^p - \lambda(1-\alpha)} (r_t - \pi_{t+1}) \\
&\quad - \frac{\lambda(1-\alpha)(1+\varphi)}{\gamma_c \mu^p - \lambda(1-\alpha)} (\Delta n_{t+1}) + \frac{\lambda \mu^p}{\gamma_c \mu^p - \lambda(1-\alpha)} (\Delta t_{t+1}^r)
\end{aligned}$$

or, more compactly:

$$\begin{aligned}
c_t &= c_{t+1} - \frac{1}{\tilde{\sigma}} (r_t - E_t \{ \pi_{t+1} \}) - \Theta_n E_t \{ \Delta n_{t+1} \} + \Theta_t E_t \{ \Delta t_{t+1}^r \} \\
\text{where } : \quad \frac{1}{\tilde{\sigma}} &= \gamma_c \Phi \mu^p (1-\lambda) \\
\Phi &= (\gamma_c \mu^p - \lambda(1-\alpha))^{-1} \\
\Theta_n &= \lambda \Phi (1-\alpha) (1+\varphi) \\
\Theta_t &= \lambda \Phi \mu^p
\end{aligned}$$

which are the coefficients of this expression in the text.

**FIRMS.** Log-linearization of expressions (Eq. 16) and (Eq. 17) gives the inflation equation (Eq. 39). Indeed, from expression (Eq. 17):

$$P_t = \left[ \theta P_{t-1}^{1-\varepsilon_p} + (1-\theta) (P_t^*)^{1-\varepsilon_p} \right]^{\frac{1}{1-\varepsilon_p}}$$

we have that in steady state:

$$\begin{aligned}
P^{1-\varepsilon_p} &= \theta P^{1-\varepsilon_p} + (1-\theta) (P^*)^{1-\varepsilon_p} \Rightarrow \\
P^{1-\varepsilon_p} (1-\theta) &= (1-\theta) (P^*)^{1-\varepsilon_p} \Rightarrow \\
P &= P^*
\end{aligned}$$

Again from (Eq. 17):

$$P_t^{1-\varepsilon_p} = \theta P_{t-1}^{1-\varepsilon_p} + (1-\theta) (P_t^*)^{1-\varepsilon_p}$$

dividing by  $P_t^{1-\varepsilon_p}$ , we obtain:

$$\frac{P_t^{1-\varepsilon_p}}{P_t^{1-\varepsilon_p}} = \theta \frac{P_{t-1}^{1-\varepsilon_p}}{P_t^{1-\varepsilon_p}} + (1-\theta) \frac{(P_t^*)^{1-\varepsilon_p}}{P_t^{1-\varepsilon_p}}$$

defining:

$$\begin{aligned}
V_t &= \frac{P_t^*}{P_t} \\
\text{and} \quad &: \quad V = 1
\end{aligned}$$

we have:

$$1 = \theta \left( \frac{P_{t-1}}{P_t} \right)^{1-\varepsilon_p} + (1-\theta) V_t^{1-\varepsilon_p}$$

log-linearizing this expression:

$$\begin{aligned}
(1-\varepsilon_p) \theta \left( \frac{P}{P} \right)^{-\varepsilon_p} \frac{1}{P} p_{t-1} P - (1-\varepsilon_p) \theta \left( \frac{P}{P} \right)^{-\varepsilon_p} \frac{P}{P^2} p_t P + (1-\varepsilon_p) (1-\theta) V^{-\varepsilon_p} v_t V &= 0 \Rightarrow \\
\theta p_{t-1} - \theta p_t + (1-\theta) v_t &= 0 \Rightarrow \\
-\theta (p_t - p_{t-1}) + (1-\theta) v_t &= 0 \Rightarrow \\
(1-\theta) v_t &= \theta \pi_t \Rightarrow
\end{aligned}$$



thus:

$$v_t = \frac{\theta}{1-\theta} \pi_t \quad (\text{Eq. A35})$$

Moreover, from (Eq. 16):

$$\begin{aligned} E_t \sum_{k=0}^{\infty} \theta^k \left\{ \Lambda_{t,t+k} Y_{t+k}(j) \left( \frac{P_t^*}{P_{t+k}} - \mu^p MC_{t+k} \right) \right\} &= 0 \Rightarrow \\ E_t \sum_{k=0}^{\infty} \theta^k \left\{ \Lambda_{t,t+k} Y_{t+k}(j) \left( \frac{P_t^*}{P_{t+k}} - \mu^p MC_{t+k} \right) \right\} &= 0 \end{aligned}$$

in steady state we have that:

$$\begin{aligned} \theta \Lambda Y \left( \frac{P^*}{P} - \mu^p MC \right) &= 0 \Rightarrow \\ 1 - \mu^p MC &= 0 \Rightarrow \\ MC &= \frac{1}{\mu^p} \end{aligned}$$

Again from (Eq. 16):

$$\begin{aligned} \sum_{k=0}^{\infty} \theta^k \left\{ \Lambda_{t,t+k} Y_{t+k}(j) \left( \frac{P_t^*}{P_{t+k}} - \mu^p MC_{t+k} \right) \right\} &= 0 \Rightarrow \\ \sum_{k=0}^{\infty} \theta^k \left\{ \beta^k \left( \frac{C_{t+k}^o}{C_t^o} \right)^{-1} Y_{t+k}(j) \left( \frac{P_t^*}{P_{t+k}} - \mu^p MC_{t+k} \right) \right\} &= 0 \end{aligned}$$

multiplying both sides for  $\frac{P_t}{P_t^*}$  gives:

$$\begin{aligned} \sum_{k=0}^{\infty} \theta^k \left\{ \beta^k \left( \frac{C_{t+k}^o}{C_t^o} \right)^{-1} Y_{t+k}(j) \frac{P_t}{P_t^*} \left( \frac{P_t^*}{P_{t+k}} - \mu^p MC_{t+k} \right) \right\} &= 0 \Rightarrow \\ \sum_{k=0}^{\infty} \theta^k \left\{ \beta^k \left( \frac{C_{t+k}^o}{C_t^o} \right)^{-1} Y_{t+k}(j) \left( \frac{P_t^*}{P_{t+k}} \frac{P_t}{P_t^*} - \mu^p MC_{t+k} \frac{P_t}{P_t^*} \right) \right\} &= 0 \Rightarrow \\ \sum_{k=0}^{\infty} \theta^k \left\{ \beta^k \left( \frac{C_{t+k}^o}{C_t^o} \right)^{-1} Y_{t+k}(j) \left( \frac{P_t}{P_{t+k}} - \mu^p MC_{t+k} \frac{1}{V_t} \right) \right\} &= 0 \end{aligned}$$

