DISCUSSION PAPER SERIES

DP13219
(v. 3)

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INTERNATIONAL TRADE AND REGIONAL ECONOMICS

ISSN 0265-8003

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Discussion Paper DP13219
First Published 05 October 2018
This Revision 30 May 2020

Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

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Should trade agreements also constrain domestic policies? We analyze this question from the perspective of heterogeneous-firm models with monopolistic-competition and multiple sectors. We propose a welfare decomposition based on principles from welfare economics to show that, in a very broad class of models, welfare changes induced by trade and domestic policies can be exactly decomposed into consumption-efficiency, production-efficiency and terms-of-trade effects. Using this decomposition, we compare trade agreements with different degrees of cooperation on domestic policies and show how their performance is affected by the interaction between firm heterogeneity and the relative importance of production efficiency versus terms-of-trade effects.

JEL Classification: F12, F13, F42

Keywords: Heterogeneous Firms, trade policy, Domestic Policy, Trade agreements, terms of trade, efficiency, Tariffs and Subsidies

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Acknowledgements
We thank Costas Arkolakis, Paola Conconi, Jan Haaland, Eckhard Janeba, Ahmad Lashkaripour, Monika Mrazova, Gonzague Vanoorenberghe, and seminar participants in Glasgow, Mannheim, Norwegian School of Economics, Nottingham, Paris School of Economics, Sciences Po, Southampton and conference participants at EEA, ETSG, the CRC TR 224 conference, the Midwest International Trade Conference, ESEM, the ITSG, and at the workshop on Trade Policy and Firm Performance for useful comments. Funding by the German Research Foundation (DFG) through CRC TR 224 is gratefully acknowledged.

Trade and Domestic Policies Under Monopolistic Competition*

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Abstract

Should trade agreements also constrain domestic policies? We analyze this question from the perspective of heterogeneous-firm models with monopolistic-competition and multiple sectors. We propose a welfare decomposition based on principles from welfare economics to show that, in a very broad class of models, welfare changes induced by trade and domestic policies can be exactly decomposed into consumption-efficiency, production-efficiency and terms-of-trade effects. Using this decomposition, we compare trade agreements with different degrees of cooperation on domestic policies and show how their performance is affected by the interaction between firm heterogeneity and the relative importance of production efficiency versus terms-of-trade effects.

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1 Introduction

We are witnessing a change in the way countries approach trade policy. In the past, regional and multilateral trade agreements were mostly ”shallow”, i.e. focused on the reduction of import tariffs and export taxes. More recently, there has been a shift to ”deeper” agreements, which, in addition to traditional trade policies, cover various domestic policies, such as production subsidies, product and labor standards, intellectual property rights, competition policy, and many other subjects (e.g., Horn, Mavroidis and Sapir, 2010; Dür, Baccini and Elsig, 2014; Rodrik, 2018).[1] Despite these fundamental changes in countries' actual approach to trade agreements, much of the theoretical literature still focuses on classical trade policies: import and export taxes (see Bagwell and Staiger, 2016, for a survey). Moreover, it misses a common tool to analyze the incentives for trade and domestic policies and to design deep trade agreements within the standard trade framework featuring monopolistic competition and firm heterogeneity.

To fill this gap, in this paper we develop a welfare decomposition for policy analysis based on efficiency principles from welfare economics that is valid in a broad class of trade models and allows us to jointly analyze the incentives for trade and domestic policies. Our approach is inspired by Arkolakis, Costinot and Rodríguez-Clare (2012) and Costinot, Rodríguez-Clare and Werning (2020) who showed, respectively, that the effects of trade cost reductions and trade policy have a common aggregate representation within a wide class of one-sector trade models. We use a general version of the modern workhorse trade model with monopolistic competition and free entry Krugman (1980), firms that are either homogeneous or heterogeneous in terms of productivity Melitz (2003) and operate potentially in mutiple sectors with CES demand. This model is particularly well suited for studying domestic policies (sector-specific production taxes/subsidies) and thus deep trade agreements because it features a clear motive for domestic regulation, even in the absence of international trade: without sector-specific production

[1] To illustrate the increasing depth and complexity of trade agreements, Rodrik (2018) compares the US trade agreements with Israel and Singapore, signed two decades apart. The US-Israel Free Trade Agreement, which went into force in 1985, was the first bilateral trade agreement the US concluded in the postwar period. It contains 22 articles and three annexes, the bulk of which are devoted to free-trade issues such as tariffs, agricultural restrictions, import licensing, and rules of origin. The US-Singapore Free Trade Agreement went into effect in 2004 and contains 20 chapters (each with many articles), more than a dozen annexes, and multiple side letters. Of its 20 chapters, only seven cover conventional trade topics. Other chapters deal with behind-the-border topics such anti-competitive business conduct, electronic commerce, labor, the environment, investment rules, financial services, and intellectual property rights.

subsidies, market outcomes are distorted by monopolistic price setting due to multiple sectors with different markups. At the same time, our setup allows us to study to what extent policies might be affected by the presence of firm heterogeneity.

We proceed as follows. We first derive a welfare decomposition and use it to tackle the long-standing theoretical debate on the motives for trade policy and trade agreements. A key advantage of the decomposition is that it allows us to identify beggar-thy-neighbor incentives[2] and to separate them clearly from efficiency considerations of policies. We then use our welfare decomposition to study the relative performance of trade agreements with different levels of integration: a *shallow trade agreement* (free trade but no coordination of domestic policies); a *deep trade agreement* (free trade and coordinated domestic policies); and a *laissez-faire agreement* (free trade and a commitment to abstain from using domestic policies). We find that achieving the full benefits of integration always requires signing a deep trade agreement and that firm heterogeneity crucially affects the costs and benefits of a shallow relative to a laissez-faire agreement.

The key idea of our approach is to rewrite the model in terms of aggregate CES bundles and to express welfare (indirect utility) changes induced by policy instruments in terms of the wedges between market prices and those that would implement the allocation chosen by a social planner. These efficiency wedges are present whenever consumer prices of aggregate bundles deviate from marginal production costs. In the context of our model, such wedges can be either due to monopolistic markups or due to policy distortions. In the spirit of Meade (1955) and Harberger (1971), we split these efficiency effects into consumption-efficiency (given by wedges between consumer and producer prices due to trade taxes) and production-efficiency terms (given by wedges between the marginal value product of labor at producer prices and its marginal cost). The general-equilibrium welfare effects induced by trade or domestic policies can then be exactly decomposed into (i) consumption-efficiency and (ii) production-efficiency effects and (iii) terms-of-trade effects that operate via changes in international prices.[3] Crucially, our welfare decomposition is valid independently of whether the set of available policy instruments

[2]A beggar-thy-neighbor or zero-sum incentive means that one player/country is made better off at the expense of the other player/country.

[3]The terms of trade are defined in terms of ideal price indices of exportables and importables. As a consequence, the terms of trade are affected both by changes in the international prices of individual varieties (intensive margin) and by changes in the set of firms active in foreign markets (extensive margin).

is sufficient to implement the planner allocation, which makes it well suited for studying policy in second-best environments.[4]

We show that the terms-of-trade motive is the only beggar-thy-neighbor incentive in our framework and thus the only reason to sign a trade agreement, thereby extending the result from the neoclassical framework (Grossman and Helpman, 1995; Bagwell and Staiger, 1999) to the "new" trade theory. This implies that the delocation effect[5] (Venables, 1987) is not a policy motive on its own. As discussed below, however, it can be at work when the set of available policy instruments is limited. Unilateral and strategic policies are governed by production-efficiency and terms-of-trade considerations, while consumption-efficiency effects are always an unwanted consequence of using trade policy instruments.[6]

To clarify the trade-off between production-efficiency and terms-of-trade effects associated with each policy instrument, the impact of firm heterogeneity in governing this trade off, as well as the role of delocation effects, we first consider unilateral deviations from the laissez-faire equilibrium in the two-sector CES framework with an outside good. When starting from this equilibrium, production efficiency can be improved with a small import tariff, a small export subsidy or a small production subsidy in the differentiated sector that triggers entry of firms at home and increases the amount of labor allocated to this sector. However, this comes at the expense of worsening the terms of trade via the extensive margin. When firms are homogeneous and sell to all markets, production-efficiency considerations always prevail and countries gain from such a policy. By contrast, when firms are heterogeneous and self-select into exporting, this is no longer the case. There exists a sufficient statistic, the variable profit share of the average active firm from sales in its domestic market, that determines which effect dominates. When the profit share from domestic sales is larger than the one from export sales, the result is qualitatively the same as in the case of homogeneous firms because the terms-of-trade motive is weak relative to the production-efficiency motive: only relatively few firms select into exporting

[4]A similar decomposition is also valid under perfect competition, see Helpman and Krugman (1989), Chapter 2. Our welfare decomposition can also be used for applications not considered in the current paper, such as unilaterally optimal policies or to study the welfare implications of other tax instruments, or changes in fundamentals, such as trade costs.

[5]In models with free entry, the delocation effect (also called home-market effect) of policies can be used to attract a larger share of a sector's production by increasing local demand for the good.

[6]In the one-sector version of the model, production is always efficient so that policy makers' choices are only driven by terms-of-trade considerations. This is not the case with multiple sectors and different elasticities across them.

and most profits are made in the domestic market. By contrast, when the profit share from domestic sales is smaller than the one from export sales, production efficiency is less important to policy makers, so the terms-of-trade motive dominates. Consequently, countries can benefit from a small unilateral import subsidy, a production tax, or an export tax in the differentiated sector that delocates firms to the foreign market (an anti-delocation effect).

With an understanding of the theoretical mechanisms that govern policy makers' incentives, we then study the normative implications of trade agreements with different degrees of integration.

In the absence of any type of trade agreement, individual-country policy makers are free to set both trade and domestic policies strategically. In this case, the targeting principle applies and strategic outcomes are qualitatively independent of firm heterogeneity: in a symmetric Nash equilibrium, production subsidies exactly offset monopolistic distortions, while trade policies consist of import subsidies and export taxes. Thus, Nash trade policies aim at delocating firms to the *other* economy in order to improve countries' terms of trade via the extensive margin. This result confirms the insight gained from our welfare decomposition: when policy makers have sufficiently many instruments to deal with production distortions and terms-of-trade effects separately, the terms-of-trade motive is the only international externality and thus the only reason to enter a trade agreement.

One may thus think that a shallow trade agreement – which forbids the use of trade policies but does not require any coordination of domestic policies – leads to an efficient outcome. We show that this is not the case: when the set of policy instruments is limited, policies are governed by the trade-off between improving production efficiency and manipulating the terms of trade, and therefore strategic domestic policies are not set efficiently. When firms are homogeneous, production-efficiency considerations always prevail and thus in the symmetric Nash equilibrium policy makers set a production subsidy, albeit smaller than the efficient one. When instead firms are heterogeneous, the relative importance of the two effects is endogenous. Like in the case of unilateral deviations, whether production-efficiency or terms-of-trade effects prevail depends on whether the profit share from domestic sales is larger than the one from export sales. When it is larger, the first effect dominates, and the Nash policy is an (inefficiently low) production subsidy. When it is smaller, the second effect dominates, and the Nash policy is a production tax. Reaping the full benefits from international trade thus requires a deep trade agreement, in which member countries cooperate on trade and domestic policies. Such an agreement is necessary and sufficient to achieve the first-best outcome: policymakers implement free trade by setting trade taxes to zero and use production subsidies to completely offset monopolistic distortions.

If, however, full cooperation on domestic policies is not feasible, countries may be able to commit not to use domestic policies at all. We thus consider a laissez-faire agreement, which forbids both the use of trade and domestic policies. We show that whether or not this dominates a shallow agreement when firms are heterogeneous depends on whether the profit share from domestic sales is smaller or larger than the one from export sales. Last but not least, we show that, due to endogenous selection into exporting, the average variable profit share from domestic sales is an increasing function of fixed and variable physical trade costs. When these costs fall sufficiently, shallow agreements are more distortive than laissez-faire agreements.

The rest of the paper is structured as follows. In the next subsection we briefly discuss the related literature. In Section 2 we describe a multi-sector Melitz (2003) model expressed in terms of macro bundles. In Section 3 we then set up and solve the problem of a planner who is concerned with maximizing world welfare. We separate it into a micro, a within-sector macro and a cross-sector macro stage and compare outcomes of each stage to the market allocation in order to identify the relevant efficiency wedges. We then show how the efficiency wedges are affected by policy instruments. In Section 4 we solve the problem of a benevolent world policy maker who is concerned with maximizing world welfare and disposes of sector-specific import, export and production taxes. As an intermediate step of solving this problem, we derive a welfare decomposition that decomposes welfare effects of policy instruments into a consumption-efficiency effect, a production-efficiency effect and terms-of-trade effects. We then show that for solving the world-policy-maker problem it is sufficient to set all efficiency wedges equal to zero. Next, we turn to the problem of individual-country policy makers, derive individual-country welfare and discuss welfare effects of unilateral policy deviations from the laissez-faire equilibrium (5). Finally, in section 6 we consider strategic trade and domestic policies. We first characterize the Nash equilibrium of the policy game where individual-country policy makers set both trade and production taxes simultaneously and strategically. We then turn to the Nash equilibrium of the policy game when only production taxes can be set strategically. Section 7 presents our conclusions.

1.1 Related literature

Several theoretical contributions have studied the incentives for trade policy in specific versions of the Krugman (1980) and Melitz (2003) models and have identified numerous mechanisms through which trade policy affects outcomes.[7] We add to the literature on trade policy in the CES monopolistic competion framework by incorporating domestic policies and showing that – since they all have the same aggregate representation – these models share a common set of policy motives, which can be understood using our welfare decomposition. Studies investigating trade policy in the two-sector version of Krugman (1980) with homogeneous firms typically find that strategic tariffs are set due to a delocation motive (Venables, 1987; Helpman and Krugman, 1989; Ossa, 2011) that induces policy makers to increase the size of the domestic differentiated sector. Recently, Campolmi, Fadinger and Forlati (2014) have shown that this result is a consequence of limiting the number of policy instruments. When policy makers dispose of production, import and export taxes, the Nash equilibrium is characterized by the first-best level of production subsidies, import subsidies and export taxes that aim at delocating firms to the foreign economy. This paper generalizes their results to heterogeneous firms and interprets their finding in terms of production-efficiency and terms-of-trade effects. Also closely related to our study is Costinot et al. (2020), who consider unilateral trade policy in a generalized two-country version of Melitz (2003) with a single sector. They study optimal firm-specific and non-discriminatory policies and investigate how optimal trade taxes are affected by firm heterogeneity, emphasizing the role of terms-of-trade effects.

[7] Gros (1987) and Helpman and Krugman (1989) examine the one-sector version of the Krugman (1980) model with homogeneous firms and identify a terms-of-trade motive for tariffs. Several studies have analyzed the incentives for trade policy in the Melitz (2003) model with a single sector. Demidova and Rodríguez-Clare (2009) and Haaland and Venables (2016) investigate optimal *unilateral* trade policy in a small-open-economy version of Melitz (2003) with Pareto-distributed productivity. While Demidova and Rodríguez-Clare (2009) identify a distortion in the relative price of imported varieties (markup distortion) and a distortion on the number of imported varieties (entry distortion) as motives for unilateral policy, Haaland and Venables (2016) single out terms-of-trade effects as the only reason for individual-country trade policy. Similarly, Felbermayr, Jung and Larch (2013), who consider strategic import taxes in a two-country version of this model, identify the same motives for tariffs as Demidova and Rodríguez-Clare (2009). Turning to models with firm heterogeneity, Haaland and Venables (2016) investigate unilateral policy in the two-sector small-open-economy variant of Melitz (2003) with Pareto-distributed productivities. They identify terms-of-trade externalities and monopolistic distortions as drivers of unilateral policy.

Our paper is also connected to the vast literature on trade policy in perfectly competitive trade models (Dixit, 1985). We show that many insights from this literature carry over to the framework with monopolistic competition and firm heterogeneity. Specifically, we find that the result that trade agreements solve a terms-of-trade externality, which has been forcefully argued by Grossman and Helpman (1995) and Bagwell and Staiger (1999), also applies in the CES monopolistic competition context.[8] Moreover, also the Bhagwati-Johnson principle of targeting, which states that optimal policy should use the instrument that operates most effectively on the appropriate margin, remains valid. Finally, our welfare decomposition establishes a tight link between the policy incentives in the CES monopolistic competition framework and those in the neoclassical model. Meade (1955) has developed a partial-equilibrium decomposition of welfare incentives in neoclassical trade models that splits welfare effects of policies into efficiency wedges and terms-of-trade effects. We show how to apply this decomposition to general-equilibrium welfare effects of policies in CES monopolistic competition models.