log-linearizing this expression:

$$\begin{aligned}
& \sum_{k=0}^{\infty} \theta^k \left\{ \beta^k \left( \frac{C_{t+k}^o}{C_t^o} \right)^{-1} Y_{t+k}(j) \left( \frac{P_t}{P_{t+k}} - \mu^p MC_{t+k} \frac{1}{V_t} \right) \right\} = 0 \Rightarrow \\
& - \sum_{k=0}^{\infty} \theta^k \beta^k \left( \frac{C^o}{C^o} \right)^{-2} \frac{1}{C} Y \left( \frac{P}{P} - \mu^p MC \frac{1}{V} \right) c_{t+k}^o C \\
& + \sum_{k=0}^{\infty} \theta^k \beta^k \left( \frac{C^o}{C^o} \right)^{-2} \frac{C}{C^2} Y \left( \frac{P}{P} - \mu^p MC \frac{1}{V} \right) c_t^o C \\
& + \sum_{k=0}^{\infty} \theta^k \beta^k \left( \frac{C^o}{C^o} \right)^{-1} \left( \frac{P}{P} - \mu^p MC \frac{1}{V} \right) y_{t+k}(j) Y \\
& + \sum_{k=0}^{\infty} \theta^k \beta^k \left( \frac{C^o}{C^o} \right)^{-1} Y \left( \frac{1}{P} \right) p_t P \\
& - \sum_{k=0}^{\infty} \theta^k \beta^k \left( \frac{C^o}{C^o} \right)^{-1} Y \left( \frac{P}{P^2} \right) p_{t+k} P \\
& - \sum_{k=0}^{\infty} \theta^k \beta^k \left( \frac{C^o}{C^o} \right)^{-1} Y \mu^p \frac{1}{V} m c_{t+k} MC \\
& + \sum_{k=0}^{\infty} \theta^k \beta^k \left( \frac{C^o}{C^o} \right)^{-1} Y \mu^p MC \frac{1}{V^2} v_t V \\
& = 0 \Rightarrow
\end{aligned}$$

$$\sum_{k=0}^{\infty} \theta^k \beta^k p_t - \sum_{k=0}^{\infty} \theta^k \beta^k p_{t+k} - \sum_{k=0}^{\infty} \theta^k \beta^k m c_{t+k} + \sum_{k=0}^{\infty} \theta^k \beta^k v_t = 0 \Rightarrow$$

$$\begin{aligned}
\sum_{k=0}^{\infty} \theta^k \beta^k v_t &= \sum_{k=0}^{\infty} \theta^k \beta^k (p_{t+k} - p_t + m c_{t+k}) \Rightarrow \\
v_t \sum_{k=0}^{\infty} \theta^k \beta^k &= \sum_{k=0}^{\infty} \theta^k \beta^k (p_{t+k} - p_t + m c_{t+k})
\end{aligned}$$

since:

$$0 < \theta^k \beta^k < 1 \Rightarrow$$

$$\sum_{k=0}^{\infty} \theta^k \beta^k = \frac{1}{1 - \theta\beta}$$

Thus, we can rewrite:

$$v_t \frac{1}{1 - \theta\beta} = \sum_{k=0}^{\infty} \theta^k \beta^k (p_{t+k} - p_t + mc_{t+k}) \Rightarrow$$

$$v_t \frac{1}{1 - \theta\beta} = \sum_{k=0}^{\infty} \theta^k \beta^k (p_{t+k} + mc_{t+k}) - \sum_{k=0}^{\infty} \theta^k \beta^k p_t \Rightarrow$$

$$v_t \frac{1}{1 - \theta\beta} = \sum_{k=0}^{\infty} \theta^k \beta^k (p_{t+k} + mc_{t+k}) - p_t \sum_{k=0}^{\infty} \theta^k \beta^k \Rightarrow$$

$$v_t \frac{1}{1 - \theta\beta} = \sum_{k=0}^{\infty} \theta^k \beta^k (p_{t+k} + mc_{t+k}) - p_t \frac{1}{1 - \theta\beta} \Rightarrow$$

$$v_t \frac{1}{1 - \theta\beta} + p_t \frac{1}{1 - \theta\beta} = \sum_{k=0}^{\infty} \theta^k \beta^k (p_{t+k} + mc_{t+k}) \Rightarrow$$

$$v_t + p_t = (1 - \theta\beta) \sum_{k=0}^{\infty} \theta^k \beta^k (p_{t+k} + mc_{t+k}) \quad (\text{Eq. A36})$$

Last expression can be rewritten as:

$$v_t + p_t = (1 - \theta\beta) (p_t + mc_t) + \theta\beta (p_{t+1} + v_{t+1}) \quad (\text{Eq. A37})$$

Now we demonstrate that expressions (Eq. A36) and (Eq. A37) are equivalent.

Indeed, taking equation (Eq. A37) at  $t - 1$ :

$$v_{t-1} + p_{t-1} = (1 - \theta\beta) (p_{t-1} + mc_{t-1}) + \theta\beta (p_t + v_t) \Rightarrow$$

$$-(v_{t-1} + p_{t-1}) + \theta\beta (p_t + v_t) = -(1 - \theta\beta) (p_{t-1} + mc_{t-1}) \Rightarrow$$

$$(p_t + v_t) - \frac{1}{\theta\beta} (v_{t-1} + p_{t-1}) = -\frac{(1 - \theta\beta)}{\theta\beta} (p_{t-1} + mc_{t-1})$$

defining:

$$\begin{aligned}l_t &= p_t + v_t \\u_{t-1} &= p_{t-1} + mc_{t-1} \\ \zeta &= \frac{1}{\theta\beta} \\ \xi &= -\frac{(1-\theta\beta)}{\theta\beta}\end{aligned}$$

we have:

$$\begin{aligned}l_t - \zeta l_{t-1} &= \xi u_{t-1} \Rightarrow \\ l_{t-1} &= \zeta^{-1} l_t - \xi \zeta^{-1} u_{t-1}\end{aligned}$$