Finally, we also contribute to the literature on trade and domestic policies. Within a model with perfect competition and a local production externality, Copeland (1990) discusses the idea that in the presence of a shallow trade agreement – that limits the strategic use of tariffs – individual-country policy makers may use domestic policies to manipulate the terms of trade. Bagwell and Staiger (2001) use a similar model to study the gains from integrating agreements on domestic standards into trade agreements within a dynamic setup. They argue against integrating rules on domestic policies into trade agreements since in their view WTO rules are sufficient to sustain efficient levels of domestic policies: they prohibit changes in domestic policies that undo the effects of previously granted tariff concessions. Differently, Ederington (2001), who considers the optimal design of joint agreements on trade and domestic policies in the absence of commitment, establishes that deep trade agreements should require full coordination of domestic policies, while allowing countries to set positive levels of tariffs in order to reduce deviation incentives. While our paper abstracts from these dynamic considerations and considers a situation with full commitment, it is the first to study shallow and deep trade agreements within the workhorse heterogeneous-firm model. In a recent contribution, Lashkaripour and Lugovsky (2019) analyze a quantitative multi-sector Krugman (1980) model with trade policies and domestic production

[8] Maggi and Rodríguez-Clare (1998) show that in the presence of political-economy motives for protection, commitment to abstain from using distortive policies may be an additional reason to sign a trade agreement.

subsidies to assess welfare gains from a deep trade agreement relative to unilaterally optimal policies. Grossman, McCalman and Staiger (2019) investigate deep trade agreements with a focus on harmonization of horizontal production standards in a monopolistic competition model with homogeneous firms, while Ossa and Maggi (2019) consider a political-economy model of agreements on standards in the presence of consumption or production externalities.

2 The Model

The setup follows Melitz and Redding (2015). The world economy consists of two countries i: Home (H) and Foreign (F). The only factor of production is labor which is supplied inelastically in amount L in each country, perfectly mobile across firms and sectors and immobile across countries. Each country has either one or two sectors. The first sector produces a continuum of differentiated goods under monopolistic competition with free entry. If present, the other sector is perfectly competitive and produces a homogeneous good.[9] Labor markets are perfectly competitive. Differentiated goods are subject to iceberg transport costs. Both countries are identical in terms of preferences, production technology, market structure and size. All variables are indexed such that the first sub-index corresponds to the location of consumption and the second sub-index to the location of production.

2.1 Technology and Market Structure

2.1.1 Differentiated sector

Firms in the differentiated sector operate under monopolistic competition with free entry. They pay a fixed cost in terms of labor, f_E, to enter the market and to pick a draw of productivity φ from a cumulative distribution $G(\varphi)$.[10] After observing their productivity draw, they decide whether to pay a fixed cost f in terms of domestic labor to become active and produce for the domestic market. Active firms then decide whether to pay an additional market access cost f_X (in terms of domestic labor) to export to the other country, or to produce only for the domestic market. Therefore, labor demand of firm φ located in market i for a variety sold in market j is

[9] The generalization of the model to multiple monopolistically competitive sectors is straightforward.

[10] We assume that φ has support $[0, \infty)$ and that $G(\varphi)$ is continuously differentiable with derivative $g(\varphi)$.

given by:

$$l_{ji}(\varphi) = \frac{q_{ji}(\varphi)}{\varphi} + f_{ji}, \quad i, j = H, F \tag{1}$$

where

$$f_{ji} = \begin{cases} f & j = i \\ f_X & j \neq i \end{cases}$$

Here $q_{ji}(\varphi)$ is the production of a firm with productivity φ located in country i for market j. Varieties sold in the foreign market are subject to an iceberg transport cost $\tau > 1$. We thus define:

$$\tau_{ji} = \begin{cases} 1 & j = i \\ \tau & j \neq i \end{cases}$$

2.1.2 Homogenous sector

In case the homogeneous-good sector is present, labor demand L_{Zi} for the homogenous good Z, which is produced in both countries i with identical production technology, is given by:

$$L_{Zi} = Q_{Zi}, \tag{2}$$

where Q_{Zi} is the production of the homogeneous good. Since this good is sold in a perfectly competitive market without trade costs, its price is identical in both countries and equals the marginal cost of production W_i. We assume that the homogeneous good is always produced in both countries in equilibrium. This implies equalization of wages $W_i = W_j$ for $i \neq j$ (factor price equalization).

We also consider a version of the model without the homogeneous sector. In this case, wages across the two countries will not necessarily be equalized.

2.2 Preferences

Households' utility function is given by:

$$U_i \equiv \alpha \log C_i + (1 - \alpha) \log Z_i, \quad i = H, F, \tag{3}$$

where C_i aggregates over the varieties of differentiated goods and α is the expenditure share of the differentiated bundle in the aggregate consumption basket. When α is set to unity, we go back to a one-sector model (Melitz, 2003). Z_i represents consumption of the homogeneous good (Krugman, 1980). The differentiated varieties produced in the two countries are aggregated with a CES function given by:[11]

$$C_i = \left[\sum_{j=H,F} C_{ij}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad , \quad i = H, F \tag{4}$$

$$C_{ij} = \left[N_j \int_{\varphi_{ij}}^{\infty} c_{ij}(\varphi)^{\frac{\varepsilon-1}{\varepsilon}} dG(\varphi) \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad , \quad i, j = H, F \tag{5}$$

Here, C_{ij} is the aggregate consumption bundle of country-i consumers of varieties produced in country j, $c_{ij}(\varphi)$ is consumption by country-i consumers of a variety φ produced in country j, N_j is the measure of varieties produced by country j. φ_{ij} is the cutoff-productivity level, such that a country-j firm with this productivity level makes exactly zero profits when selling to country i, while firms with strictly larger productivity levels make positive profits from selling to this market, so that all country-j firms with $\varphi \geq \varphi_{ij}$ export to country i. Finally, $\varepsilon > 1$ is the elasticity of substitution between domestic and foreign bundles and between different varieties.

2.3 Government

The government of each country disposes of the following fiscal instruments: a sector-specific production tax/subsidy (τ_{Li}) on the fixed and marginal costs of firms in the differentiated sector,[12] a sector-specific tariff/subsidy on imports in the differentiated sector (τ_{Ii}) and a sector-specific tax/subsidy on exports in the differentiated sector (τ_{Xi}).[13] Note that τ_{mi} indicates a gross tax for $m \in \{L, I, X\}$, i.e., $\tau_{mi} < 1$ indicates a subsidy and $\tau_{mi} > 1$ indicates a tax.

[11] Notice that we can index consumption of differentiated varieties by firms' productivity level φ since all firms with a given level of φ behave identically. Note also that our definitions of C_{ij} imply $C_i = \left[N_i \int_{\varphi_{ii}}^{\infty} c_{ii}(\varphi)^{\frac{\varepsilon-1}{\varepsilon}} dG(\varphi) + N_j \int_{\varphi_{ij}}^{\infty} c_{ij}(\varphi)^{\frac{\varepsilon-1}{\varepsilon}} dG(\varphi) \right]^{\frac{\varepsilon}{\varepsilon-1}}$ i.e., the model is the standard one considered in the literature. However, it is convenient to define optimal consumption indices.

[12] Since the only production factor in the model is labor, this is equivalent to a sector-specific labor tax/subsidy. We impose that the same production tax is levied on both fixed and marginal costs (including also the fixed entry cost f_E). This assumption is necessary to keep firm size unaffected by production taxes, which turns out to be optimal, as we will show in Section 3.2.

[13] Note that we could easily allow for tax instruments in the perfectly competitive sector but these would be completely redundant. We do not explicitly introduce sector-specific consumption taxes/subsidies but they can be replicated with a combination of production subsidies and import tariffs.

In what follows, we employ the word *tax* whenever we refer to a policy instrument without specifying whether τ_{mi} is smaller or larger than one and we use the following notation:

$$\tau_{Tij} = \begin{cases} 1 & i = j \\ \tau_{Ii}\tau_{Xj} & i \neq j \end{cases} \tag{6}$$

Moreover, we assume that taxes are paid directly by the firms[14] and that all government revenues are redistributed to consumers through a lump-sum transfer. We use the term *laissez-faire allocation* to refer to the market allocation in which both countries refrain from using any of the policy instruments, i.e., $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$.

2.4 Equilibrium

Since the model is standard, we relegate a more detailed description of the setup and the derivation of the market equilibrium to Appendix A. In a market equilibrium, households choose consumption of the differentiated bundles and – when available –the homogeneous good in order to maximize utility subject to their budget constraint; firms in the differentiated sector choose quantities in order to maximize profits given their residual demand schedules and enter the differentiated sector until their expected profits – before productivity realizations are drawn – are zero; they produce for the domestic and export markets if their productivity draw is weakly above the market-specific survival-cutoff level at which they make exactly zero profits; if present, firms in the homogeneous-good sector price at marginal cost; governments run balanced budgets and all markets clear. Similarly to Arkolakis et al. (2012), Campolmi et al. (2014) and Costinot et al. (2020), we write the equilibrium in terms of sectoral aggregates. Specifically, the one-sector model can be represented in terms of three aggregate goods: a good that is domestically produced and consumed (non-tradable good); a domestic exportable good and a domestic importable good. The two-sector model additionally features a homogeneous good. This representation in terms of aggregate bundles (i) highlights that models with monopolistic competition and CES preferences have a common macro representation and (ii) makes the connection to standard neoclassical trade models visible. It will also be useful for interpreting

[14] In particular, following the previous literature (Venables (1987), Ossa (2011)), we assume that tariffs and export taxes are charged ad valorem on the factory gate price augmented by transport costs. This implies that transport services are taxed.

the wedges between the planner and the market allocations and for our welfare decomposition. Finally, the macro representation will make clear that the welfare-relevant terms of trade that policy makers consider in their objective are defined in terms of ideal price indices of sectoral exportables relative to importables.

The market equilibrium is described by the following conditions:

$$\widetilde{\varphi}_{ji} = \left[\int_{\varphi_{ji}}^{\infty} \varphi^{\varepsilon-1} \frac{dG(\varphi)}{1-G(\varphi_{ji})} \right]^{\frac{1}{\varepsilon-1}}, \quad i,j = H,F \tag{7}$$

$$\delta_{ji} = \frac{f_{ji}(1-G(\varphi_{ji}))\left(\frac{\widetilde{\varphi}_{ji}}{\varphi_{ji}}\right)^{\varepsilon-1}}{\sum\limits_{k=H,F} f_{ki}(1-G(\varphi_{ki}))\left(\frac{\widetilde{\varphi}_{ki}}{\varphi_{ki}}\right)^{\varepsilon-1}}, \quad i,j = H,F \tag{8}$$

$$\frac{\varphi_{ii}}{\varphi_{ij}} = \left(\frac{f_{ii}}{f_{ij}}\right)^{\frac{1}{\varepsilon-1}} \left(\frac{\tau_{Li}}{\tau_{Lj}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \left(\frac{W_i}{W_j}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tau_{ij}^{-1} \tau_{Tij}^{-\frac{\varepsilon}{\varepsilon-1}} \quad i = H,F, \quad i \neq j \tag{9}$$

$$\sum_{j=H,F} f_{ji}(1-G(\varphi_{ji}))\left(\frac{\widetilde{\varphi}_{ji}}{\varphi_{ji}}\right)^{\varepsilon-1} = f_E + \sum_{j=H,F} f_{ji}(1-G(\varphi_{ji})), \quad i = H,F \tag{10}$$

$$C_{ij} = \frac{\varepsilon-1}{\varepsilon} (\varepsilon f_{ij})^{\frac{-1}{\varepsilon-1}} \tau_{ij}^{-1} \varphi_{ij} (\delta_{ij} L_{Cj})^{\frac{\varepsilon}{\varepsilon-1}}, \quad i,j = H,F \tag{11}$$

$$P_{ij} = \frac{\varepsilon}{\varepsilon-1} (\varepsilon f_{ij})^{\frac{1}{\varepsilon-1}} \tau_{ij} \tau_{Tij} \tau_{Lj} W_j \varphi_{ij}^{-1} (\delta_{ij} L_{Cj})^{\frac{-1}{\varepsilon-1}}, \quad i,j = H,F \tag{12}$$

$$L - L_{Ci} - \frac{1-\alpha}{\alpha} \sum_{k=H,F} P_{ik} C_{ik} + \tau_{Ij}^{-1} P_{ji} C_{ji} = \tau_{Ii}^{-1} P_{ij} C_{ij}, \quad i = H, \quad j = F \tag{13}$$

$$\sum_{i=H,F} (L - L_{Ci}) = \frac{1-\alpha}{\alpha} \sum_{i=H,F} \sum_{j=H,F} P_{ij} C_{ij} \tag{14}$$

$$Z_i = \frac{1-\alpha}{\alpha} \sum_{j=H,F} P_{ij} C_{ij} \quad i = H,F \tag{15}$$

Condition (7) defines $\tilde{\varphi}_{ji}$, the average productivity of country-i firms active in market j, which is given by the harmonic mean of productivity of those firms that operate in the respective market. Condition (8) defines δ_{ji}, the variable-profit share of a country-i firm with average productivity $\tilde{\varphi}_{ji}$ arising from sales in market j – henceforth called *domestic profit share.*[15] Equivalently, δ_{ji} is also the share of total labor used in the differentiated sector in country i that is allocated to production for market j. Condition (9) follows from dividing the zero-profit conditions defining the survival-productivity cutoffs – which imply zero profits for a country-i firm with the cutoff-productivity level φ_{ij} from selling in market j – for firms in their domestic

[15]It can be shown that $f_{ji}(1-G(\varphi_{ji}))\left(\frac{\widetilde{\varphi}_{ji}}{\varphi_{ji}}\right)^{\varepsilon-1}$ are variable profits of a the average country-i firm active in market j.

market by the one for foreign firms that export to the domestic market. Condition (10) is the free-entry condition combined with the zero-profit conditions. In equilibrium, expected variable profits (left-hand side) have to equal the expected overall fixed cost bill (right-hand side).

Condition (11) can be interpreted as a sectoral aggregate production function $C_{ij} = Q_{Cij}(L_{Cj})$ in terms of aggregate labor allocated to the differentiated sector, L_{Cj}, measuring the amount of production of the aggregate bundle produced in country j for consumption in market i. Condition (12) defines the equilibrium consumer price index P_{ij} of the aggregate differentiated bundle produced in country j and sold in country i.[16]

Importantly, condition (13) defines the trade-balance condition that states that the value of net imports of the homogeneous good plus the value imports of the differentiated bundle (left-hand side) must equal the value of exports of the differentiated bundle (right-hand side). Note that imports and exports of differentiated bundles are evaluated at international prices (before tariffs are applied). The model-consistent definition of the terms of trade then follows directly from this equation.[17] The international price of imports $\tau_{Ii}^{-1} P_{ij}$ defines the inverse of the terms of trade of the differentiated importable bundle (relative to the homogeneous good), while the international price of exports $\tau_{Ij}^{-1} P_{ji}$ defines the terms of trade of the differentiated exportable bundle (relative to the homogeneous good). In addition, the terms of trade of the differentiated exportable relative to the importable bundle are given by $(\tau_{Ij}^{-1} P_{ji})/(\tau_{Ii}^{-1} P_{ij})$, which is the only relevant relative price when $\alpha = 1$. Given that terms of trade are defined in terms of sectoral ideal price indices of exportables relative to importables, they will be affected not only by changes in the prices of individual varieties but also by changes in the measure of exporters and importers and their average productivity levels. We will discuss this in detail in Section 5.

Finally, (14) is the market-clearing condition for the homogeneous good[18] and condition (15) defines demand for the homogeneous good, presented here for future reference.

We normalize the foreign wage, W_i, $i = F$, to unity. Thus, we have a system of 24 equilibrium equations in 25 unknowns, namely δ_{ji}, φ_{ji}, $\widetilde{\varphi}_{ji}$, C_{ji}, P_{ij}, L_{Ci}, Z_i for $i, j = H, F$ and W_i for

[16] More precisely, if $\alpha < 1$, P_{ij} should be interpreted as a relative aggregate price index in terms of the homogeneous good.

[17] This definition is also consistent with Campolmi, Fadinger and Forlati (2012) and Costinot et al. (2020), who also define terms of trade in terms of aggregate international price indices of exportables and importables.

[18] Alternatively if $\alpha = 1$ it states the domestic labor-market-clearing condition.

$i = H$. Note that if $\alpha < 1$, so that the homogeneous sector is present, $W_i = 1$ for $i = H$, since factor prices must be equalized in equilibrium; by contrast, if $\alpha = 1$, so that there is only a single sector, $L_{Ci} = L$ for $i = F$, since the domestic labor market must clear.

Observe that when $\alpha < 1$ the equilibrium can be solved recursively. First, we can use conditions (7), (9) and (10) to implicitly determine the four productivity cutoffs φ_{ji} as well as the average productivity levels in the domestic and export market for both countries $\tilde{\varphi}_{ji}$, given the values of the policy instruments. Then, we can determine the variable profit shares of the average firm in each market δ_{ji}, since they are a function of the productivity cutoffs only. Finally, given both φ_{ji} and δ_{ji}, we can recover C_{ji} and P_{ij} and plug them into the trade-balance condition and the homogeneous-good-market-clearing condition to solve for the equilibrium levels of L_{Cj}.