lagging one period:

$$\begin{aligned}l_t &= \zeta^{-1} l_{t+1} - \xi \zeta^{-1} u_t \Rightarrow \\ (1 - \zeta^{-1} L^{-1}) l_t &= -\xi \zeta^{-1} u_t \\ \text{where } : \quad \zeta^{-1} &< 1 \\ \text{and } : \quad L &\text{ is the lag operator}\end{aligned}$$

thus:

$$\begin{aligned}l_t &= -\xi \zeta^{-1} \frac{1}{(1 - \zeta^{-1} L^{-1})} u_t \Rightarrow \\ l_t &= -\xi \frac{1}{\zeta} \sum_{k=0}^{\infty} \zeta^{-k} u_{t+k}\end{aligned}$$

substituting, we obtain expression (Eq. A36):

$$\begin{aligned}
p_t + v_t &= -\left(-\frac{(1-\theta\beta)}{\theta\beta}\right) (\theta\beta) \sum_{k=0}^{\infty} \theta^k \beta^k (p_{t+k} + mc_{t+k}) \Rightarrow \\
p_t + v_t &= (1-\theta\beta) \sum_{k=0}^{\infty} \theta^k \beta^k (p_{t+k} + mc_{t+k})
\end{aligned}$$

Thus, taking equation (Eq. A37):

$$\begin{aligned}
v_t + p_t &= (1-\theta\beta)(p_t + mc_t) + \theta\beta(p_{t+1} + v_{t+1}) \Rightarrow \\
v_t + p_t &= p_t + mc_t - \theta\beta p_t - \theta\beta mc_t + \theta\beta(p_{t+1} + v_{t+1}) \Rightarrow \\
v_t &= mc_t(1-\theta\beta) + \theta\beta(p_{t+1} - p_t + v_{t+1}) \Rightarrow \\
v_t &= mc_t(1-\theta\beta) + \theta\beta(\pi_{t+1} + v_{t+1}) \tag{Eq. A38}
\end{aligned}$$

putting expression (Eq. A35) into (Eq. A38) gives:

$$\begin{aligned}
\frac{\theta}{1-\theta}\pi_t &= mc_t(1-\theta\beta) + \theta\beta\left(\pi_{t+1} + \frac{\theta}{1-\theta}\pi_{t+1}\right) \Rightarrow \\
\frac{\theta}{1-\theta}\pi_t &= mc_t(1-\theta\beta) + \theta\beta\left(1 + \frac{\theta}{1-\theta}\right)\pi_{t+1} \Rightarrow \\
\frac{\theta}{1-\theta}\pi_t &= mc_t(1-\theta\beta) + \beta\frac{\theta}{1-\theta}\pi_{t+1} \Rightarrow \\
\pi_t &= \beta\pi_{t+1} + (1-\theta\beta)(1-\theta)\frac{1}{\theta}mc_t \Rightarrow
\end{aligned}$$

Moreover, starting from the production function:

$$Y_t(j) = K_t(j)^\alpha N_t(j)^{1-\alpha}$$

$$\begin{aligned}
\frac{\partial Y_t(j)}{\partial K_t(j)} &= R_t^k \Rightarrow \\
\alpha K_t(j)^\alpha N_t(j)^{1-\alpha} \frac{1}{K_t(j)} \lambda_t(j) &= R_t^k \Rightarrow \\
\alpha \frac{Y_t(j)}{K_t(j)} \lambda_t(j) &= R_t^k
\end{aligned}$$

since we know that:

$$MC_t = \lambda_t(j)$$

we can rewrite:

$$\alpha \frac{Y_t(j)}{K_t(j)} MC_t = R_t^k$$

log-linearizing the last expression:

$$\begin{aligned}
\alpha \frac{1}{K} MC y_t Y - \alpha \frac{Y}{K^2} MC k_t K + \alpha \frac{Y}{K} m c_t MC &= r_t^k R^k \Rightarrow \\
y_t - k_t + m c_t &= r_t^k \Rightarrow \\
m c_t &= -y_t + k_t + r_t^k \Rightarrow \\
-m c_t &= (y_t - k_t) - r_t^k
\end{aligned}$$

Equivalently, starting from the production function:

$$Y_t(j) = K_t(j)^\alpha N_t(j)^{1-\alpha}$$

$$\begin{aligned}
\frac{\partial Y_t(j)}{\partial N_t(j)} &= W_t \Rightarrow \\
(1-\alpha) K_t(j)^\alpha N_t(j)^{-\alpha} \lambda_t(j) &= W_t \Rightarrow \\
(1-\alpha) K_t(j)^\alpha N_t(j)^{1-\alpha} \lambda_t(j) &= N_t(j) W_t \Rightarrow \\
(1-\alpha) Y_t(j) \lambda_t(j) &= N_t(j) W_t
\end{aligned}$$

since we know that:

$$MC_t = \lambda_t(j)$$

we can rewrite:

$$(1 - \alpha) Y_t(j) MC_t = N_t(j) W_t$$

log-linearizing the last expression:

$$\begin{aligned} (1 - \alpha) MC y_t Y + (1 - \alpha) Y m c_t MC &= W n_t N + N w_t W \Rightarrow \\ y_t + m c_t &= n_t + w_t \Rightarrow \\ m c_t &= -y_t + n_t + w_t \Rightarrow \\ -m c_t &= (y_t - n_t) - w_t \end{aligned}$$