Moreover, note that under some additional assumptions the equilibrium equations also nest the one-sector and the multi-sector versions of the Krugman (1980) model with homogeneous firms and exogenous productivity level $\varphi = 1$. A sufficient set of assumptions is that $f_{ji} = 0$ for $i, j = H, F$ and that $G(\varphi)$ is degenerate at unity. In this case, conditions (7), (8) (9) and (10) need to be dropped from the set of equilibrium conditions and (11) and (12) are replaced by

$$C_{ij} = \frac{\varepsilon - 1}{\varepsilon} L_{Cj}^{\frac{\varepsilon}{\varepsilon-1}} (\varepsilon f_E)^{\frac{-1}{\varepsilon-1}} \frac{(\tau_{ij}\tau_{Tij})^{-\varepsilon} \left[\left(\frac{W_k}{W_j} \frac{\tau_{Lk}}{\tau_{Lj}} \right)^{\varepsilon} - \left(\frac{W_i}{W_j} \frac{\tau_{Li}}{\tau_{Lj}} \right)^{\varepsilon} \tau_{ki}^{\varepsilon-1} \tau_{Tki}^{\varepsilon} \right]}{\tau_{Tik}^{-\varepsilon} \tau_{ki}^{1-\varepsilon} - \tau_{Tki}^{\varepsilon} \tau_{ki}^{\varepsilon-1}}, \quad i, j = H, F, \quad k \neq i \tag{16}$$

$$P_{ij} = \frac{\varepsilon}{\varepsilon - 1} (\varepsilon f_E)^{\frac{1}{\varepsilon-1}} \tau_{ij} \tau_{Tij} \tau_{Lj} W_j L_{Cj}^{\frac{-1}{\varepsilon-1}}, \quad i, j = H, F. \tag{17}$$

The remaining equilibrium conditions (13)-(15) remain valid.

3 The Planner Allocation

In this section we solve the problem of a social planner who maximizes total world welfare[19] given the constraints imposed by the production technology in each sector and the aggregate labor resources available to each country. The solution to this problem provides a benchmark against which one can compare any market allocation. Moreover, and more importantly, it identifies the wedges that need to be closed in order to implement the planner allocation in a market equilibrium. As explained in Section 4, these wedges will exactly correspond to the

[19] World welfare is defined as the unweighted sum of individual countries' welfare. In this way we single out the symmetric Pareto-efficient allocation, which can then be compared with the symmetric market allocation.

ones in our welfare decomposition.

We solve the planner problem in three stages,[20] using the total-differential approach. First, we determine the amount of consumption and labor allocated to each variety of the differentiated good in each location. Next, we solve for the optimal average productivity of firms active in the domestic and export markets, the optimal allocation of consumption and labor across aggregate bundles within sectors, and the measure of differentiated varieties in each sector given the allocation across sectors. Finally, in the third stage we find the optimal allocation of consumption and labor across aggregate sectors. There are two main advantages in following this three-stage approach: (i) it highlights the various trade-offs that the planner faces at the micro and the macro level; (ii) it allows comparing the planner and the market allocation in a transparent way by pointing to the specific wedges arising in the market allocation at each level of aggregation.[21]

3.1 First Stage: Optimal Production of Individual Varieties

At the first stage the planner chooses $c_{ij}(\varphi)$, $l_{ij}(\varphi)$ and φ_{ij} for $i, j = H, F$ by solving the following problem:

$$\begin{aligned} & \max u_{ij} \\ s.t. \quad & c_{ij}(\varphi) = q_{ij}(l_{ij}(\varphi)), \quad i, j = H, F \\ & L_{Cij} = N_j \int_{\varphi_{ij}}^{\infty} l_{ij}(\varphi) dG(\varphi), \quad i, j = H, F, \end{aligned} \tag{18}$$

where $u_{ij} \equiv C_{ij}$, $C_{ij} = \left[N_j \int_{\varphi_{ij}}^{\infty} c_{ij}(\varphi)^{\frac{\varepsilon-1}{\varepsilon}} dG(\varphi) \right]^{\frac{\varepsilon}{\varepsilon-1}}$, $q_{ij}(l_{ij}(\varphi)) = (l_{ij}(\varphi) - f_{ij}) \frac{\varphi}{\tau_{ij}}$, and where N_j and L_{Cij} – defining the amount of labor allocated in country j to produce differentiated goods consumed by country i – are taken as given since they are determined at the second stage. The optimality conditions of the problem (18) imply the equalization of the marginal value product (measured in terms of marginal utility) between any two varieties φ_1 and $\varphi_2 \in [\varphi_{ij}, \infty)$.[22]

$$\frac{\partial u_{ij}}{\partial c_{ij}(\varphi_1)} \frac{\partial q_{ij}(\varphi_1)}{\partial l_{ij}(\varphi_1)} = \frac{\partial u_{ij}}{\partial c_{ij}(\varphi_2)} \frac{\partial q_{ij}(\varphi_2)}{\partial l_{ij}(\varphi_2)}, \quad i, j = H, F \tag{19}$$

[20] Our approach is similar to the one of Costinot et al. (2020) for the unilateral optimal-policy problem.

[21] The results of this Section are derived in Appendix C.

[22] Equivalently, this condition sets the marginal rate of substitution between any two varieties equal to their marginal rate of transformation.

The solution to this problem also determines the consumption of individual varieties $c_{ij}(\varphi)$, the amount of labor allocated to the production of each variety $l_{ij}(\varphi)$ and the optimal sectoral labor aggregator L_{Cij} and allows us to obtain the sectoral aggregate production function[23]

$$Q_{Cij}(\tilde{\varphi}_{ij}, N_i, L_{Cij}) \equiv \frac{\tilde{\varphi}_{ij}}{\tau_{ij}} \left\{ [N_j(1-G(\varphi_{ij})]^{\frac{1}{\varepsilon-1}} L_{Cij} - f_{ij}[N_j(1-G(\varphi_{ij}))]^{\frac{\varepsilon}{\varepsilon-1}} \right\}, \quad i,j = H, F. \tag{20}$$

3.2 Second Stage: Optimal Choice of Bundles Within Sectors

At the second stage, the planner chooses C_{ij}, L_{Cij}, N_i and $\widetilde{\varphi}_{ij}$ for $i,j = H, F$ in order to solve the following problem:

$$\max \sum_{i=H,F} u_i \tag{21}$$

$$s.t. \quad L_{Ci} = N_i f_E + \sum_{j=H,F} L_{Cji}, \quad i = H, F$$

$$C_{ij} = Q_{Cij}(\tilde{\varphi}_{ij}, N_i, L_{Cij}), \quad i,j = H, F,$$

where $u_i = \log C_i$, C_i is given by (4) and $Q_{Cij}(\tilde{\varphi}_{ij}, N_i, L_{Cij})$ is defined in (20). The first-order conditions of the above problem lead to the following conditions:

$$\frac{\partial u_i}{\partial C_{ii}} \frac{\partial Q_{Cii}}{\partial L_{Cii}} = \frac{\partial u_j}{\partial C_{ji}} \frac{\partial Q_{Cji}}{\partial L_{Cji}}, \quad i,j = H, F, \quad i \neq j \tag{22}$$

$$f_E = \sum_{j=H,F} \frac{\partial Q_{Cji}/\partial N_i}{\partial Q_{Cji}/\partial L_{Cji}}, \quad i = H, F \tag{23}$$

$$\frac{\partial Q_{Cji}}{\partial \tilde{\varphi}_{ji}} = 0, \quad i,j = H, F \tag{24}$$

Condition (22) states that the marginal value product of labor of the domestic non-tradable bundle (measured in terms of domestic marginal utility) has to equal the marginal value product of labor of the domestic exportable bundle (measured in terms of foreign marginal utility).[24]

Condition (23) captures the trade-off between the extensive and intensive margins of production. Creating an additional variety (firm) requires f_E units of labor in terms of entry cost. This additional variety marginally increases output of the non-tradable and the exportable bundles

[23]Note that condition (19) is also satisfied in the case of homogeneous firms. In this case equation (20) holds with $\tilde{\varphi}_{ij} = 1$, $(1 - G(\varphi_{ij})) = 1$ and $f_{ij} = 0$.

[24]Equivalently, this condition states that the marginal rate of substitution (in terms of home versus foreign utility) between the domestic nontradable bundle and the domestic exportable bundle has to equal the marginal rate of transformation of these bundles.

at the extensive margin but comes at the opportunity cost of reducing the amount of production of existing varieties (intensive margin), since aggregate labor has to be withdrawn from these production activities.

Condition (24) reveals the trade-off between increasing average productivity and reducing the number of active firms. From the aggregate production function (20), an increase in $\tilde{\varphi}_{ji}$ on the one hand increases sectoral production by making the average firm more productive, on the other hand it decreases sectoral production by reducing the measure of firms that are above the cutoff-productivity level φ_{ji}. At the margin, these two effects have to offset each other exactly.

By combining (20), (23) and (24), we obtain a sectoral production function for Q_{Cji} in terms of aggregate labor L_{Ci}:

$$Q_{Cji}(L_{Ci}) = \frac{\varepsilon - 1}{\varepsilon} \left(\varepsilon f_{ji}\right)^{\frac{-1}{\varepsilon-1}} \tau_{ji}^{-1} \varphi_{ji} \left(\delta_{ji} L_{Ci}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad i,j = H,F \tag{25}$$

3.3 Third Stage: Allocation Across Sectors

The third stage is present only in the case of multiple sectors ($\alpha < 1$). In this stage, the planner chooses C_{ij} and Z_i for $i,j = H,F$, and the amount of aggregate labor allocated to the differentiated sector L_{Ci} to solve the following maximization problem:[25]

$$\max \sum_{i=H,F} U_i \tag{26}$$

$$\begin{aligned} s.t. \quad C_{ij} &= Q_{Cij}(L_{Cj}), \quad i,j = H,F \\ Q_{Zi} &= Q_{Zi}(L - L_{Ci}), \quad i = H,F \\ \sum_{i=H,F} Q_{Zi} &= \sum_{i=H,F} Z_i, \end{aligned}$$

where U_i is given by (3) and (4), $Q_{Zi}(L - L_{Ci}) = L - L_{Ci}$ and $Q_{Cij}(L_{Cj})$ is defined in (25).

[25] We state the third stage of the planner problem as a choice between C_{ij} and Z_i (instead of a choice between C_i and Z_i) because this enables us to identify the efficiency wedges in the welfare decomposition, as will become clear below.

The first-order conditions of this problem are given by:

$$\frac{\partial U_i}{\partial Z_i} = \frac{\partial U_j}{\partial Z_j}, \quad i = H, j = F \tag{27}$$

$$\sum_{j=H,F} \frac{\partial U_j}{\partial C_{ji}} \frac{\partial Q_{Cji}}{\partial L_{Ci}} = -\frac{\partial U_i}{\partial Z_i} \frac{\partial Q_{Zi}}{\partial L_{Ci}}, \quad i = H, F \tag{28}$$

Condition (27) states that the marginal utility of the homogeneous good has to be equalized across countries, implying that $Z_i = Z_j$, so that there is no inter-industry trade due to symmetry. Since $\partial Q_{Zi}/\partial L_{Ci} = -\partial Q_{Zi}/\partial L_{Zi}$, condition (28) states that the social marginal value product of each country's aggregate labor,[26] (evaluated with the marginal utility of the consuming country), has to be equalized across sectors. Note that since the aggregate representation of the model does not depend on firm heterogeneity, the planner's optimality conditions for the third stage with homogeneous firms are identical to those above.

The following proposition characterizes the properties of the planner allocation.

Proposition 1 ***The planner allocation***

The planner allocation is unique and symmetric.

Proof See Appendix C.5. ■

3.4 Relationship between Planner and Market Allocation

We now discuss whether the optimality conditions of the planner problem are fulfilled in the market allocation. Observe that the optimality conditions of the first stage are satisfied in any market allocation and are independent of policy instruments. This implies that the relative production levels of individual varieties are optimal in any market allocation.

The second stage is more complicated. One can show that conditions (23)[27] and (24) are satisfied in any market allocation. Equation (25) also corresponds exactly to condition (11) for the market allocation. Thus, consumption of the aggregate differentiated bundles is efficient in any market allocation conditional on the cutoffs φ_{ji}[28] and the amount of aggregate labor

[26] By construction, aggregate labor already incorporates the optimal split of labor in the differentiated sector between the domestically produced and consumed and the exportable bundles.

[27] This condition is satisfied in any market allocation as long as the same production tax is charged on marginal and fixed costs.

[28] Remember that, from (7) and (8), δ_{ji} is pinned down once φ_{ji} has been chosen.

allocated to the differentiated sector L_{Ci}.[29] By contrast, even in a symmetric market allocation condition (22) is not satisfied, i.e., there is a wedge between the marginal value product of labor of the non-tradable bundle and the one of the exportable bundle (both evaluated in terms of marginal utility of the consuming country) whenever $\tau_{Tji} \neq 1$:[30]

$$\frac{\partial u_i}{\partial C_{ii}} \frac{\partial Q_{Cii}}{\partial L_{Cii}} = \frac{\partial u_j}{\partial C_{ji}} \frac{\partial Q_{Cji}}{\partial L_{Cji}} (\tau_{Tji})^{1-\varepsilon}, \quad i,j = H,F, \quad i \neq j. \tag{29}$$

Thus, a foreign tariff ($\tau_{Ij} > 1$) or a domestic export tax ($\tau_{Xi} > 1$) imply that the marginal value product on the right-hand side must increase relative to the one on the left-hand side, which happens when foreign consumers reduce imports or home consumers increase consumption of the domestically produced bundle. As a result, production and consumption is inefficiently tilted towards the domestically produced bundle. Note that condition (29) can be rewritten as (9) evaluated at the symmetric equilibrium. Consequently, the cutoff-productivity levels φ_{ii} and φ_{ji} are efficient whenever policymakers from both countries refrain from using trade taxes, i.e., when $\tau_{Tji} = 1$ for $i,j = H,F, \quad i \neq j$. If instead $\tau_{Tji} > 1$ for some country, then $\varphi_{ii}/\varphi_{ji}$ is too small compared to the efficient level, so that the marginal exporter is too productive relative to the least productive domestic producer.

Turning to the third stage, condition (27) is satisfied in a symmetric market allocation while condition (28) is in general violated. Specifically, in the market allocation the following condition holds:

$$\sum_{j=H,F} \frac{\partial U_j}{\partial C_{ji}} \frac{\partial Q_{Cji}}{\partial L_{Ci}} = -\Omega_{3Pi} \frac{\partial U_i}{\partial Z_i} \frac{\partial Q_{Zi}}{\partial L_{Ci}}, \quad i = H,F$$

where Ω_{3Pi} is the wedge between the planner and the market allocation. In a symmetric allocation Ω_{3Pi} can be expressed as $\Omega_{3Pi} = \frac{\varepsilon}{\varepsilon-1} \tau_{Li} \sum_{j=H,F} \tau_{Tij} \delta_{ij}, \quad i = H,F$:[31] The wedge reflects the fact that the monopolistic markup in the differentiated sector depresses its relative demand. It also indicates distortions induced by trade taxes and, potentially, domestic policies.

[29]Note that the planner's optimality conditions are also valid for the case of homogeneous firms with $\tilde{\varphi}_{ij} = 1$ exogenous, so that the first-order conditions are given by (22) and (23) only. Equation (25) also holds with $f_{ij} = f_E$, $\varphi_{ij} = 1$ and $\delta_{ji} = [1 + \tau_{ij}^{\varepsilon-1} (\frac{C_j}{C_i})^{\varepsilon-1}]^{\frac{1-\varepsilon}{\varepsilon}}$.

[30]Observe that this wedge is present independently of firm heterogeneity.

[31] In the Krugman model the wedge in a symmetric allocation is given by:

$$\Omega_{3Pi} = \tau_{Li} \frac{\varepsilon}{\varepsilon - 1} \sum_{j=H,F} (\tau_{Tij} \tau_{ij})^{1-\varepsilon} \frac{\left[1 - \tau_{ij}^{\varepsilon-1} \tau_{Tij}^{\varepsilon}\right]}{\left[\tau_{Tij}^{-\varepsilon} \tau_{ij}^{1-\varepsilon} - \tau_{Tij}^{\varepsilon} \tau_{ij}^{\varepsilon-1}\right]}$$

Thus, the allocation of labor across aggregate sectors is generally not efficient.

The next lemma summarizes all the conditions that need to be satisfied in order for the market allocation to coincide with the planner allocation.

Lemma 1 ***Relationship between the planner and the market allocation***
The market allocation coincides with the planner allocation if and only if in the market equilibrium:
(a)

$$\frac{\partial u_i}{\partial C_{ii}} \frac{\partial Q_{Cii}}{\partial L_{Cii}} = \frac{\partial u_j}{\partial C_{ji}} \frac{\partial Q_{Cji}}{\partial L_{Cji}}, \quad i, j = H, F \quad i \neq j \tag{30}$$

(b) and (for the multi-sector model only)

$$\frac{\partial U_i}{\partial Z_i} = \frac{\partial U_j}{\partial Z_j}, \quad i = H, \quad j = F \tag{31}$$

$$\sum_{j=H,F} \frac{\partial U_j}{\partial C_{ji}} \frac{\partial Q_{Cji}}{\partial L_{Ci}} = -\frac{\partial U_i}{\partial Z_i} \frac{\partial Q_{Zi}}{\partial L_{Ci}}, \quad i = H, F \tag{32}$$

Proof See Appendix C.6. ■

Lemma 1 implies that the market allocation is efficient whenever the social marginal value product of labor is equalized across markets within the differentiated sector as well as across sectors.

3.5 Efficiency Wedges in the Market Equilibrium

We now investigate the efficiency wedges in the market equilibrium induced by (i) monopolistic distortions, (ii) production taxes and (iii) trade policy at each side of the border in more detail. This will allow us to give a clearcut interpretation of the terms in the welfare decomposition in the next section. For this purpose it is useful to re-state the efficiency conditions of Lemma 1 in terms of efficiency wedges between consumer prices and aggregate marginal costs.

Proposition 2 ***Efficiency wedges***
Conditions (30), (31) *and* (32) *hold in the market equilibrium if and only if:*

(a) countries have the same level of income:

$$I_i = I_j \quad j \neq i, \tag{33}$$

(b) the consumer price indices of the differentiated importable bundles must correspond to the monopolistic markup over the aggregate marginal costs of the differentiated exportable bundles:

$$P_{ij} - \frac{\varepsilon}{\varepsilon - 1} \tau_{Lj} W_j \frac{\partial L_{Cij}}{\partial Q_{Cij}} = 0, \quad i = H, F, \quad j \neq i, \tag{34}$$

(c) and (for the multi-sector model only) the marginal value product of labor in the differentiated sector evaluated at producer prices must equal the price of labor:

$$\sum_{j=H,F} \tau_{Tji}^{-1} P_{ji} \frac{\partial Q_{Cji}}{\partial L_{Ci}} - W_i = 0, \qquad i = H, F, \tag{35}$$

Proof See Appendix C.7. ■

In general, conditions (34) and (35) are not satisfied in a market allocation. The corresponding conditions in the market equilibrium are:

$$P_{ij} = \tau_{Tij} \frac{\varepsilon}{\varepsilon - 1} \tau_{Lj} W_j \frac{\partial L_{Cij}}{\partial Q_{Cij}}, \;\; i, j = H, F \;\; j \neq i, \quad \sum_{j=H,F} \tau_{Tji}^{-1} P_{ji} \frac{\partial Q_{Cji}}{\partial L_{Ci}} = \frac{\varepsilon}{\varepsilon - 1} \tau_{Li} W_i, \;\; i = H, F \tag{36}$$

Note that the terms on the left-hand side of conditions (34) and (35) can be interpreted as efficiency wedges or distortions[32] The wedge in (34) measures the distortions induced by trade policies in the exportable and importable markets. It compares the market value of a marginal unit of output of the aggregate tradable bundles in their destination market with the (markup over the) marginal cost of producing them. The wedge in (35) measures the distortions induced by production taxes and monopolistic competition in the allocation of labor across aggregate sectors. It compares the marginal value product of labor measured at producer prices with its cost. This wedge is present only in the multi-sector model.

In the one-sector model, imposing $I_i = I_j$ for $j \neq i$ and (34) is necessary and sufficient to implement an efficient allocation. Comparing this with the corresponding condition in (36), this implies that τ_{Tij} must equal unity, while monopolistic markups charged on aggregate marginal costs do not impact on the allocation because they do not distort relative prices of tradables.

[32] This definition of distortions goes back at least to Harberger (1971).

By contrast, in the multi-sector model, both (34) and (35) must hold together with $I_i = I_j$ for $j \neq i$. Together, these conditions imply that consumer prices must equal aggregate marginal costs in all markets, i.e., $P_{ij} = W_j \frac{\partial L_{Cij}}{\partial Q_{Cij}}$ for $i, j = H, F$ and thus social marginal benefits have to equal to social marginal costs for all aggregate bundles.[33]

The wedges in (34) and (35) represent distortions from the global perspective, capturing the joint effects of trade and domestic policies of both countries. However, our analysis requires to identify to what extent these wedges are affected by: (a) decisions of both home and foreign policy makers; (b) the impact of individual policy instruments; (c) interaction effects between policy instruments. We thus decompose them accordingly.

Lemma 2 ***The efficiency-wedge decomposition***
The efficiency wedges in (34) *and* (35) *can be decomposed into domestic and foreign components as follows:*

$$P_{ij} - \frac{\varepsilon}{\varepsilon-1}\tau_{Lj} W_j \frac{\partial L_{Cij}}{\partial Q_{Cij}} = \underbrace{(\tau_{Ii}-1)}_{\substack{\text{domestic}\\ \text{consumption-efficiency}\\ \text{wedge, home tariff}}} \tau_{Ii}^{-1} P_{ij} + \underbrace{(\tau_{Xj}-1)\,\tau_{Tij}^{-1} P_{ij}}_{\substack{\text{domestic}\\ \text{consumption-effiency}\\ \text{wedge, foreign export tax}}} = \underbrace{\left(1-\tau_{Tij}^{-1}\right) P_{ij},}_{\substack{\text{domestic}\\ \text{consumption-efficiency}\\ \text{wedge}}} \qquad i = H, F \quad j \neq i \tag{37}$$

$$\sum_{j=H,F} \tau_{Tji}^{-1} P_{ji} \frac{\partial Q_{Cji}}{\partial L_{Ci}} - W_i = \underbrace{\frac{\varepsilon}{\varepsilon-1}\tau_{Li} - 1,}_{\substack{\text{domestic}\\ \text{production-efficiency}\\ \text{wedge}}} \qquad i = H, F \tag{38}$$

Moreover, the efficiency wedge induced by an export tax can be decomposed as:

$$(\tau_{Xi}-1)\,\tau_{Tji}^{-1} P_{ji} \frac{\partial Q_{Cji}}{\partial L_{Ci}} = \underbrace{(1-\tau_{Xi})}_{\substack{\text{domestic}\\ \text{consumption-efficiency}\\ \text{wedge, home export tax}}} P_{ii} \frac{\partial Q_{Cii}}{\partial L_{Ci}} + \underbrace{\left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi} - 1\right) - \left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li} - 1\right),}_{\substack{\text{domestic}\\ \text{production-efficiency}\\ \text{wedges, home export tax}}} \qquad i = H, F \quad j \neq i \tag{39}$$

Proof See Appendix C.8. ■

Condition (37) decomposes the wedge between the consumer price and the markup over the aggregate marginal production cost of the differentiated importable bundle into two components:

[33] Notice that in the market equilibrium $P_{ij} = \frac{\partial U_i}{\partial C_{ij}} I_i$ for $i, j = H, F$, where $I_i = \left(\frac{\partial U_i}{\partial I_i}\right)^{-1}$ is the inverse of the marginal utility of of income. Then condition (34) and (35) equate the marginal benefits of consuming an additional unit of the differentiated bundles to the marginal costs of producing them.

(i) the consumption-efficiency wedge induced by a domestic tariff, consisting of the difference between the domestic consumer price and the international price of the imported bundle due to the tariff;[34] (ii) the consumption-efficiency wedge induced by a foreign export tax, consisting of the difference between the international price and the foreign producer price of the importable bundle due to the foreign export tax. A domestic tariff, or a foreign export tax both reduce domestic imports of the foreign differentiated bundle inefficiently.

Condition (38) shows how the wedge between the marginal value product of aggregate labor in the domestic differentiated sector and the wage depends on the monopolistic markup and domestic production taxes. The domestic production-efficiency wedge is open whenever the monopolistic markup is not completely offset by a production subsidy.

Finally, condition (39) decomposes the distortions induced by an export tax. According to the left-hand side, an export tax induces a wedge between the marginal value product of labor in the differentiated exportable market evaluated at the international price and at the domestic producer price. The right-hand side of condition (39) decomposes this distortion into the sum of two wedges: First, the consumption-efficiency wedge induced by an export tax in the market of non-tradable differentiated goods. It implies a consumption distortion between the differentiated non-tradable bundle and the importable bundle. Second, a production-efficiency wedge induced by the domestic export tax τ_{Xi} on aggregate production in the differentiated sector.

Using condition (39) and summing the wedges in (37) and (38) we can then decompose the overall welfare effects of a small policy change.

Proposition 3 ***Global and individual-country efficiency effects of a policy change***

(a) The total efficiency effects of a small policy change can be decomposed as:

[34] In particular, $P_{ij}(1-\tau_{Ii}^{-1}) = \frac{\partial U_i / \partial C_{ij}}{\partial U_i / \partial I_i} - \tau_{Ii}^{-1} P_{ij}, \quad i = H, F \quad j \neq i$, and $P_{ij}(1-\tau_{Xj}^{-1})\tau_{Ii}^{-1} = \tau_{Ii}^{-1}(\frac{\partial U_i / \partial C_{ij}}{\partial U_i / \partial I_i} - \tau_{Xj}^{-1} P_{ij})$, $i = H, F \quad j \neq i$.

$$\sum_{i=H,F} dE_i \equiv \sum_{\substack{i=H,F\\ j\neq i}} \left[\left(P_{ij} - \frac{\varepsilon}{\varepsilon-1}\tau_{Lj} W_j \frac{\partial L_{Cij}}{\partial Q_{Cij}} \right) dC_{ij} + \left(\sum_{k=H,F} \tau_{Tki}^{-1} P_{ki} \frac{\partial Q_{Cki}}{\partial L_{Ci}} - 1 \right) dL_{Ci} \right]$$

$$= \sum_{\substack{i=H,F\\ j\neq i}} \underbrace{\left[\underbrace{(\tau_{Ii}-1)\tau_{Ii}^{-1} P_{ij} dC_{ij} + (1-\tau_{Xi}) P_{ii} dC_{ii}}_{\text{individual consumption-efficiency effect}} + \underbrace{\left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi} - 1 \right) dL_{Ci}}_{\text{individual production-efficiency effect}} \right]}_{dE_i} \tag{40}$$

$$= \underbrace{\sum_{\substack{i=H,F\\ j\neq i}} (\tau_{Tij}-1)\, \tau_{Tij}^{-1} P_{ij} dC_{ij}}_{\text{global consumption-efficiency effect}} + \underbrace{\sum_{i=H,F} \left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li} - 1 \right) dL_{Ci}}_{\text{global production-efficiency effect}} \tag{41}$$

Condition (40) *disentangles the efficiency effects of a small policy change in each country as a function of the distortions induced by its own policy instruments. Condition* (41) *expresses the same effects in terms of global distortions induced by trade and domestic policies of both countries.*

(b) The total effects of a small policy change are zero and the market allocation is efficient: (i) if each wedge on the right-hand side of condition (40) *is zero; (ii) if and only if $I_i = I_j$ for $j \neq i$ and each wedge on the right-hand side of condition* (41) *is zero.*

Proof See Appendix C.9.2. ■

Proposition 3 provides an overall measure of the efficiency effects caused by a small policy change from the individual-country and from the global perspective. Specifically, condition (40) breaks down the efficiency effects of policy according to the efficiency gains and distortions a country imposes on itself by using its own instruments and shows that in the aggregate some of the wedges of Lemma 2 cancel out. From the perspective of an individual country, both the use of a tariff and an export tax cause distortions whenever they are used. In the presence of a tariff ($\tau_{Ii} > 1$), the first consumption-efficiency wedge in (40) is positive. This implies that, in general, the consumption of importables is inefficiently low. Thus, an increase in C_{ij} ($dC_{ij} > 0$) improves efficiency. By contrast, in the presence of an export tax, the second consumption-effiency wedge in (40) is negative and then, everything else equal, the consumption of the domestically non-tradable bundle is inefficiently high. As a result, a reduction in C_{ii} potentially increases consumption efficiency. Finally, even in the absence of trade taxes, in the multi-sector model the production-efficiency wedge is positive due to the monopolistic markup, which implies that too little labor is employed in the aggregate differentiated sector. Hence, an increase in L_{Ci} ($dL_{Ci} > 0$) improves production efficiency. Closing the production-efficiency wedge requires a production subsidy equal to the inverse of the markup. At the same time, when $\tau_{Xi} > 1$ the production-efficiency wedge in (40) is also positive (unless τ_{Xi} is more than compensated by a production subsidy) because an export tax shifts labor out of the differentiated sector. Then, an increase in L_{Ci} ($dL_{Ci} > 0$) again improves efficiency. By contrast, in the one-sector model, $dL_{Ci} = 0$, so production-efficiency effects are absent, as the monopolistic markup does not induce any distortions in this case.

When instead considering the perspective of a global policy maker, who can control policy instruments of both countries, the relevant efficiency effects of policies are given by (41). Indeed, the global policy maker – who can set all policy instruments at once – realizes that what matters in terms of consumption efficiency is the difference between the price in the producer country and the one paid by consumers in the other country, i.e., what matters is τ_{Tij}.

4 Policy-Maker Problem and Welfare Decomposition

In the previous section we have derived the planner allocation and have compared it with the market allocation. We now study two policy problems: the one faced by a benevolent world policy maker and the one faced by individual-country policy makers. In doing so, we derive a welfare decomposition which identifies policymakers' incentives.

4.1 Policy and Welfare from the Global Perspective

We now solve the problem of the world policy maker who maximizes the sum of individual-country welfare and has all three policy instruments (production, import and export taxes in the differentiated sector) at her disposal. We show that, by solving the world-policy-maker problem using the total-differential approach, we obtain a welfare decomposition that: (i) incorporates all general-equilibrium effects of setting policy instruments under cooperation; (ii) and allows separating policy makers' incentives into efficiency effects and beggar-thy-neighbor motives.

The world policy maker sets domestic and foreign policy instruments τ_{Li}, τ_{Ii} and τ_{Xi} in order to solve the following problem:

$$\max_{\substack{\{\delta_{ji}, \varphi_{ji}, \widetilde{\varphi}_{ji}, C_{ji}, W_i \\ P_{ij}, L_{Ci}, \tau_{Li}, \tau_{Ii}, \tau_{Xi}\}_{i,j=H,F}}} \sum_{i=H,F} U_i \tag{42}$$

$$\textit{subject to conditions } (7)\text{-}(14).$$

where U_i is defined in (3), (4) and (15) with the additional restrictions that $\tau_{T,i}$ for $i = H, F$ is as defined in (6), $W_i = 1$ for $i = H, F$ if $\alpha < 1$ and that $W_F = 1$ and $L_{Ci} = L$ for $i = H, F$ if $\alpha = 1$.[35]

As a first step, solving the world policy maker problem using the total-differential approach involves taking total differentials of (42) and the equilibrium equations. We then substitute the total differentials of the trade balance and the other equilibrium equations into the objective function to obtain the following representation of world welfare changes induced by changes in one or several policy instruments:[36]

Proposition 4 ***Decomposition of world welfare***[37]
The total differential of world welfare in (42) *in response to domestic or foreign policy changes can be decomposed as follows:*

$$\sum_{i=H,F} dU_i = \sum_{i=H,F} \frac{dE_i}{I_i} + \underbrace{\sum_{\substack{i=H,F \\ j \neq i}} \frac{C_{ji} d(\tau_{Ij}^{-1} P_{ji}) - C_{ij} d(\tau_{Ii}^{-1} P_{ij})}{I_i}}_{\textit{terms-of-trade effect}} \tag{43}$$

which, if $I_i = I_j$, implies that

$$\sum_{i=H,F} dV_i = \sum_{i=H,F} dE_i \tag{44}$$

where $dV_i \equiv dU_i / \frac{\partial U_i}{\partial I_i}$, dE_i is defined in Proposition 3, and $I_i = W_i L + T_i$ is household income.

Proof See Appendix D.1. ■

[35] In the case of homogeneous firms, conditions (7)-(10) need to be dropped and (11)-(12) are replaced by (16) and (17).

[36] See Appendix B for an explanation how to solve constrained optimization problems using total differentials.

[37] A predecessor of this welfare decomposition can be found in Meade (1955) and in chapter 2 of Helpman and Krugman (1989).

In general, changes in world welfare induced by policies consist of the sum of efficiency effects and terms-of-trade effects in both countries. Changes in world welfare due to changes in policy instruments are given by the sum of the proportional differentials of individual countries' welfare and can be written as the sum of three terms: (i) a *consumption-efficiency effect*; (ii) a *production-efficiency effect* and (iii) *terms-of-trade effects*. We have already discussed the first two effects in detail, which correspond to efficiency wedges between the planner and the market allocation. The only additional incentive – not driven by efficiency considerations – is the *terms-of-trade effect* of policies. This term consists of the consumption of the differentiated exportable bundle times the differential of its international price minus the consumption of the differentiated importable bundle times the differential of its international price. An increase in the price of exportables raises domestic welfare and decreases welfare of the other country, while an increase in the price of importables has the opposite effects. The domestic and foreign terms-of-trade effects exactly compensate each other because they are beggar-thy-neighbor effects and make one country better off at the expense of the other. Consequently, the differential of world welfare consists exclusively of the consumption-efficiency and the production-efficiency terms. Note that the welfare decomposition is valid (i) both for changes in world welfare induced by changes in all policy instruments or just a subset of them and (ii) for the cases of heterogeneous and homogeneous firms. The next Proposition characterizes the optimal policies from the global perspective.