With this procedure we have obtained equations (Eq. 39), (Eq. 40) and (Eq. 41):

$$\begin{aligned} \pi_t &= \beta \{ \pi_{t+1} \} - \lambda_p \hat{\mu}_t^p \\ \text{where} &: \lambda_p = (1 - \beta\theta) (1 - \theta) \frac{1}{\theta} \\ \text{and} &: m c_t = \hat{\mu}_t^p \end{aligned}$$

knowing that:

$$\hat{\mu}_t^p = (y_t - n_t) - w_t$$

or, equivalently:

$$\hat{\mu}_t^p = (y_t - k_t) - r_t^k$$

The log-linearized aggregate production function (Eq. 42) is obtained from expression (Eq. 15):

$$\begin{aligned}
Y_t &= K_t^\alpha N_t^{1-\alpha} \Rightarrow \\
y_t Y &= \alpha K^{\alpha-1} N^{1-\alpha} k_t K + (1-\alpha) K^\alpha N^{-\alpha} n_t N \Rightarrow \\
y_t Y &= \alpha K^\alpha N^{1-\alpha} k_t + (1-\alpha) K^\alpha N^{1-\alpha} n_t \Rightarrow \\
y_t Y &= (K^\alpha N^{1-\alpha}) (\alpha k_t + (1-\alpha) n_t)
\end{aligned}$$

knowing that in steady state:

$$Y = K^\alpha N^{1-\alpha}$$

finally, we have:

$$y_t = (1-\alpha) n_t + \alpha k_t$$

### ***FISCAL POLICY.***

***AGGREGATE GOVERNMENT SPENDING.*** The log-linearized government budget constraint (Eq. 43) is obtained from expression (Eq. 19). From (Eq. 19):

$$P_t T_t + R_t^{-1} B_{t+1} = B_t + P_t G_t$$

dividing for  $P_t Y$ :

$$\begin{aligned}
T_t \frac{1}{Y} + R_t^{-1} \frac{B_{t+1}}{P_t} \frac{1}{Y} &= \frac{B_t}{P_t} \frac{1}{Y} + G_t \frac{1}{Y} \Rightarrow \\
\frac{T_t - T}{Y} - \frac{1}{R^2} \frac{B}{P} \frac{1}{Y} (R_t - R) + \frac{1}{R} \frac{1}{P} \frac{1}{Y} (B_{t+1} - B) - \frac{1}{R} \frac{B}{P^2} \frac{1}{Y} (P_t - P) \\
&= \frac{1}{P} (B_t - B) \frac{1}{Y} - \frac{B}{P^2} \frac{1}{Y} (P_t - P) + \frac{G_t - G}{Y} \Rightarrow
\end{aligned}$$



assuming steady state with zero debt:

$$\begin{aligned}\frac{T_t - T}{Y} + \beta \frac{B_{t+1}}{P} \frac{1}{Y} &= \frac{B_t}{P} \frac{1}{Y} + \frac{G_t - G}{Y} \Rightarrow \\ \beta \frac{B_{t+1}}{P} \frac{1}{Y} &= \frac{B_t}{P} \frac{1}{Y} + \frac{G_t - G}{Y} - \frac{T_t - T}{Y} \Rightarrow\end{aligned}$$

assuming a balanced primary budget:

$$P_{t-1} = P_t = P$$

we obtain:

$$\begin{aligned}b_{t+1} &= \frac{1}{\beta} (b_t + g_t - t_t) \\ \text{where } : \quad b_t &= \left( \frac{B_t}{P_{t-1}} \right) \frac{1}{Y} \\ g_t &= \frac{G_t - G}{Y} \\ t_t &= \frac{T_t - T}{Y}\end{aligned}$$

**MILITARY AND NON-MILITARY EXPENDITURES.** The log-linearized government budget constraint (Eq. 44) is obtained from expression (Eq. 22). From (Eq. 22):

$$\begin{aligned}P_t T_t + R_t^{-1} B_{t+1} &= B_t + P_t (NM_t + M_t) \Rightarrow \\ P_t T_t + R_t^{-1} B_{t+1} &= B_t + P_t NM_t + P_t M_t\end{aligned}$$

dividing for  $P_t Y$ :

$$T_t \frac{1}{Y} + R_t^{-1} \frac{B_{t+1}}{P_t} \frac{1}{Y} = \frac{B_t}{P_t} \frac{1}{Y} + NM_t \frac{1}{Y} + M_t \frac{1}{Y} \Rightarrow$$

$$\begin{aligned} & \frac{T_t - T}{Y} - \frac{1}{R^2} \frac{B}{P} \frac{1}{Y} (R_t - R) + \frac{1}{R} \frac{1}{P} \frac{1}{Y} (B_{t+1} - B) - \frac{1}{R} \frac{B}{P^2} \frac{1}{Y} (P_t - P) \\ = & \frac{1}{P} (B_t - B) \frac{1}{Y} - \frac{B}{P^2} \frac{1}{Y} (P_t - P) + \frac{M_t - M}{Y} + \frac{NM_t - NM}{Y} \Rightarrow \end{aligned}$$

assuming steady state with zero debt:

$$\begin{aligned} \frac{T_t - T}{Y} + \beta \frac{B_{t+1}}{P} \frac{1}{Y} &= \frac{B_t}{P} \frac{1}{Y} + \frac{M_t - M}{Y} + \frac{NM_t - NM}{Y} \Rightarrow \\ \beta \frac{B_{t+1}}{P} \frac{1}{Y} &= \frac{B_t}{P} \frac{1}{Y} + \frac{M_t - M}{Y} + \frac{NM_t - NM}{Y} - \frac{T_t - T}{Y} \Rightarrow \end{aligned}$$

assuming a balanced primary budget:

$$P_{t-1} = P_t = P$$

we obtain:

$$\begin{aligned} b_{t+1} &= \frac{1}{\beta} (b_t + nm_t + m_t - t_t) \\ \text{where } : \quad b_t &= \left( \frac{B_t}{P_{t-1}} \right) \frac{1}{Y} \\ m_t &= \frac{M_t - M}{Y} \\ nm_t &= \frac{NM_t - NM}{Y} \\ t_t &= \frac{T_t - T}{Y} \end{aligned}$$

**MARKET CLEARING.** Log-linearization of the market clearing condition of the final good around the steady state yields the following expression:

$$\begin{aligned}
Y_t &= C_t + I_t + G_t \Rightarrow \\
y_t Y &= c_t C + i_t I + (G_t - G) \Rightarrow \\
y_t \frac{Y}{\bar{Y}} &= c_t \frac{C}{\bar{Y}} + i_t \frac{I}{\bar{Y}} + (G_t - G) \frac{1}{\bar{Y}} \Rightarrow \\
y_t &= \gamma_c c_t + \gamma_i i_t + g_t \\
\text{since } : \quad \gamma_c &= \frac{C}{\bar{Y}} \\
\gamma_i &= \frac{I}{\bar{Y}} \\
g_t &= \frac{G_t - G}{\bar{Y}}
\end{aligned}$$