Proposition 5 ***Optimal world policies and Pareto efficiency***

(a) When production, import and export taxes are available in the differentiated sector, solving the world-policy-maker problem in (42) *by using the total-differential approach is equivalent to setting $I_i = I_j$ and the efficiency wedges in* (44) *individually equal to zero.*

(b) As a result, the world policy maker implements the planner allocation and the global policy is optimal if and only if:

(i) when $\alpha = 1$ (one-sector model): $\tau_{Tij} = \tau_{Ii}\tau_{Xj} = 1$, $\tau_{Ii} = \tau_{Ij}$ (or $\tau_{Xi} = \tau_{Xj}$) for $i = H, F$ and $j \neq i$.

(ii) when $\alpha < 1$ (multi-sector model): $\tau_{Tij} = \tau_{Ii}\tau_{Xj} = 1$, $\tau_{Ii} = \tau_{Ij}$ (or $\tau_{Xi} = \tau_{Xj}$) and $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ for $i = H, F$ and $j \neq i$.

Proof See Appendix D.2. ■

The global policy maker realizes that the distortion of a domestic import tariff can be completely offset with a foreign export subsidy, so that only $\tau_{Tij} = \tau_{Ii}\tau_{Xj}$ needs to be set to unity in order to avoid opening a consumption-efficiency wedge.[38] This guarantees that consumer prices indices of importable differentiated bundles are equal to the corresponding monopolistic markup over aggregate marginal costs (condition (34)). Thus, zero trade taxes are sufficient but not necessary to achieve consumption efficiency. Finally, the global policy maker can implement global production efficiency by setting the production subsidy in both countries equal to the inverse of the markup. This guarantees that the marginal value product of labor in the differentiated sector evaluated at producer prices equals the price of labor (condition (35)).

4.2 Policy and Welfare from the Individual-country Perspective

We now turn to the welfare incentives of policy makers that are concerned with maximizing the welfare of individual countries and have either all policy instruments (production and trade taxes in the differentiated sector) orjust a subset of them available.

The individual-country policy maker sets domestic policy instruments $\mathcal{T}_i \subseteq \{\tau_{Li}, \tau_{Ii}, \tau_{Xi}\}$ in order to solve the following problem:

$$\max_{\substack{\{\delta_{ji}, \varphi_{ji}, \widetilde{\varphi}_{ji}, C_{ji}, W_i \\ P_{ij}, L_{Ci}\}_{i,j=H,F}, \mathcal{T}_i}} \quad U_i \tag{45}$$

$$\textit{subject to conditions } (7)\text{-}(14).$$

where $\mathcal{T}_i \subseteq \{\tau_{Li}, \tau_{Ii}, \tau_{Xi}\}$ for $i = H, F$ and taking as given $\mathcal{T}_j \subseteq \{\tau_{Lj}, \tau_{Ij}, \tau_{Xj}\}$, with $j \neq i$. U_i is defined in (3), (4) and (15) with the additional restrictions that $\tau_{T,i}$ for $i = H, F$ is as defined in (6), $W_i = 1$ for $i = H, F$ if $\alpha < 1$ and that $W_F = 1$ and $L_{Ci} = L$ for $i = H, F$ if $\alpha = 1$.[39]

Again, as a first step for solving the individual-country policy maker problem given foreign

[38]Our formulation of the trade balance implies that tariffs are applied to the international value of exports (including export taxes). This implies that also the *level* of trade taxes needs to be identical across countries to avoid distortions. With the alternative assumption that tariffs are applied to the producer value of exports just the product of the tariff and the export tax needs to equal unity, e.g. Costinot et al. (2020).

[39]In the case of homogeneous firms, conditions (7)-(10) need to be dropped and (11)-(12) are replaced by (16) and (17).

policy instruments, we take total differentials of the objective function and the constraints and substitute them into the differential of the objective in order to obtain the welfare decomposition for individual countries. We will then use this decomposition to analyze unilateral deviations from the laissez-faire equilibrium in Section 5 and to solve for the Nash equilibria of the policy games implied by different institutional arrangements in Section 6.

Proposition 6 ***Decomposition of individual-country welfare*** *The total differential of individual-country welfare in* (45) *can be decomposed as follows:*

$$dV_i = dE_i + \underbrace{C_{ji}d(\tau_{Ij}^{-1}P_{ji}) - C_{ij}d(\tau_{Ii}^{-1}P_{ij})}_{\textit{domestic terms-of-trade effect}} =, \quad j \neq i \tag{46}$$

$$\underbrace{(1-\tau_{Xi})P_{ii}dC_{ii} + (\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}dC_{ij}}_{\substack{\textit{domestic}\\ \textit{consumption-efficiency effect}}} + \underbrace{\left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi}-1\right)dL_{Ci}}_{\substack{\textit{domestic}\\ \textit{production-efficiency effect}}} + \underbrace{C_{ji}d(\tau_{Ij}^{-1}P_{ji}) - C_{ij}d(\tau_{Ii}^{-1}P_{ij})}_{\substack{\textit{domestic}\\ \textit{terms-of-trade effect}}}, \quad j \neq i$$

where $dV_i \equiv dU_i/\frac{\partial U_i}{\partial I_i}$, dE_i *is defined in Proposition 3 and* $I_i = W_iL + T_i$ *is household income.*

Proof See Appendix D.3. ■

This welfare decomposition is again valid both with homogeneous and heterogeneous firms and independently of the number of policy instruments that the individual-country policy makers have at their disposal. Like the world policy maker, they care about domestic consumption efficiency and production efficiency. Moreover, unlike the world policy maker, they also take into account the terms-of-trade effects of their policy choice, as these are not zero.

From Proposition 5 (b) (i) we know that in the one-sector model the laissez-faire allocation is optimal from the perspective of the world policy maker. It is implementable from the perspective of the individual-country policy maker by abstaining from the use of trade taxes, which allows setting the domestic consumption-efficiency wedges of both countries equal to zero. However, from her perspective, this allocation is not optimal due to the presence of terms-of-trade effects. It thus follows that: first, any deviation from the laissez-faire allocation must be due to terms-of-trade effects; second, any such deviation is a *beggar-thy-neighbor* policy, defined as an increase in domestic welfare that is compensated by an equal fall in the foreign one, i.e., a zero-sum game, because foreign terms-of-trade effects equal the opposite of their domestic counterpart.[40]

[40]Costinot et al. (2020) also emphasize that in the one-sector heterogeneous-firm model terms-of-trade effects are the only externality driving the incentives of individual-country policy makers.

For the case of the multi-sector model, we know from Proposition 5 (b) (ii) that the world policy maker can achieve the planner allocation by setting a production subsidy in the differentiated sector while abstaining from the use of trade taxes. By contrast, the individual-country policy maker not only has the objective of achieving domestic production efficiency but also tries to manipulate domestic terms of trade in her favor. Thus, her incentives to deviate from the laissez-faire equilibrium are due to a combination of production-efficiency and terms-of-trade incentives.

The following Corollary summarizes these observations:

Corollary 1 ***Individual-country incentives***

(a) In the one-sector model, deviations of individual-country policy makers from the laissez-faire equilibrium are exclusively due to terms-of-trade effects.

(b) In the multi-sector model, individual-country policy makers' deviations from the laissez-faire equilibrium are driven by terms-of-trade and production-efficiency effects.

(c) Terms-of-trade effects are the only beggar-thy-neighbor effects.

Thus, the welfare decomposition in (46) provides a common framework for analyzing the incentives for trade and domestic policies in a general CES monopolistic competition setup. It remains valid independently of the presence of firm heterogeneity and of the set of policy instruments available to the individual-country policy maker. It clearly identifies the terms-of-trade effect as the only beggar-thy-neighbor incentive arising in this class of models and thus as the only reason to sign a trade agreement. This insight extends the result of the neoclassical framework (Bagwell and Staiger, 1999) to the "new" trade theory. Importantly, it also implies that the delocation effect (Venables, 1987) is not a policy motive on its own. We will investigate this in more detail in the next section.

5 How Policy Instruments affect the Terms of Trade and Production Efficiency

Before studying strategic policies in Section 6, we first analyze how unilateral policy choices affect the terms of trade and the efficiency wedges and thereby the welfare of individual countries.

We are particularly interested in explaining the different micro channels through which policy instruments impact on the terms of trade and production efficiency. As mentioned previously, the terms of trade can be influenced both through changes in the international prices of individual exportable and importable varieties and through changes in the measure of exporters and importers.

Note that, when starting from a symmetric allocation, the impact of a unilateral policy change on the terms of trade can be written as:

$$C_{ij}[d(\tau_{Ij}^{-1}P_{ji}) - d(\tau_{Ii}^{-1}P_{ij})] = \tag{47}$$

$$\tau_{Ii}^{-1}P_{ij}C_{ij}\left[\frac{d\tau_{Li}}{\tau_{Li}} + \frac{d\tau_{Xi}}{\tau_{Xi}} + \underbrace{\left(\frac{dW_i}{W_i} - \frac{dW_j}{W_j}\right)}_{(i)} + \frac{1}{\varepsilon-1}\left(\underbrace{\frac{dL_{Cj}}{L_{Cj}} - \frac{dL_{Ci}}{L_{Ci}}}_{(ii)} + \underbrace{\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}}}_{(iii)}\right) + \underbrace{\left(\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}}\right)}_{(iv)}\right],$$

where deviations are defined as $dX_i/X_i = \frac{\partial X_i}{\partial \tau_{mi}}\frac{1}{X_i}d\tau_{mi}$. We discuss the impact of tariffs (i.e.,$d\tau_{Ii} > 0, d\tau_{Li} = d\tau_{Xi} = 0$) in more detail and then provide results for the other instruments. A domestic tariff influences the terms of trade (i) by changing the relative wage; (ii) by affecting the amount of labor allocated to the differentiated sector in both countries; (iii) by impacting on the average variable profit share of domestic and foreign firms in their respective export markets; (iv) by moving the cutoff productivity levels of domestic and foreign exporters. Here, (i) corresponds to a a change in the price of individual varieties, while (ii)-(iii) correpond to changes in the measure of exportables and importables. Finally, (iv), the change in the cutoff productivity levels, impacts both on the average price of individual varieties and the measure of domestic and foreign exporters.[41] In particular, an increase in the domestic relative wage raises the price of exported varieties relative to imported ones and improves the terms of trade. By contrast, an increase in the amount of labor allocated to the domestic differentiated sector worsens the terms of trade by reducing the price index of exportables via an increase in the number of varieties, while an increase in foreign labor in this sector improves them by reducing the price index of importables. Domestic terms of trade worsen with an increment in the profit share of domestic firms from exports and improve in the corresponding share of foreign firms by

[41] An alternative decomposition splits the price index of exportables and importables into an extensive margin $[N_i(1 - G(\varphi_{ji}))]^{\frac{1}{1-\varepsilon}} = \left(\frac{\delta_{ji}L_{Ci}}{\varepsilon f_{ji}}\right)^{\frac{1}{\varepsilon-1}}\left(\frac{\varphi_{ji}}{\tilde{\varphi}_{ji}}\right)$ and an intensive margin $\tau_{Ij}^{-1}p_{ji}(\tilde{\varphi}_{ji}) = \frac{\varepsilon}{\varepsilon-1}(\tau_{ji}\tau_{Xi}\tau_{Li})\left(\frac{W_i}{\tilde{\varphi}_{ji}}\right)$. Thus, δ_{ji} and L_{Ci} only impact on the extensive margin, and W_i only impacts on the intensive margin, while φ_{ji} affects both margins.

changing the measure of firms that export to each market. Finally, an increase in the domestic cutoff-productivity level for exports worsens the terms of trade both by making the average exportable variety cheaper and by affecting the measure of exporters, whereas an increase in the foreign productivity cutoff has the opposite effect.

We first discuss the impact of a small unilateral tariff (i.e., $d\tau_{Li} = d\tau_{Xi} = 0$) in the one-sector model (i.e., $dL_{Cj} = dL_{Ci} = 0$), starting from the laissez-faire equilibrium. In the presence of a single sector, the terms-of-trade effects of a small tariff are *positive* and given by[42]

$$P_{ij}C_{ij}\left[\underbrace{\left(\frac{dW_i}{W_i} - \frac{dW_j}{W_j}\right)}_{(i)>0} + (\varepsilon - 1)^{-1}\underbrace{\left(\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}}\right)}_{(iii)>0} + \underbrace{\left(\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}}\right)}_{(iv)<0}\right] > 0. \tag{48}$$

A tariff raises home's demand for domestically produced varieties and thus, ceteris paribus, home firms' profits and the demand for domestic labor. Since labor supply is completely inelastic in this model, home's relative wage needs to adjust upward in response ((i)>0), thereby reducing equilibrium profits of domestic firms. Moreover, the increase in relative domestic income increases the share of profit firms from both countries make in home's domestic market, which improves home's terms of trade via the extensive margin by reducing the measure of domestic exporters and increasing the measure of foreign exporters ((iii)>0). Finally, the increase in the relative domestic wage leads to tougher selection into exporting at home and less selection in the other country, which negatively impacts on home's terms of trade ((iv)<0). In the absence of firm heterogeneity, the tariff exclusively raises home's relative wage. Firm heterogeneity leads to two additional and opposing effects: if heterogeneity mostly affects the profit share from exports, terms of trade respond more to tariffs compared to the case of homogeneous firms; by contrast, if selection effects are large, firm heterogeneity tends to reduce the response of the terms of trade by reducing the average price of exported varieties relative to the one of imported varieties. Note also that in the one-sector model production efficiency is always guaranteed, so the only incentive to deviate from the laissez-faire equilibrium is the positive terms-of-trade effect of the tariff.

Finally, we consider the impact of a unilateral export tax. Differently from a tariff, an export

[42] In Appendix E, Lemma 8, we sign the contribution of each component to the terms-of-trade effect for the one-sector model.

tax increases the international price of individual varieties directly but reduces the demand for domestic labor and thus home's relative wage. In the presence of homogeneous firms, the first effect dominates, leading to a terms-of-trade improvement. By contrast, with heterogeneous firms the direct increase in the international price of individual varieties is completely offset by a drop in home's relative wage, while the impact on domestic and foreign export profit shares and export cutoffs is symmetric, so that these effects compensate each other. Thus, the total terms-of-trade-effect of an export tax is zero. The following Lemma summarizes these results.

Lemma 3 ***Unilateral deviations from laissez-faire in one-sector model***

Consider a marginal unilateral increase in each trade policy instrument at a time, starting from the laissez-faire equilibrium, i.e., with $\tau_{Ii} = \tau_{Xi} = 1$ and $\tau_{Li} = 1$ for $i = H, F$. Then:

(a) the production-efficiency effect is zero for all policy instruments.

(b) the consumption-efficiency effect is zero for all policy instruments.

(c) the terms-of-trade effect is positive for τ_{Ii}, positive for τ_{Xi} when firms are homogeneous and zero for τ_{Xi} when firms are heterogeneous.

(d) the total welfare effect is positive for τ_{Ii}, positive for τ_{Xi} when firms are homogeneous and zero for τ_{Xi} when firms are heterogeneous.

Proof See Appendix E.3. ■

We now turn to the multi-sector model (i.e., $dW_i = dW_j = 0$). In this case the terms-of-trade effect of a small tariff starting from the laissez-faire equilibrium is *negative* and given by:[43]

$$P_{ij}C_{ij}\left[(\varepsilon-1)^{-1}\underbrace{\left(\frac{dL_{Cj}}{L_{Cj}}-\frac{dL_{Ci}}{L_{Ci}}\right)}_{(ii)<0}+(\varepsilon-1)^{-1}\underbrace{\left(\frac{d\delta_{ij}}{\delta_{ij}}-\frac{d\delta_{ji}}{\delta_{ji}}\right)}_{(iii)<0\Leftrightarrow\delta_{ii}>1/2}+\underbrace{\left(\frac{d\varphi_{ij}}{\varphi_{ij}}-\frac{d\varphi_{ji}}{\varphi_{ji}}\right)}_{(iv)>0\Leftrightarrow\delta_{ii}>1/2}\right]<0 \qquad (49)$$

A small tariff increases home's demand for domestically produced varieties and thus, ceteris paribus, the profits of home firms and the demand for domestic labor. Since wages are pinned down by the linear outside sector and workers can freely move across sectors, labor supply is perfectly elastic. Therefore, home labor in the differentiated sector surges in response to the

[43] In Appendix E, Lemma 8, we sign the contribution of each component to the terms-of-trade effect for the multi-sector model.

increase in labor demand, raising the measure of domestic firms and reducing their equilibrium profits. At the same time, foreign firms experience a drop in demand and profits, leading to a reduction in foreign labor employed in the differentiated sector. These effects impact negatively on home's terms of trade via the extensive margin ((ii)$<$ 0). Moreover, in the presence of heterogeneous firms there are two additional effects, the sign of which depends on whether firms make the larger share of profits in their domestic ($\delta_{ii} > 1/2$) or in their export market ($\delta_{ii} < 1/2$).[44]

In the first case, the tariff increases the profit share of home firms and decreases the profit share of foreign firm made in their respective export markets, which worsens home's terms of trade ((iii)$<$ 0) (more home exporters and less foreign exporters). In addition, the tariff leads to less stringent selection into exporting at home and more selection in the other country, which positively impacts on home's terms of trade ((iv)$>$ 0). When $\delta_{ii} < 1/2$ the signs of the last two effects switch, but the overall terms-of-trade effect of a small tariff deviation from the laissez-faire equilibrium remains negative.

Because the tariff increases the amount of labor allocated to the differentiated sector, it induces a positive production-efficiency effect when starting from the laissez-faire allocation and thus creates a trade-off between increasing production efficiency and worsening the terms of trade. Which of the two effects dominates in welfare terms depends again on δ_{ii}: when δ_{ii} is larger than one half, so that the domestic market is more important in terms of profits, production-efficiency effects are dominant. Intuitively, when firms sell mostly to their domestic market, welfare gains from improving the terms of trade are relatively small and policy makers care mostly about domestic production efficiency.

Analogous results hold for export and production taxes: they improve domestic terms of trade by shifting labor away from the differentiated sector, which simultaneously worsens domestic production efficiency. Again, the total welfare effect depends on the magnitude of δ_{ii}. The

[44] As made clear by Lemma 7 in Appendix E.1 if we impose the standard assumption $f_{ji} > f_{ii}\tau_{ij}^{1-\varepsilon}$ then in the laissez-faire allocation the export cutoff φ_{ji} for $j \neq i$ must be larger than the domestic survival cutoff φ_{ii} and also δ_{ii} is always larger than 1/2. In general, in the presence of trade taxes, at a symmetric allocation exporters might still be more productive than firms serving only the domestic market even when $f_{ji} \leq f_{ii}\tau_{ij}^{1-\varepsilon}$. In this case $\varphi_{ji} > \varphi_{ii}$ as long as $f_{ji} > f_{ii}\tau_{ij}^{1-\varepsilon}\tau_{Tij}^{-\varepsilon}$. By contrast, when this condition is not satisfied $\delta_{ii} < 1/2$ is possible. In general, at a symmetric allocation exporters are not necessarily more productive than firms serving only the domestic market even when $f_{ji} > f_{ii}\tau_{ij}^{1-\varepsilon}$. When τ_{Tij} is close to zero (high import or export subsidies) the export cutoff φ_{ji} for $j \neq i$ might smaller than the domestic survival cutoff φ_{ii} .

following Lemma summarizes these findings.

Lemma 4 ***Unilateral deviations from laissez-faire in multi-sector model***

Consider a marginal unilateral increase in each policy instrument at a time starting from the laissez-faire equilibrium, i.e., with $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ *for* $i = H, F$. *Then:*

(a) the production-efficiency effect is positive for τ_{Ii} *and negative for* τ_{Xi} *and* τ_{Li}.

(b) the consumption-efficiency effect is zero for all policy instruments.

(c) the terms-of-trade effect is negative for τ_{Ii} *and positive for* τ_{Xi} *and* τ_{Li}.

(d) the total welfare effect is positive for τ_{Ii} *and negative for* τ_{Xi} *and* τ_{Li} *if and only if* $1/2 < \delta_{ii} < 1$ *or when firms are homogeneous.*

Proof See Appendix E.4. ■

To summarize, the direction in which a particular policy instrument moves the terms of trade depends crucially on the elasticity of labor supply. Moreover, when considering unilateral deviations in the multi-sector model, the qualitative impact of policy instruments on welfare also depends on firm heterogeneity. In particular, whether individual-country policy makers benefit from a unilateral tariff or an import subsidy depends on the profit share from domestic relative to export sales (analogous statements hold for the other policy instruments). This results makes clear that policy makers may exploit the delocation effect to increase production efficiency when the set of policy instruments is limited and the positive production-efficiency effect of the policy dominates its negative terms-of-trade effect.[45] As we show next, when strategic policy makers have sufficiently many instruments to address production-efficiency and terms-of-trade effects separately, they always optimally set trade policy instruments in a way that shifts firms to the other economy (an anti-delocation effect) in order to improve the terms of trade.

[45] This is the case in Ossa (2011), who considers the homogeneous-firm model and an import tariff in the differentiated sector as the only available policy instrument.

6 The Design of Trade Agreements in the Presence of Domestic Policies

After having analyzed individual-country incentives to set taxes in the absence of retaliation, we now move to strategic policies in order to study how trade agreements should be designed. We know from the planner problem that in the one-sector model the laissez-faire allocation is efficient. We thus focus here on the more interesting case of the two-sector model in which production in the laissez-faire equilibrium is inefficient, so that there exists a motive for domestic policy intervention even in the absence of international trade.

6.1 Trade and Domestic Policies in the Absence of a Trade Agreement

We first consider a situation without any type of agreement, so that individual-country policy makers can set both trade and domestic policies non-cooperatively. We thus allow domestic policies τ_{Li} and trade policies τ_{Ii}, τ_{Xi}, for $i = H, F$ to be set strategically and simultaneously by the policy makers of both countries. Individual-country policy makers solve the problem described in (45). Similarly to the world policy maker problem, the welfare decomposition in (46) holds independently of the number of instruments at the disposal of the individual-country policy maker and corresponds to the policy maker's objective. After substituting additional equilibrium conditions, it can be grouped into three terms that are all individually equal to zero at the optimum. Proposition 7 states this more formally and characterizes the symmetric Nash equilibrium of this policy game.

Proposition 7 ***Strategic trade and domestic policies***

When production, import and export taxes are available in the differentiated sector,

(a) it is possible to rewrite (46) *as follows:*

$$dV_i = [\Omega_{Cii} dC_{ii} + \Omega_{Cij} dC_{ij} + \Omega_{LCi} dL_{Ci}] \tag{50}$$

where $dV_i \equiv dU_i / \frac{\partial U_i}{\partial I_i}$ *and the wedges* Ω_{Cii}, Ω_{Cij} *and* Ω_{LCi} *are defined in Appendix F.1.*

(b) *Solving the individual-country policy maker problem stated in* (45) *by using the total-differential approach requires setting* $\Omega_{Cii} = \Omega_{Cij} = \Omega_{LCi} = 0$.

(c) *As a result, any symmetric Nash equilibrium in the two-sector model with heterogeneous firms when both countries can simultaneously set all policy instruments entails the first-best level of production subsidies, and inefficient import subsidies and export taxes in the differentiated sector. Formally,*

$$\tau_L^N = \tfrac{\varepsilon-1}{\varepsilon},\ \tau_I^N < 1 \text{ and } \tau_X^N > 1.$$

Proof See Appendix F.1 ■

Our welfare decomposition allows us to interpret the Nash policy outcome stated in Proposition 7. Domestic policies are set efficiently even under strategic interaction and do not cause any beggar-thy-neighbor effects, while trade policy instruments are set with the intention to manipulate the terms of trade. As made clear in Section 5, an import subsidy or an export tax both aim at improving the terms of trade by delocating firms to the other economy. Because there are two international relative prices (the one of the differentiated exportable bundle and the one of the differentiated importable bundle relative to the homogeneous good) two trade-policy instruments are necessary to target both.

The result that production subsidies are set so as to completely offset monopolistic distortions is an application of the targeting principle in public economics (Dixit, 1985). It states that an externality or distortion is best countered with a tax instrument that acts directly on the appropriate margin. If the policy maker disposes of sufficiently many instruments to deal with each incentive separately, she uses the production subsidy to address production efficiency. The trade policy instruments are instead used to exploit the terms-of-trade effect, which is the only remaining incentive according to our welfare decomposition. Consequently, even in the presence of domestic policies, terms-of-trade externalities remain the only motive for signing a trade agreement.

Proposition 7 extends the result of Campolmi et al. (2014) – who find that in the two-sector model with homogeneous firms strategic trade policy consists of first-best wage subsidies and inefficient import subsidies and export taxes – to the case of heterogeneous firms. This implies that firm heterogeneity neither adds further motives for signing a trade agreement beyond the classical terms-of-trade effect nor changes the qualitative results (import subsidies and export taxes in the differentiated sector) of the equilibrium outcome compared to the case with homogeneous firms. This finding is different from the unilateral case (Lemma 4), where the sign of the welfare impact of unilateral deviations in individual instruments depends on the domestic profit share. However, firm heterogeneity does play a role for the qualitative effects of strategic policy when the set of policy instruments is limited, as we show in Section 6.2 below.

Our finding that domestic policies do not introduce new motives for trade policy coordination is closely related to the conclusion of Bagwell and Staiger (1999) and Bagwell and Staiger (2016), who uncover that in a large class of perfectly competitive trade models terms-of-trade motives are the only reason for signing a trade agreement. Proposition 7 extends their result (i) to models with monopolistic competition and heterogeneous firms; and (ii) the presence of domestic policies.

6.2 A Deep Trade Agreement – Globally Efficient Trade and Domestic Policies

Proposition 7 implies that some type of trade agreement is necessary to prevent countries from trying to exploit the terms-of-trade effects of their policies. Thus, the question arises how to design such an agreement and how much cooperation is necessary to achieve a globally efficient outcome. Indeed, a deep trade agreement, which requires full coordination of trade and domestic policies, is sufficient to implement the symmetric Pareto-optimal allocation. In fact, this result is implied directly by Propostion 5.

Corollary 2 ***Efficiency of a deep trade agreement***

In the two-sector model, a deep trade agreement with cooperation on trade and domestic policies implements the symmetric Pareto-efficient allocation by forbidding the use of trade policy instruments ($\tau_{Ii} = 1$ and $\tau_{Xi} = 1$ for $i = H, F$) and setting production subsidies in both countries equal to the inverse of the monopolistic markup ($\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ for $i = H, F$).[46]

[46] In principle, countries could alternatively continue to use tariffs and export taxes as long as they agree to set $\tau_{Tij} = 1$ and $\tau_{Ii} = \tau_{Ij}$ and $\tau_{Xi} = \tau_{Xj}$ for i,j=H,F. Since this is not very practical, we focus on zero trade taxes. In the one-sector model the laissez-faire allocation is Pareto optimal and individual-country policy makers' only incentive is to manipulate their terms of trade. Thus, the use of any type of policy instruments (both trade and

Thus, a deep trade agreement is sufficient to achieve global efficiency. But is it also necessary to achieve it, or would a shallow trade agreement, which only forbids the use of trade-policy instruments but requires no coordination of domestic policies, achieve a similarly efficient outcome?

6.3 A Shallow Trade Agreement – Strategic Domestic Policies

We now consider a shallow trade agreement that implements free trade but allows individual-country policy makers to set domestic policies strategically.[47] For simplicity, we focus on the case of full trade liberalization (tariffs and export taxes are set to zero), as required, e.g., by a regional trade agreement under Article 24 of GATT-WTO, but one can also think of a multilateral agreement that prevents countries from using trade instruments strategically, such as GATT-WTO.[48] In the case where only domestic policies can be set strategically, individual-country policy makers face a missing-instrument problem and consequently a trade-off between increasing production efficiency (calling for a production subsidy) and improving the terms of trade (calling for a production tax). Thus, it is ex ante unclear which motive dominates in equilibrium and one has to characterize the Nash-equilibrium policies.

Proposition 8 ***Strategic domestic policies in the presence of a shallow trade agreement***

When only production taxes in the differentiated sector are available,

(a) it is possible to rewrite (46) *as follows:*

$$dV_i = \Omega_i dL_{Ci} \tag{51}$$

production taxes) should be restricted by a trade agreement.

[47] GATT-WTO does not regulate domestic policies to the extent that they do not imply outright discrimination of foreign goods. GATT Article III (1): The contracting parties recognize that internal taxes and other internal charges, and laws, regulations and requirements affecting the internal sale, offering for sale, purchase, transportation, distribution or use of products, and internal quantitative regulations requiring the mixture, processing or use of products in specified amounts or proportions, should not be applied to imported or domestic products so as to afford protection to domestic production. (...) (8 b) The provisions of this Article shall not prevent the payment of subsidies exclusively to domestic producers, including payments to domestic producers derived from the proceeds of internal taxes or charges applied consistently with the provisions of this Article and subsidies effected through governmental purchases of domestic products.

[48] GATT-WTO features tariff bindings and prohibition of export subsidies. The approximation of zero tariffs is legitimate, since, at least for high-income countries, bound rates are close to zero in manufacturing.

where $dV_i \equiv dU_i / \frac{\partial U_i}{\partial I_i}$ *and where the wedge* Ω_i *is defined in Appendix F.2.*

(b) Solving the individual-country policy maker problem in (45) *by using the total-differential approach when* $\tau_{Ii} = \tau_{Xi} = 1$, $i = H, F$ *requires setting* $\Omega_i = 0$.

(c) As a result, the symmetric Nash equilibrium of the two-sector model when trade taxes are not available and both countries can simultaneously set production taxes in the differentiated sector is characterized as follows: it exists, is unique and entails positive, but inefficiently low, production subsidies when the domestic profit share, δ_{ii}, *is larger or equal than 1/2. Otherwise, the Nash equilibrium entails positive production taxes. Formally:*

(i) If $\delta_{ii} \geq \frac{1}{2}$, *then there exists a unique symmetric Nash equilibrium with* $\frac{\varepsilon-1}{\varepsilon} \leq \tau_L^N \leq 1$;

(ii) If either $0 < \delta_{ii} < \frac{1}{2}$ *and* $\varepsilon \geq \frac{3-\alpha}{2}$ *or* $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} \leq \delta_{ii} < \frac{1}{2}$ *and* $\varepsilon < \frac{3-\alpha}{2}$, *there exist a unique symmetric Nash equilibrium with* $\tau_L^N > 1$;

Proof See Appendix F.2. ■

The domestic profit share of firms' from both countries, δ_{ii}, is a sufficient statistic for the impact of firm heterogeneity and selection. Proposition 8 says that if it is larger than their export profit share strategic domestic policies feature positive production subsidies. From Lemma 4 we know that this outcome reflects that the (positive) production-efficiency effect dominates the (negative) terms-of-trade effect. However, these subsidies are inefficiently low due the trade-off between these motives.[49] By contrast, when the domestic profit share is smaller than the export profit share, strategic domestic policies feature production taxes, which worsen the allocation compared to the laissez-faire allocation.[50] In this case, the terms-of-trade effect dominates the production-efficiency effect because firms make the bulk of their profits from exporting, so that manipulating international prices is key. In the presence of firm heterogeneity, the relative importance of the two effects thus depends on the size of the domestic profit share. Therefore, when the set of policy instruments is limited, firm heterogeneity plays a crucial role in shaping

[49] Proposition 8 extends the result of Campolmi et al. (2014) – who find that in the two-sector model with homogeneous firms strategic domestic policies feature positive but inefficiently low production subsidies – to the case of heterogeneous firms.

[50] Observe that if we impose the assumption that the export cutoff φ_{ji} for $j \neq i$ must be larger than the domestic survival cutoff φ_{ii} at the symmetric Nash equilibrium, i.e. $\left(\frac{\varphi_{ji}}{\varphi_{ii}}\right) = \left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon-1}} \tau_{ij} > 1$, then δ_{ii} is always strictly greater than 1/2.

the equilibrium policies, and thus the desirability of specific institutional arrangements, as we show next.

Consequently, since coordination that is limited to trade policies does not lead to an efficient outcome, reaping the full benefits of integration requires a deep trade agreement. However, full cooperation on domestic policies may not be feasible in practice. Alternatively, countries may be able to commit not to use domestic policies at all. We thus consider as an alternative scenario a laissez-faire agreement, which forbids both the use of trade and domestic policies. Whether or not such an arrangement dominates a shallow agreement when firms are heterogeneous depends on whether the profit share from domestic sales is smaller or larger than the one from export sales. This is straightforward: a Nash production subsidy improves equilibrium production efficiency, and thus welfare, compared to the laissez-faire allocation, while a Nash production tax worsens it. (Terms-of-trade effects of domestic policies offset each other in the symmetric Nash equilibrium.)

Finally, in the presence of firm heterogeneity and selection effects, the domestic profit share is endogenous to physical trade costs: one can show that δ_{ii} is increasing in τ_{ij} and f_{ij} for $j \neq i$. Thus, as physical trade barriers fall, the domestic profit share falls and may even become smaller than one half. Therefore, with sufficiently low physical trade barriers a laissez-faire agreement can be better than a shallow trade agreement. These insights on the welfare effects of shallow vs. laissez-faire agreements are summarized by the following Proposition.[51]

Proposition 9 ***Welfare effects of strategic domestic policies in the presence of a shallow trade agreement***

Assume that $\tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$ and let firms' average variable-profit share from sales in their domestic market be given by δ_{ii}.

(a) When $\delta_{ii} < \frac{1}{2}$ the symmetric Nash equilibrium when countries can only set domestic policies strategically is welfare-dominated by the laissez-faire allocation with $\tau_{Li} = 1$, $i = H, F$.