In the case of disaggregation of government component the log-linearized market clearing condition is given by:

$$\begin{aligned}
Y_t &= C_t + I_t + NM_t + M_t \Rightarrow \\
y_t Y &= c_t C + i_t I + (NM_t - NM) + (M_t - M) \Rightarrow \\
y_t \frac{Y}{\bar{Y}} &= c_t \frac{C}{\bar{Y}} + i_t \frac{I}{\bar{Y}} + (NM_t - NM) \frac{1}{\bar{Y}} + (M_t - M) \frac{1}{\bar{Y}} \Rightarrow \\
y_t &= \gamma_c c_t + \gamma_i i_t + nm_t + m_t \\
\text{since } : \quad nm_t &= \frac{NM_t - NM}{\bar{Y}} \\
m_t &= \frac{M_t - M}{\bar{Y}}
\end{aligned}$$

#### 4.10 APPENDIX C: NON-COMPETITIVE LABOR MARKET

In the present section we describe a model of wage determination that generate a log-linear aggregate equilibrium condition corresponding to (Eq. 37) in the text. Consider a model with a continuum of unions, each of which represents workers of a

certain type. Effective labour input hired by firm  $j$  is a CES function of the quantities of the different labour types employed:

$$N_t(j) = \left( \int_0^1 N_t(j, i)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} di \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}}$$

where  $\varepsilon_w$  is the elasticity of substitution across different types of households. The fraction of non-Ricardian and Ricardian consumers is uniformly distributed across worker types (and hence across unions). Each period, a typical union (say, representing worker of type  $z$ ) sets the wage for its workers in order to maximize the objective function:

$$\lambda \left[ \frac{1}{C_t^r(z)} W_t(z) N_t(z) - \frac{N_t^{1+\varphi}(z)}{1+\varphi} \right] + (1-\lambda) \left[ \frac{1}{C_t^o(z)} W_t(z) N_t(z) - \frac{N_t^{1+\varphi}(z)}{1+\varphi} \right]$$

subject to a labour demand schedule:

$$N_t(z) = \left( \frac{W_t(z)}{W_t} \right)^{-\varepsilon_w} N_t$$

We can write this maximization problem as:

$$\begin{aligned} \max_{W_t(z)} & \left\{ \begin{array}{l} \lambda \left[ \frac{1}{C_t^r(z)} W_t(z) N_t(z) - \frac{N_t^{1+\varphi}(z)}{1+\varphi} \right] \\ + (1-\lambda) \left[ \frac{1}{C_t^o(z)} W_t(z) N_t(z) - \frac{N_t^{1+\varphi}(z)}{1+\varphi} \right] \end{array} \right\} \\ \text{s.t.} & \quad N_t(z) = \left( \frac{W_t(z)}{W_t} \right)^{-\varepsilon_w} N_t \end{aligned}$$

thus:

$$\begin{aligned} \max_{W_t(z)} \left\{ \begin{aligned} & \lambda \left[ \frac{1}{C_t^r(z)} W_t(z) \left( \frac{W_t(z)}{W_t} \right)^{-\varepsilon_w} N_t - \frac{\left( \left( \frac{W_t(z)}{W_t} \right)^{-\varepsilon_w} N_t \right)^{1+\varphi}}{1+\varphi} \right] \\ & + (1-\lambda) \left[ \frac{1}{C_t^o(z)} W_t(z) \left( \frac{W_t(z)}{W_t} \right)^{-\varepsilon_w} N_t - \frac{\left( \left( \frac{W_t(z)}{W_t} \right)^{-\varepsilon_w} N_t \right)^{1+\varphi}}{1+\varphi} \right] \end{aligned} \right\} \Rightarrow \\ \max_{W_t(z)} \left\{ \begin{aligned} & \lambda \left[ \frac{1}{C_t^r(z)} (W_t(z))^{1-\varepsilon_w} W_t^{\varepsilon_w} N_t - \frac{(W_t(z))^{-\varepsilon_w(1+\varphi)} W_t^{\varepsilon_w(1+\varphi)} N_t^{1+\varphi}}{1+\varphi} \right] \\ & + (1-\lambda) \left[ \frac{1}{C_t^o(z)} (W_t(z))^{1-\varepsilon_w} W_t^{\varepsilon_w} N_t - \frac{(W_t(z))^{-\varepsilon_w(1+\varphi)} W_t^{\varepsilon_w(1+\varphi)} N_t^{1+\varphi}}{1+\varphi} \right] \end{aligned} \right\} \end{aligned}$$

Because consumption will generally differ between the two types of consumers, the union weighs labour income with their respective marginal utility of consumption (i.e.,  $\frac{1}{C_t^r}$  and  $\frac{1}{C_t^o}$ ). Notice that, in writing down the problem, we have assumed that the union takes into account the fact that firms allocate labour demand uniformly across different workers of type  $z$ , independently of their household type. It follows that, in the aggregate, we will have  $N_t^r = N_t^o = N_t$  for all  $t$ . The first order condition of this problem can be written as follows (after invoking symmetry, and thus dropping the  $z$  index):

$$\begin{aligned} \lambda \left[ \frac{\frac{1}{C_t^r(z)} (1-\varepsilon_w) (W_t(z))^{-\varepsilon_w} W_t^{\varepsilon_w} N_t}{+\frac{\varepsilon_w(1+\varphi)(W_t(z))^{-\varepsilon_w(1+\varphi)-1} W_t^{\varepsilon_w(1+\varphi)} N_t^{1+\varphi}}{1+\varphi}} \right] + (1-\lambda) \left[ \frac{\frac{1}{C_t^o(z)} (1-\varepsilon_w) (W_t(z))^{-\varepsilon_w} W_t^{\varepsilon_w} N_t}{+\frac{\varepsilon_w(1+\varphi)(W_t(z))^{-\varepsilon_w(1+\varphi)-1} W_t^{\varepsilon_w(1+\varphi)} N_t^{1+\varphi}}{1+\varphi}} \right] = 0 \\ \lambda \left[ \frac{\frac{1}{C_t^r(z)} (1-\varepsilon_w) (W_t(z))^{-\varepsilon_w} W_t^{\varepsilon_w} N_t}{+\varepsilon_w (W_t(z))^{-\varepsilon_w(1+\varphi)-1} W_t^{\varepsilon_w(1+\varphi)} N_t^{1+\varphi}} \right] + (1-\lambda) \left[ \frac{\frac{1}{C_t^o(z)} (1-\varepsilon_w) (W_t(z))^{-\varepsilon_w} W_t^{\varepsilon_w} N_t}{+\varepsilon_w (W_t(z))^{-\varepsilon_w(1+\varphi)-1} W_t^{\varepsilon_w(1+\varphi)} N_t^{1+\varphi}} \right] = 0 \Rightarrow \\ \lambda \left[ \frac{1}{C_t^r} (1-\varepsilon_w) N_t + \varepsilon_w \frac{1}{W_t} N_t^{1+\varphi} \right] + (1-\lambda) \left[ \frac{1}{C_t^o} (1-\varepsilon_w) N_t + \varepsilon_w \frac{1}{W_t} N_t^{1+\varphi} \right] = 0 \Rightarrow \\ \lambda \frac{1}{C_t^r} (1-\varepsilon_w) N_t + \lambda \varepsilon_w \frac{1}{W_t} N_t^{1+\varphi} + (1-\lambda) \frac{1}{C_t^o} (1-\varepsilon_w) N_t + (1-\lambda) \varepsilon_w \frac{1}{W_t} N_t^{1+\varphi} = 0 \Rightarrow \end{aligned}$$