[51] In numerical simulations with Pareto-distributed productivity we have obtained the robust result that when physical trade barriers fall Nash-equilibrium production subsidies decrease smoothly until they turn into positive taxes at a level of trade barriers that implies $\delta_{ii} = 1/2$. From that point on, production taxes strictly increase as trade barriers fall further. These results imply that the proportional welfare gains from moving from a shallow to a deep trade agreement rise as physical trade barriers fall.

(b) δ_{ii} *is increasing in* τ_{ij} *and* f_{ij}*,* $j \neq i$*.*

Proof See Appendix F.3. ■

To summarize, when $\delta_{ii} \geq \frac{1}{2}$ or in the presence of homogeneous firms, a shallow trade agreement that forbids the strategic use of trade policies and allows countries to set domestic policies freely welfare-dominates a laissez-faire agreement that forbids countries to use domestic and trade policies. When instead $\delta_{ii} < \frac{1}{2}$ a laissez-faire agreement that forbids countries to use domestic and trade policies welfare-dominates a shallow trade agreement that forbids the strategic use of trade policies and allows countries to set domestic policies freely.

7 Conclusion

In this paper we have made progress on several fronts. Starting with the observation that trade models with CES preferences and monopolistic competition have a common macro representation, we have shown that this class of models also has common welfare incentives for trade and domestic policies. Solving the problem of a world policy maker, we have derived a welfare decomposition that decomposes world welfare changes induced by trade and domestic policies into changes in consumption- and production-efficiency wedges. As long as the world policy maker disposes of a sufficient set of instruments, she closes these wedges one by one and implements the first-best allocation. In the multi-sector model this requires using production subsidies to offset monopolistic markups.

From the individual-country perspective, welfare incentives for trade and domestic policies are additionally governed by terms-of-trade incentives. This makes clear that the terms-of-trade motive is the only pure beggar-thy-neighbor externality in this class of models.

Then we have discussed that using individual policy instruments always leads to a trade-off between production-efficiency and terms-of-trade effects. Firm heterogeneity in combination with physical trade costs matter for unilateral policies because they determine the profit share from sales in each market. This variable governs how the trade-off between these motives plays out: when physical trade barriers are high, firms make most of their profits domestically, and thus production efficiency dominates.

Finally, we have studied the design of trade agreements from the perspective of the multi-sector heterogeneous-firm model. We have shown that in the absence of any trade agreement, the Nash equilibrium entails the first-best level of production subsidies and inefficient import subsidies and export taxes that aim at improving the terms of trade. Thus, even in the presence of firm heterogeneity and domestic policies terms-of-trade motives remain the only reason for signing a trade agreement. Moreover, when a shallow trade agreement prevents countries from using trade policy strategically, domestic policies are set to balance a trade-off between improving the terms of trade and increasing production efficiency. In this case, Nash-equilibrium domestic policies depend on firm heterogeneity via the profit share from domestic sales: when it is larger than the one from export sales, the Nash equilibrium features positive (albeit inefficiently low) production subsidies. By contrast, when it is smaller, the Nash equilibrium is characterized by positive production taxes. This result implies that achieving the full benefits of globalization requires a deep trade agreement that allows countries to coordinate both trade and domestic policies. Moreover, it means that shallow trade agreement are more distortive when physical trade costs are lower.

References

Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare, "New Trade Models, Same Old Gains?," *American Economic Review*, 2012, *102* (1), 94–130.

Bagwell, Kyle and Robert W. Staiger, "An Economic Theory of GATT," *American Economic Review*, 1999, *89* (1), 215–248.

_ **and** _ , "Domestic Policies, National Sovereignty, and International Economic Institutions," *Quarterly Journal of Economics*, 2001.

_ **and** _ , "Chapter 8 - The Design of Trade Agreements," in Kyle Bagwell and Robert W. Staiger, eds., *Handbook of Commercial Policy*, Vol. 1A, Elsevier, 2016.

Campolmi, Alessia, Harald Fadinger, and Chiara Forlati, "Trade Policy: Home Market Versus Terms of Trade Externality," *Center for Fiscal Policy Working Paper*, 2012.

_ **,** _ **, and** _ , "Trade Policy: Home Market Versus Terms of Trade Externality," *Journal of International Economics*, 2014, *93*, 92–107.

Copeland, Brian R., "Strategic Interaction among Nations: Negotiable and Non-negotiable Trade Barriers," *Canadian Journal of Economics*, 1990, *21* (1), 84–108.

Costinot, Arnaud, Andrés Rodríguez-Clare, and Iván Werning, "Micro to Macro: Optimal Trade Policy with Firm Heterogeneity," *Econometrica*, 2020, *(Forthcoming)*.

Demidova, Svetlana and Andrés Rodríguez-Clare, "Trade Policy Under Firm-level Heterogeneity in a Small Open Economy," *Journal of International Economics*, 2009, *78*, 100–112.

Dixit, Avinash K., "Tax policy in Open Economies," in Allan. J. Auerbach and Martin Feldstein, eds., *Handbook of Public Economics*, Vol. 1, Elsevier, 1985.

Dür, Andreas, Leornardo Baccini, and Martin Elsig, "The Design of International Trade Agreements: Introducing a New Dataset," *Review of International Organization*, 2014, *9* (3), 353–375.

Ederington, Josh, "International Coordination of Trade and Domestic Policies," *American Economic Review*, 2001, *91* (5), 1580–1593.

Felbermayr, Gabriel, Benjamin Jung, and Mario Larch, "Optimal Tariffs, Retaliation, and the Welfare Loss from Tariff Wars in the Melitz Model," *Journal of International Economics*, 2013, *89*, 13–25.

Gros, Daniel, "A Note on the Optimal Tariff Retaliation and the Welfare Loss from Tariff Wars in a Framework with Intra-Industry Trade," *Journal of International Economics*, 1987, *23* (3-4), 357–367.

Grossman, Gene M. and Elhanan Helpman, "Trade Wars and Trade Talks," *Journal of Political Economy*, 1995, *103* (4), 675–708.

Grossman, Gene, Phillip McCalman, and Robert Staiger, "The "New" Economics of Trade Agreements: From Trade Liberalization to Regulatory Convergence?," *NBER Working Paper*, 2019, *26132.*

Haaland, Jan I. and Anthony J. Venables, "Optimal Trade Policy with Monopolistic Competition and Heterogeneous Firms," *Journal of International Economics*, 2016, *102*, 85–95.

Harberger, Arnold, "Three Basic Postulates for Applied Welfare Economics: An Interpretive Essay," *Journal of Economic Perspectives*, 1971, *9* (3), 785–797.

Helpman, Elhanan and Paul Krugman, *Trade Policy and Market Structure*, MIT Press, 1989.

Horn, Henrik, Petros Mavroidis, and André Sapir, "Beyond the WTO? An Anatomy of EU and US Preferential Trade Agreements," *The World Economy*, 2010, *33* (11), 1565–1588.

Krugman, Paul, "Scale Economics, Product Differentiation, and Pattern of Trade," *American Economic Review*, 1980, *70* (5), 950–959.

Lashkaripour, Ahmad and Volodymyr Lugovsky, "Scale Economies and the Structure of Trade and Industrial Policy," *mimeo*, 2019.

Maggi, Giovanni and Andrés Rodríguez-Clare, "The Value of Trade Agreements in the Presence of Political Pressures," *Journal of Political Economy*, 1998, *106* (3), 574–601.

Meade, James, *The Theory of International Economic Policy Vol. 2 - Trade and Welfare: Mathematical Supplement*, Oxford University Press, 1955.

Melitz, Marc J., "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 2003, *71* (6), 1695–1725.

_ **and Stephen J. Redding**, "Heterogeneous Firms and Trade," in Elhanan Helpman Gita Gopinath and Kenneth Rogoff, eds., *Handbook of International Economics*, Vol. 4, Elsevier, 2015.

Ossa, Ralph, "A 'New Trade' Theory of GATT/WTO Negotiations," *Journal of Political Economy*, 2011, *119* (1), 122–152.

_ **and Giovanni Maggi**, "Are Trade Agreements Good for You?," *mimeo*, 2019.

Rodrik, Dani, "What Do Trade Agreements Really Do?," *Journal of Economic Perspectives*, 2018, *32* (2), 73–90.

Venables, Anthony, "Trade and Trade Policy with Differentiated Products: A Chamberlinian-Ricardian Model," *The Economic Journal*, 1987, *97* (387), 700–717.

APPENDIX - FOR ONLINE PUBLICATION

A The Model

In this appendix we first we lay out the general model set-up. Then, we explain how to recover the set of equilibrium conditions (i) in the presence and in the absence of the homogeneous sector and (ii) for the cases of heterogeneous and homogeneous firms. Finally, we derive the free-trade allocation for the two-sector model.

A.1 Households

Given the Dixit-Stiglitz structure of preferences in (4), the households' maximization problem can be solved in three stages. At the first two stages, households choose how much to consume of each domestically produced and foreign produced variety, and how to allocate consumption between the domestic and the foreign bundles. The optimality conditions imply the following demand functions and price indices:

$$c_{ij}(\varphi) = \left[\frac{p_{ij}(\varphi)}{P_{ij}}\right]^{-\varepsilon} C_{ij}, \quad C_{ij} = \left[\frac{P_{ij}}{P_i}\right]^{-\varepsilon} C_i, \quad i,j = H,F \tag{A-1}$$

$$P_i = \left[\sum_{j\in H,F} P_{ij}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \quad P_{ij} = \left[N_j \int_{\varphi_{ij}}^{\infty} p_{ij}(\varphi)^{1-\varepsilon} dG(\varphi)\right]^{\frac{1}{1-\varepsilon}}, \quad i,j = H,F \tag{A-2}$$

Here P_i is the price index of the differentiated bundle in country i, P_{ij} is the country-i price index of the bundle of differentiated varieties produced in country j, and $p_{ij}(\varphi)$ is the country-i consumer price of variety φ produced by country j.

In the last stage, households choose how to allocate consumption between the homogeneous good and the differentiated bundle. Thus, they maximize (3) subject to the following budget constraint:

$$P_i C_i + p_{Zi} Z_i = I_i, \quad i = H,F$$

where $I_i = W_i L + T_i$ is total income and T_i is a lump sum transfer which depends on the tax scheme adopted by the country-i government. The solution to the consumer problem implies that the marginal rate of substitution between the homogeneous good and the differentiated bundle equals their relative price:

$$\frac{\alpha}{1-\alpha}\frac{Z_i}{C_i} = \frac{P_i}{p_{Zi}}, \quad i = H,F \tag{A-3}$$

Then following Melitz and Redding (2015), we can rewrite the demand functions as

$$c_{ij}(\varphi) = p_{ij}(\varphi)^{-\varepsilon} A_i, \quad C_{ij} = P_{ij}^{-\varepsilon} A_i, \quad C_i = P_i^{-\varepsilon} A_i, \quad i,j = H,F, \tag{A-4}$$

where $A_i \equiv P_i^{\varepsilon-1}\alpha I_i$. A_i can be interpreted as an index of market (aggregate) demand.

A.2 Firms

A.2.1 Firms' behavior in the differentiated sector

Given the constant price elasticity of demand, optimal prices charged by country-i firms in their domestic market are a fixed markup over their perceived marginal cost ($\tau_{Li}\frac{W_i}{\varphi}$), and optimal prices charged to country-j

consumers for varieties produced in country i equal country-i prices augmented by transport costs and trade taxes

$$p_{ji}(\varphi) = \tau_{ji}\tau_{Tji}\tau_{Li}\frac{\varepsilon}{\varepsilon-1}\frac{W_i}{\varphi}, \quad i,j = H,F \tag{A-5}$$

The optimal pricing rule implies the following firm revenues:

$$r_{ji}(\varphi) \equiv \tau_{Tji}^{-1}p_{ji}(\varphi)c_{ji}(\varphi) = \tau_{Tji}^{-1}p_{ji}(\varphi)^{1-\varepsilon}A_j = \varepsilon\tau_{ji}^{1-\varepsilon}\tau_{Tji}^{-\varepsilon}\tau_{Li}^{1-\varepsilon}W_i^{1-\varepsilon}\varphi^{\varepsilon-1}B_j, \quad i,j = H,F, \tag{A-6}$$

where $B_i \equiv \left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}\frac{1}{\varepsilon}A_i$. Profits are given by:

$$\pi_{ji}(\varphi) \equiv B_j\left(\frac{\tau_{Li}W_i}{\varphi}\right)^{1-\varepsilon}\tau_{ji}^{1-\varepsilon}\tau_{Tji}^{-\varepsilon} - \tau_{Li}W_i f_{ji} = \frac{r_{ji}(\varphi)}{\varepsilon} - \tau_{Li}W_i f_{ji}, \quad i,j = H,F \tag{A-7}$$

A.2.2 Zero-profit conditions

Firms choose to produce for the domestic (export) market only when this is profitable. Since profits are monotonically increasing in φ, we can determine the equilibrium productivity cutoffs for firms active in the domestic market and export market, φ_{ji}, by setting $\pi_{ji}(\varphi_{ji}) = 0$, namely:

$$\pi_{ji}(\varphi_{ji}) = 0 \Rightarrow \frac{r_{ji}(\varphi_{ji})}{\varepsilon} = \tau_{Li}W_i f_{ji}, \quad i,j = H,F \tag{A-8}$$

As in Melitz (2003), we call these conditions the *zero profit* (**ZCP**) conditions. Using (A-7) we rewrite (A-8) as follows:

$$B_j = \tau_{ji}^{\varepsilon-1}\tau_{Li}^{\varepsilon}\tau_{Tji}^{\varepsilon}W_i^{\varepsilon}\varphi_{ji}^{1-\varepsilon} \quad j = H,F, \quad i \neq j \tag{A-9}$$

A.2.3 Free-entry conditions (FE)

The *free entry* (**FE**) conditions require expected profits (before firms know the realization of their productivity) in each country to be zero in equilibrium:

$$\sum_{j=H,F}\int_{\varphi_{ji}}^{\infty}\pi_{ji}(\varphi)dG(\varphi) = \tau_{Li}W_i f_E, \quad i = H,F$$

Substituting optimal profits (A-7), we obtain

$$\sum_{j=H,F}\int_{\varphi_{ji}}^{\infty}\left[B_j\left(\frac{\tau_{Li}W_i}{\varphi}\right)^{1-\varepsilon}\tau_{ji}^{1-\varepsilon}\tau_{Tji}^{-\varepsilon} - \tau_{Li}W_i f_{ji}\right]dG(\varphi) = \tau_{Li}W_i f_E, \quad i = H,F \tag{A-10}$$

A.2.4 Firms' behavior in the homogeneous sector

Since the homogeneous good is sold in a perfectly competitive market without trade costs, price equals marginal cost and is the same in both countries. We assume that the homogeneous good is produced in both countries in equilibrium. Given the production technology, this implies factor price equalization in the presence of the homogeneous sector:

$$p_{Zi} = p_{Zj} = W_i = W_j = 1, \quad i = H, j = F$$

A.3 Government

The government is assumed to run a balanced budget. Hence, country-i government's budget constraint is given by:

$$T_i = (\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}C_{ij} + (\tau_{Xi}-1)\tau_{Tji}^{-1}P_{ji}C_{ji}+$$
$$+ (\tau_{Li}-1)N_iW_i\left[\sum_{k=H,F}\int_{\varphi_{ki}}^{\infty}\left(\frac{q_{ki}(\varphi)}{\varphi}+f_{ki}\right)dG(\varphi)+f_E\right], \quad i=H,F, \quad j\neq i \tag{A-11}$$

Government income consists of import tax revenues charged on imports of differentiated goods gross of transport costs and foreign export taxes (thus, tariffs are charged on CIF values of foreign exports), export tax revenues charged on exports gross of transport costs, and production tax revenues.