$$\begin{aligned}
& \lambda \frac{1}{C_t^r} (1 - \varepsilon_w) N_t \frac{1}{\varepsilon_w} W_t \frac{1}{N_t^{1+\varphi}} + \lambda \varepsilon_w \frac{1}{W_t} N_t^{1+\varphi} \frac{1}{\varepsilon_w} W_t \frac{1}{N_t^{1+\varphi}} \\
& + (1 - \lambda) \frac{1}{C_t^o} (1 - \varepsilon_w) N_t \frac{1}{\varepsilon_w} W_t \frac{1}{N_t^{1+\varphi}} + (1 - \lambda) \varepsilon_w \frac{1}{W_t} N_t^{1+\varphi} \frac{1}{\varepsilon_w} W_t \frac{1}{N_t^{1+\varphi}} \\
= & 0 \Rightarrow
\end{aligned}$$

$$\begin{aligned}
& \lambda \frac{1}{C_t^r N_t^\varphi} \frac{1 - \varepsilon_w}{\varepsilon_w} W_t + \lambda + (1 - \lambda) \frac{1}{C_t^o N_t^\varphi} \frac{1 - \varepsilon_w}{\varepsilon_w} W_t + (1 - \lambda) = 0 \Rightarrow \\
& \lambda \frac{1}{C_t^r N_t^\varphi} \frac{1 - \varepsilon_w}{\varepsilon_w} W_t + (1 - \lambda) \frac{1}{C_t^o N_t^\varphi} \frac{1 - \varepsilon_w}{\varepsilon_w} W_t = -1 \Rightarrow \\
& \frac{1 - \varepsilon_w}{\varepsilon_w} W_t \left[ \frac{\lambda}{C_t^r N_t^\varphi} + \frac{1 - \lambda}{C_t^o N_t^\varphi} \right] = -1 \Rightarrow \\
& \left[ \frac{\lambda}{C_t^r N_t^\varphi} + \frac{1 - \lambda}{C_t^o N_t^\varphi} \right] W_t = \frac{\varepsilon_w}{\varepsilon_w - 1} \Rightarrow
\end{aligned}$$

$$\left[ \frac{\lambda}{MRS_t^r} + \frac{1 - \lambda}{MRS_t^o} \right] W_t = \mu^w \quad (\text{Eq. A39})$$

$$\text{where : } MRS_t^r = C_t^r N_t^\varphi$$

$$MRS_t^o = C_t^o N_t^\varphi$$

$$\mu^w = \frac{\varepsilon_w}{\varepsilon_w - 1}$$

knowing that:

$$\left[ \frac{\lambda}{MRS^r} + \frac{1 - \lambda}{MRS^o} \right] W = \mu^w$$

Log-linearizing expression (Eq. A39) and ignoring constant terms yields the wage schedule:

$$\begin{aligned}
-\frac{\lambda W}{(MRS^r)^2} mrs_t^r MRS^r - \frac{(1-\lambda)W}{(MRS^o)^2} mrs_t^o MRS^o + \left[ \frac{\lambda}{MRS^r} + \frac{1-\lambda}{MRS^o} \right] w_t W &= 0 \Rightarrow \\
-\frac{\lambda W}{MRS^r} mrs_t^r - \frac{(1-\lambda)W}{MRS^o} mrs_t^o + \left[ \frac{\lambda W}{MRS^r} + \frac{(1-\lambda)W}{MRS^o} \right] w_t &= 0 \Rightarrow \\
-\frac{\lambda W}{MRS^r} mrs_t^r - \frac{(1-\lambda)W}{MRS^o} mrs_t^o + \left[ \frac{\lambda W}{MRS^r} + \frac{(1-\lambda)W}{MRS^o} \right] w_t &= 0 \Rightarrow \\
-\frac{\lambda W}{MRS^r} mrs_t^r - \frac{(1-\lambda)W}{MRS^o} mrs_t^o + \mu^w w_t &= 0
\end{aligned}$$

since:

$$\begin{aligned}
\chi_r &= \frac{\lambda W}{MRS^r \mu^w} \\
\chi_o &= \frac{(1-\lambda)W}{MRS^o \mu^w}
\end{aligned}$$

we can rewrite:

$$w_t = \chi_r mrs_t^r + \chi_o mrs_t^o$$

where:

$$\begin{aligned}
MRS^r &= C^r N^\varphi \Rightarrow \\
mrs_t^r MRS^r &= N^\varphi c_t^r C^r + \varphi C^r N^{\varphi-1} n_t N \Rightarrow \\
mrs_t^r MRS^r &= C^r N^\varphi c_t^r + \varphi C^r N^\varphi n_t \Rightarrow \\
mrs_t^r MRS^r &= C^r N^\varphi (c_t^r + \varphi n_t) \Rightarrow \\
mrs_t^r &= c_t^r + \varphi n_t
\end{aligned}$$

and:

$$\begin{aligned}
MRS^o &= C^o N^\varphi \Rightarrow \\
mrs_t^o MRS^o &= N^\varphi c_t^o C^o + \varphi C^o N^{\varphi-1} n_t N \Rightarrow \\
mrs_t^o MRS^o &= C^o N^\varphi c_t^o + \varphi C^o N^\varphi n_t \Rightarrow \\
mrs_t^o MRS^o &= C^o N^\varphi (c_t^o + \varphi n_t) \Rightarrow \\
mrs_t^o &= c_t^o + \varphi n_t
\end{aligned}$$

thus:

$$\begin{aligned}
w_t &= \chi_r (c_t^r + \varphi n_t) + \chi_o (c_t^o + \varphi n_t) \Rightarrow \\
w_t &= \chi_r c_t^r + \chi_o c_t^o + \chi_r \varphi n_t + \chi_o \varphi n_t
\end{aligned}$$

since:

$$\tilde{c}_t = \chi_r c_t^r + \chi_o c_t^o$$

we can rewrite:

$$w_t = \tilde{c}_t + \varphi (\chi_r + \chi_o) n_t$$

We note that, to the extent that tax policy equates steady state consumption across household types (i.e.,  $C^r = C^o$ ) we will have:

$$MRS^r = MRS^o$$

and, hence:

$$\begin{aligned}
\chi_r &= \lambda \\
\chi_o &= 1 - \lambda
\end{aligned}$$



We can then rewrite the previous equilibrium condition as:

$$w_t = c_t + \varphi n_t$$

which corresponds to the equation (Eq. 37) in the text. Under the present scenario we assume that the wage mark-up  $\mu^w$  is sufficiently large (and the shocks sufficiently small) so that the conditions  $W_t > MRS_t^j = C^o N^\varphi$  for  $j = r, o$  are satisfied for all  $t$ . Both conditions guarantee that both type of households will be willing to meet firms' labour demand at the prevailing wage.

## References

- [1] Aiyagari, S., R., Christiano, L., & Eichenbaum, M. (1992) "The output, employment, and interest rate effects of government consumption", *Journal of Monetary Economics*, 30(1), 73-86.
- [2] Auerbach, A. (2000). "Formation of Fiscal Policy: the Experience of the Past Twenty-Five years". *Federal Reserve Bank of New York Economic Policy Review*, April 2000, 9-32.
- [3] Barro, R. (1981). "Output effects of government purchases", *Journal of Political Economy*, 89(6), 1115.
- [4] Baxter, M., & King, R., G. (1993). "Fiscal policy in general equilibrium", *American Economic Review*, 83(3), 315-334.
- [5] Beetsma, R., Giuliodori M., & Klaassen, F. (2006). 'Trade Spill-Overs of Fiscal Policy in the European Union: A Panel Analysis', *Economic Policy*, 21(48): 639-687.
- [6] Beetsma, R. (2008). 'A Survey of the Effects of Discretionary Policy', *Working paper, University of Amsterdam, Amsterdam School of Economics*.
- [7] Benetrix, A., & Lane, P., (2009). "Fiscal Shocks and the Real Exchange Rate". *IIIS Discussion Paper N. 286*.
- [8] Blanchard, O., & Perotti, R. (2002). "An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output". *Quarterly Journal of Economics*, 117, 1329-1368.
- [9] Burnside, C., Eichenbaum, M., & Fisher, j. (2004). "Fiscal shocks and their consequences", *Journal of Economic Theory*, 115, 89-117.
- [10] Brunner-meier, M. (2009). "Deciphering the Liquidity and Credit Crunch 2007-2008". *Journal of Economic Perspectives*, 23(1), 77-100.
- [11] Calvo, G., A. (1983). "Staggered Prices in a Utility-Maximizing Framework", *Journal of Monetary Economics*, 13, 383-98.

- [12] Campbell, J., & Mankiw, N. (1989). "Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence". *NBER Macroeconomics Annual 1989*, edited by O. J. Blanchard and S. Fischer. MIT Press, 185-216.
- [13] Canova, F., & Pappa, E. (2007). "Price Differentials in Monetary Unions: The Role of Fiscal Shocks". *Economic Journal*, 117: 713-737.
- [14] Castelnuovo, E. (2010). "Monetary Policy Shocks, Cholesky Identification, and DSGE models: An Empirical Investigation for the U.S", *mimeo*.
- [15] Cavallo, M. (2005). 'Government Employment Expenditure and the Effects of Fiscal Policy Shocks', *Working paper 16, Federal Reserve Bank of San Francisco*.
- [16] Cavallo, M. (2007). 'Government Consumption Expenditure and the Current Account', *Public Finance and Management*, 7, 73-115.
- [17] Clarida, R., Galí, J., & Gertler, M. (1999). "The Science of Monetary Policy: A New Keynesian Perspective". *Journal of Economic Literature*, 37, 1661-1707.
- [18] Enders, Z., Muller, G., & Scholl, A. (2008). "How Do Fiscal and Technology Shocks Affect Real Exchange Rates? New Evidence for the United States". *Center of Financial Studies*, N. 2008/22.
- [19] Fatás, A. & Mihov, I. (2001). "The Effects of Fiscal Policy on Consumption and Employment: Theory and Evidence", *INSEAD, mimeo*.
- [20] Fisher, J., & Peters, R. (2009). "Using Stock Returns to Identify Government Spending Shocks" *mimeo, Federal Reserve Bank of Chicago*.
- [21] Galí, J., López-Salido, D., & Vallés, J. (2007). "Understanding the Effects of Government Spending on Consumption", *Journal of the European Economic Association*, 5, 227-70.
- [22] Giordano, R., Momigliano S., Neri, S., & Perotti, R. (2007). 'The Effects of Fiscal Policy in Italy: Evidence from a VAR Model', *European Journal of Political Economy*, 23(3), 707-733.

- [23] Hall, R., E. (1986) "The Role of Consumption in Economic Fluctuations," in *Robert J. Gordon, ed. The American Business Cycle: Continuity and Change Chicago: NBER and The University of Chicago Press.*
- [24] Hemming, R., Michael, K., & Mahfouz, S. (2002). "The Effectiveness of Fiscal Policy in Stimulating Economic Activity-A Review of the Literature". *IMF Working Paper 02/208.*
- [25] Ilzetzki, E., Mendoza, E., & Vegh, C., A. (2009). "How big (small?) are fiscal multipliers?," *mimeo, University of Maryland.*
- [26] King, R., & Watson, M. (1996). "Money, Prices, Interest Rates and the Business Cycle". *Review of Economics and Statistics, 78, 35-53.*
- [27] Mankiw, N., G. (2000). 'The Savers-Spenders Theory of Fiscal Policy', *American Economic Review, 90, 120-125.*
- [28] Marcellino, M. (2006). "Some Stylized Facts on Non-Systematic Fiscal Policy in the Euro Area". *Journal of Macroeconomics, 28, 461-479.*
- [29] Monacelli, T., & Perotti, R. (2006). "Fiscal Policy, the Trade Balance, and the Real Exchange Rate: Implications for International Risk Sharing", *manuscript, IGIER, Università Bocconi.*
- [30] Monacelli, T., & Perotti, R. (2008). "Fiscal policy, wealth effects and markups", *National Bureau of Economic Research, Working Paper 14584.*
- [31] Monacelli, T., & Perotti, R. (2008). "Openness and the Sectoral Effects of Fiscal Policy". *Journal of the European Economic Association, 6 (2.3): 395-403*
- [32] Mountford, A., & Uhlig, H. (2005). "What Are the Effects of Fiscal Policy Shocks?". *SFB 649 Discussion Papers SFV649DP2005-039, Sonderforschungsbereich 649, Humboldt University, Berlin, Germany.*
- [33] Muller, G. (2008). "Understanding the Dynamic Effects of Government Spending on Foreign Trade". *Journal of International Money and Finance, 27 (3), 345-371.*

- [34] Perron, P. (1989). "The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis". *Econometrica*, Econometric Society, 57(6), 1361-1401,
- [35] Ramey, V., A., & Shapiro, M., D. (1998). "Costly capital reallocation and the effects of government spending", *Carnegie-Rochester Conference Series on Public Policy*, 48, 145-194.
- [36] Perotti, R. (2005). "Estimating the effects of fiscal policy in OECD countries". Proceedings, *Federal Reserve Bank of San Francisco*.
- [37] Ramey, V., A. (2008). "Identifying government spending shocks: it's all in the timing", *mimeo*.
- [38] Rotemberg, J., Woodford, M., (1997). "An Optimization Econometric Framework for the Evaluation of Monetary Policy". *NBER Macroeconomics Annual 1997*, edited by O. J. Blanchard and S. Fischer. MIT Press, pp. 297-346.
- [39] Rotemberg, J., & Woodford, M. (1999). "Interest Rate Rules in an Estimated Sticky Price Model". *Monetary Policy Rules*, edited by J.B. Taylor, University of Chicago Press and NBER.
- [40] Sims, C. (1988). "Identifying Policy Effects". Bryant, R.C. et al., *Empirical Macroeconomics for Interdependent Economies*, The Brookings Institutions, Washington, D.C, 305-321.
- [41] Sims, C., Stock, J., & Watson, M. (1990). "Inference in Linear Time Series Models with Some Unit Roots". *Econometrica*, 58, 113-44.
- [42] Taylor, J., B. (1993). "Discretion versus Policy Rules in Practice", *Carnegie-Rochester Conference Series on Public Policy*, 39, 195-214.
- [43] Uhlig, H., (2005). "What Are the Effects of Monetary Policy on Output? Results from An Agnostic Identification Procedure". *Journal of Monetary Economics*, 52 (2), 381-419.
- [44] Woodford, M. (2003). "Interest and Prices". Chap. 4, *Princeton University Press*.

## 5 *CONCLUSIONS*

In this thesis we analyse the impact of government expenditure on U.S. economy, with particular attention on the spending devolved to the defence sector. In the three chapters presented above, we have obtained several interesting results that can be summarized as follows. In chapter 1, we empirically test the so called military Keynesianism hypothesis taking into account two different aspects. First, our inferences are adjusted for a structural break exhibited by the data concerning fiscal and monetary variables. Second, we show that the results are sensitive to a sub sample choice. The estimated elasticity of government defence spending on output shows a lack of significance in the more recent years of the sample, indicating that the effect of government spending defence on output is very weak.

This result is confirmed in chapter 2 where our findings indicate a much larger positive effect of non-military spending on the economy with respect to the rise of resources devolved to military sector. In this chapter, we propose a New Keynesian DSGE model featuring limited asset market participation and taking into account a fiscal policy composed by civilian and military sectors. Bayesian estimates of the theoretical model provide consistent evidence that defence spending has a weaker effect on private consumption and wages with respect to civilian spending. In this chapter, we also show that an increase in total government spending and its components leads to a sustained rise in consumption and wages in the period 1954-1979, but has less important effects on these variables after 1982. Our empirical results confirm that this relevant change in the fiscal shocks can be related to the increase of agents' participation in asset market.

Finally, chapter 3 shows two relevant aspects concerning the impact of public spending on the economy. First, we analyse the "within" relationship of the different public spending components, finding a clear substitute effect in the resources devolved to defence sector with respect to the resources devolved to non-defence sec-

tor. The second aspect is related to the way of financing total public expenditure and its components. In particular, our results show that the military spending is financed by an increase in government budget deficit, contrary to the civilian spending case. Starting from these aspects we find two main results using a structural VAR method and a DSGE model simulation. On one side civilian spending has a positive effect on GDP and private consumption. On the other side, output and consumption respond negatively to an increase of military expenditure. As a consequence, the first result strengthens the new-Keynesian theoretical approach whereas the second finding seems to confirm the standard neoclassical wealth effect.

Overall, these results suggest that trying to measure the impact on private consumption by considering the whole government expenditure aggregate - and not its decomposition according to features and goals - can be misleading. Moreover, U.S. economy seems to have greater returns from non-military spending. Thus, giving government priorities in favour of supplying civilian goods and services rather than financing federal defence spending may be a good recovery plan after the severity of the last economic downturn followed the global financial crisis.

As further extensions of the present thesis, we believe interesting to focus in two directions. First, we would like to compare the three different approaches to the identification of fiscal policy shocks presented in VAR literature and well summarized in the paper of Perotti (2005). Apart from the identification issue, our analysis should show that the effects of public spending on the economy depends on the "within" substitutivity/complementarity of government spending components. It should be also interesting to assess the existence of an indirect and contrasting channel for the effects of specific government components of expenditure on private consumption.

A second aspect that we should investigate concerns a comparison between the effects of fiscal policies in U.S. and Euro area. As it is well known, the overwhelming majority of the studies are concerned with U.S. economy, with scarce evidence collected for other countries. Thus, focusing on the Euro area we could analyse the

similarities and the differences with the American economy.