A.4 Equilibrium

A.4.1 Equilibrium of the two-sector model

Substituting **ZCP** (A-9) into **FE** (A-10), we obtain:

$$\sum_{j=H,F} f_{ji}(1-G(\varphi_{ji}))\left(\frac{\widetilde{\varphi}_{ji}}{\varphi_{ji}}\right)^{\varepsilon-1} = f_E + \sum_{j=H,F} f_{ji}(1-G(\varphi_{ji})), \quad i=H,F, \tag{A-12}$$

where

$$\widetilde{\varphi}_{ji} = \left[\int_{\varphi_{ji}}^{\infty}\varphi^{\varepsilon-1}\frac{dG(\varphi)}{1-G(\varphi_{ji})}\right]^{\frac{1}{\varepsilon-1}}, \quad i,j=H,F, \tag{A-13}$$

which correspond to (10) and (7) in the main text. Moreover, dividing the **ZCP** conditions (A-9), we obtain condition (9) in the main text:

$$\frac{\varphi_{ii}}{\varphi_{ij}} = \left(\frac{f_{ii}}{f_{ij}}\right)^{\frac{1}{\varepsilon-1}}\left(\frac{\tau_{Li}}{\tau_{Lj}}\right)^{\frac{\varepsilon}{\varepsilon-1}}\left(\frac{W_i}{W_j}\right)^{\frac{\varepsilon}{\varepsilon-1}}\tau_{ij}^{-1}\tau_{Tij}^{-\frac{\varepsilon}{\varepsilon-1}}, \quad i,j=H,F \tag{A-14}$$

The remaining equilibrium equations are then given as follows:

Consumption sub-indices, which can be determined using (A-4) jointly with (A-9):

$$C_{ij} = P_{ij}^{-\varepsilon}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\varepsilon-1}\varepsilon\tau_{Lj}^{\varepsilon}\tau_{ij}^{\varepsilon-1}\tau_{Tij}^{\varepsilon}\varphi_{ij}^{1-\varepsilon}W_j^{\varepsilon}f_{ij}, \quad i,j=H,F \tag{A-15}$$

Price sub-indices, which emerge from substituting (A-5) into (A-2):

$$P_{ij}^{1-\varepsilon} = \left(\frac{\varepsilon}{\varepsilon-1}\right)^{1-\varepsilon}N_j(1-G(\varphi_{ij}))(\tau_{ij}\tau_{Tij}\tau_{Lj})^{1-\varepsilon}\widetilde{\varphi}_{ij}^{\varepsilon-1}W_j^{1-\varepsilon}, \quad i,j=H,F \tag{A-16}$$

Aggregate profits Π_i are given by $\Pi_i = R_i - \tau_{Li}W_iL_{Ci} + \tau_{Li}W_iN_if_E$, where R_i is aggregate revenue, $R_i \equiv N_i\sum_{j=H,F}\int_{\varphi_{ji}}^{\infty}r_{ji}(\varphi)dG(\varphi)$. From the **FE** condition (A-10) it then follows that $\Pi_i = \tau_{Li}W_iN_if_E$ and thus

$R_i = \tau_{Li} W_i L_{Ci}$. Substituting the definition of optimal revenues (A-6) into the previous condition, we get

$$\tau_{Li} W_i L_{Ci} = \varepsilon N_i \sum_{j=H,F} \int_{\varphi_{ji}}^{\infty} B_j \tau_{ji}^{1-\varepsilon} \tau_{Tji}^{-\varepsilon} \tau_{Li}^{1-\varepsilon} W_i^{1-\varepsilon} \varphi^{\varepsilon-1} dG(\varphi), \quad i = H, F$$

Combining this condition with (10) and (A-9), we obtain:

Labor market clearing in the differentiated sector

$$L_{Ci} = \varepsilon N_i \sum_{j=H,F} f_{ji}(1 - G(\varphi_{ji})) + \varepsilon f_E N_i, \quad i = H, F \tag{A-17}$$

This can be solved for the equilibrium level of N_i:

$$N_i = \frac{L_{Ci}}{\varepsilon \sum_{j=H,F} f_{ji}(1 - G(\varphi_{ji})) + \varepsilon f_E}, \quad i = H, F \tag{A-18}$$

Combining this last condition with (10), plugging into (A-15) and (A-16) and taking into account the definition (7), allows us to recover (11) and (12) in the main text.

In the presence of the homogeneous sector, the trade-balance condition is given by:[52]

$$Q_{Zi} - Z_i + \tau_{Ij}^{-1} P_{ji} C_{ji} = \tau_{Ii}^{-1} P_{ij} C_{ij}, \quad i = H, j = F \tag{A-19}$$

We can use the fact that $\sum_{j=H,F} P_{ij} C_{ij} = P_i C_i$ to rewrite (A-3) as:

$$Z_i = \frac{1-\alpha}{\alpha} \sum_{j=H,F} P_{ij} C_{ij}, \quad i = H, F$$

We can combine this equation with the trade-balance condition above and the aggregate labor market clearing $L = L_{Ci} + L_{Zi}$ to obtain:

Trade-balance condition

$$L - L_{Ci} - \frac{1-\alpha}{\alpha} \sum_{k=H,F} P_{ik} C_{ik} + \tau_{Ij}^{-1} P_{ji} C_{ji} = \tau_{Ii}^{-1} P_{ij} C_{ij}, \quad i = H, j = F,$$

which corresponds to condition (13).

Finally, when the homogeneous sector is present, we also require equilibrium in the market for the homogenous good, i.e. $\sum_{i=H,F} Q_{Zi} = \sum_{i=H,F} Z_i$. Combining this condition with aggregate labor market clearing and demand for the homogeneous good (A-3) we obtain:

Homogeneous-good market clearing condition

$$\sum_{i=H,F} (L - L_{Ci}) = \frac{1-\alpha}{\alpha} \sum_{i=H,F} \sum_{j=H,F} P_{ij} C_{ij},$$

which coincides with condition (14).

[52] Import taxes are collected directly by the governments at the border so they do not enter into this condition.

A.4.2 Equilibrium of the one-sector model

When there is no homogeneous sector, i.e., when $\alpha = 1$, then from (15) $Z_i = 0$ for $i = H, F$, and $L_{Ci} = L$ for $i = H$ from the labor market clearing $L = L_{Ci} + Q_{Zi}$. Conditions (7)-(12) remain the same. Condition (13) simplifies to:

$$\tau_{Ij}^{-1} P_{ji} C_{ji} = \tau_{Ii}^{-1} P_{ij} C_{ij}, \quad i = H, \quad j = F \tag{A-20}$$

Then, $L_{Ci} = L$ for $i = F$ from (14).

Finally note that, as well known, in the one-sector model the allocation of labor is efficient. Thus, we assume that policymakers abstain from strategically using the labor subsidy. For convenience, we assume that in any symmetric allocation the labor cost is equal across countries i.e., labor subsidies are such that $\tau_{Li} W_i = \tau_{Lj} W_j$ for $i = H$ and $j = F$. As it will become clear in the following sections, this assumption will simplify the comparison between the planner and the market allocation.

A.4.3 From Melitz to Krugman (1980)

The Melitz model encompasses the Krugman (1980) model with homogeneous firms under the assumptions that there are no fixed market access costs (i.e, $f_{ij} = 0$ for $i, j = H, F$) and that $G(\varphi)$ is a degenerate distribution. Then, without loss of generality, we normalize $\tilde{\varphi} = \varphi = 1$. Under this parametrization, conditions (7), (8), (9) and (10) should be dropped from the set of equilibrium conditions.

In addition, the free-entry conditions are given by:

$$\sum_{j=H,F} \pi_{ji} = \tau_{Li} W_i f_E, \quad i = H, F,$$

and profits are given by:

$$\pi_{ji} \equiv B_j \left(\tau_{Li} W_i\right)^{1-\varepsilon} \tau_{ji}^{1-\varepsilon} \tau_{Tji}^{-\varepsilon}, \quad i, j = H, F$$

By combining these two last conditions we can solve for B_i and B_j as functions of W_i, W_j and the policy instruments:

$$B_i = f_E W_j^{\varepsilon} \frac{\tau_{Lj}^{\varepsilon} - \tau_{Li}^{\varepsilon} \tau_{ij}^{\varepsilon-1} \tau_{Tji}^{\varepsilon} \left(\frac{W_i}{W_j}\right)^{\varepsilon}}{\tau_{Tij}^{-\varepsilon} \tau_{ij}^{1-\varepsilon} - \tau_{Tji}^{\varepsilon} \tau_{ij}^{\varepsilon-1}}, \quad i = H, F, \quad j \neq i$$

Moreover, by substituting the optimal pricing decision into the definition of the price indices and observing that $N_j = L_{Cj}/(\varepsilon f_E)$ we get:

$$P_{ij} = \frac{\varepsilon}{\varepsilon - 1} (\varepsilon f_E)^{\frac{1}{\varepsilon-1}} \tau_{ij} \tau_{Tij} \tau_{Lj} W_j L_{Cj}^{\frac{-1}{\varepsilon-1}}, \quad i, j = H, F \tag{A-21}$$

At the same time, from the definition of C_{ij}, it follows that:

$$C_{ij} = P_{ij}^{-\varepsilon} \left(\frac{\varepsilon}{\varepsilon - 1}\right)^{\varepsilon-1} \varepsilon B_i, \quad i, j = H, F$$

Substituting the expressions above for P_{ij} and B_i into the above condition, leads to:

$$C_{ij} = \frac{\varepsilon - 1}{\varepsilon} L_{Cj}^{\frac{\varepsilon}{\varepsilon-1}} (\varepsilon f_E)^{\frac{-1}{\varepsilon-1}} \frac{(\tau_{ij} \tau_{Tij})^{-\varepsilon} \left[\left(\frac{W_k}{W_j} \frac{\tau_{Lk}}{\tau_{Lj}}\right)^{\varepsilon} - \left(\frac{W_i}{W_j} \frac{\tau_{Li}}{\tau_{Lj}}\right)^{\varepsilon} \tau_{ki}^{\varepsilon-1} \tau_{Tki}^{\varepsilon}\right]}{\tau_{Tik}^{-\varepsilon} \tau_{ki}^{1-\varepsilon} - \tau_{Tki}^{\varepsilon} \tau_{ki}^{\varepsilon-1}}, \quad i, j = H, F, \quad k \neq i \tag{A-22}$$

Thus, if the homogeneous sector is present ($\alpha < 1$), the equilibrium is given by equations (A-21) and (A-22) together with (13),(14) and (15) and the fact that $W_i = 1$ for $i = H$. By contrast, in the absence of the homogeneous sector (i.e., when $\alpha = 1$), the equilibrium is determined by conditions (A-20), (A-21), (A-22) and the fact that $L_{Cj} = L$ for $j = H, F$.

A.4.4 The allocation under the laissez-faire agreement in the two-sector model

Using equations (11) and (12), we find that

$$P_{ij}C_{ij} = \delta_{ij}L_{Cj}\tau_{Tij}\tau_{Lj}W_j, \quad i,j = H,F$$

Substituting into the trade-balance condition (13), we obtain:

$$L - L_{Ci} - \frac{1-\alpha}{\alpha}\sum_{k=H,F}\delta_{ik}L_{Ck}\tau_{Tik}\tau_{Lk}W_k + \delta_{ji}L_{Ci}\tau_{Xi}\tau_{Li}W_i = \delta_{ij}L_{Cj}\tau_{Xj}\tau_{Lj}W_j, \quad i = H, j = F$$

Under the laissez-faire agreement, $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$. Since the countries are symmetric, the equilibrium is also symmetric and thus $L_{Ci} = L_{Cj}$, $W_i = W_j = 1$, $\delta_{ij} = \delta_{ji}$ for $i = H, F$ and $j \neq i$.

Substituting these conditions, we find that

$$L_{Ci}^{LF} = \alpha L, \quad i = H, F$$

Using this result together with (A-17) and (A-12), we obtain

$$N_i^{LF} = \frac{\alpha L}{\varepsilon \sum_{j=H,F}\left[f_{ji}(1-G(\varphi_{ji}))\left(\frac{\tilde{\varphi}_{ji}}{\varphi_{ji}}\right)^{\varepsilon-1}\right]}, \quad i = H, F$$

B The Total-Differential Approach

We use the total-differential approach to optimization to solve both the planner and the optimal-policy problems.[53] In this way, we can use the same methodology to derive all the main results of the paper: the welfare decomposition and the efficiency wedges; the world, the unilateral and the strategic policies.

We first discuss how we apply this approach to find the optimal deviations of domestic and trade policies. Then, we explain how to employ it to solve constrained optimization problems. Finally, we derive a number of preliminary results that we will use in the rest of the appendix.

B.1 How to apply the total-differential approach

B.1.1 Unilateral policy deviations

The unilateral deviations of each policy instrument can be determined following these steps:

(1) Take the total differential of the objective function and the equilibrium conditions.

[53] Observe that using this approach implies restricting our analysis to interior solutions.

(2) Use the total differential of the equilibrium conditions to solve for the total differentials of the endogenous variables as linear functions of the total differentials of the policy instruments. Since we consider each policy instrument at a time, set the total differentials of the policy instruments that are not of interest to zero.

(3) Substitute the solution of the total differentials of the endogenous variables into the total differential of the objective function and evaluate it at the laissez-faire allocation. Collect all the terms and sign the coefficient multiplying the total differential of the policy instrument to determine the direction of the optimal deviations from the laissez-faire allocation.

B.1.2 Constrained optimization problems

A constrained optimization problem in n variables given m constraints with $n > m$ can be solved using the total-differential approach according to the following steps:

(1) Take the total differential of the objective function and the constraints.

(2) Use the total differential of the constraints to solve for m total differentials as a function of the $n-m$ other total differentials.

(3) Substitute the solution of the m total differentials into the total differential of the objective function. Then the total differential of the objective function must be zero for *any* of the $n-m$ total differentials (i.e., for any *arbitrary* perturbation of the $n-m$ relevant variables). Collect the terms multiplied by the $n-m$ differentials to find the $n-m$ conditions that need to be zero at the optimum.

(4) The $n-m$ conditions found in (3) jointly with the m constraints determine the solution of the n variables.

B.2 Preliminary steps for applying the total-differential approach

In this section, we derive some preliminary results that will be useful to derive the results of Sections 3 to 6.

As explained above, the first steps to apply the total-differential approach – independently of whether the optimal policy problem or unilateral deviations are considered – is to take the total differential of the equilibrium equations (7)-(14), which we do in Section B.2.1 below. Then, we evaluate the total differentials at a symmetric allocation. Moreover, when considering policies from the individual-country perspective, as analyzed in Section 5, we set $d\tau_{Lj} = d\tau_{Ij} = d\tau_{Xj} = 0$, and combine the equations so as to be left with 3 equations, which are linear functions of 6 differentials: dL_{Ci}, dC_{ii}, dC_{ij}, $d\tau_{Li}$, $d\tau_{Ii}$ and $d\tau_{Xi}$. We can then use these 3 equations to express 3 differentials as functions of the remaining 3. For the unilateral deviations considered in Section 5, we solve for dL_{Ci}, dC_{ii} and dC_{ij} as linear functions of the deviations of the policy instruments $d\tau_{Li}$, $d\tau_{Ii}$ and $d\tau_{Xi}$. Then, we allow only a single policy instrument to vary at a time, while setting the deviations for the other two to zero. Differently, for the cases of strategic interaction in Section 6 we use the 3 equations to write the differentials of the tax instruments, $d\tau_{Li}$, $d\tau_{Ii}$ and $d\tau_{Xi}$ as linear functions of the other 3 differentials, dL_{Ci}, dC_{ii} and dC_{ij}. Finally, for the case of strategic interaction when only production taxes are available (shallow trade agreement) we set the deviations for $d\tau_{Ii}$ and $d\tau_{Xi}$ to zero. This allow us to express $d\tau_{Li}$ as a function of dL_{Ci} only.

B.2.1 Total differentials of some equilibrium conditions

Since the total differentials of the equilibrium equations (7)-(11) are extensively used in the proofs of Sections 3 to 6, and since they hold for both the one-sector and the two-sector models, we present them here for future reference in their general formulation.

The total differential of (7) gives:

$$d\widetilde{\varphi}_{ji} = \frac{1}{\varepsilon - 1} \frac{g(\varphi_{ji})}{[1 - G(\varphi_{ji})]} \widetilde{\varphi}_{ji} \left[1 - \left(\frac{\varphi_{ji}}{\widetilde{\varphi}_{ji}} \right)^{\varepsilon - 1} \right] d\varphi_{ji}, \quad i, j = H, F \tag{B-1}$$

Substituting this condition into the total differential of (10), we get:

$$d\varphi_{ji} = -\frac{f_{ii}[1 - G(\varphi_{ii})] \varphi_{ii}^{-\varepsilon} \widetilde{\varphi}_{ii}^{\varepsilon - 1}}{f_{ji}[1 - G(\varphi_{ji})] \varphi_{ji}^{-\varepsilon} \widetilde{\varphi}_{ji}^{\varepsilon - 1}} d\varphi_{ii}, \quad i = H, F, \quad i \neq j \tag{B-2}$$

Using (8) and (10), this condition can be rewritten as

$$d\varphi_{ji} = -\frac{\delta_{ii}}{1 - \delta_{ii}} \frac{\varphi_{ji}}{\varphi_{ii}} d\varphi_{ii}, \quad i = H, F, \quad i \neq j, \tag{B-3}$$

which expresses the total differential of the productivity cut-offs for the domestically produced goods in the export markets as a function of the cut-offs in the domestic markets. Taking the total differential of (8) combined with (10) and substituting (B-1) and (B-2) into the resulting condition, we get:

$$d\delta_{ji} = -\frac{\delta_{ji}}{\varphi_{ji}} \left(\Phi_i + (\varepsilon - 1) \right) d\varphi_{ji}, \quad i, j = H, F \tag{B-4}$$

where $\Phi_i \equiv \delta_{ii} \frac{g(\varphi_{ji}) \varphi_{ji}^{\varepsilon} \widetilde{\varphi}_{ji}^{1-\varepsilon}}{1 - G(\varphi_{ji})} + \delta_{ji} \frac{g(\varphi_{ii}) \varphi_{ii}^{\varepsilon} \widetilde{\varphi}_{ii}^{1-\varepsilon}}{1 - G(\varphi_{ii})} > 0$, $i = H, F$ and $j \neq i$. Condition (B-4) states that as the productivity cut-off rises, the corresponding variable-profit share shrinks.

Moreover, by totally differentiating (11), we obtain:

$$d\varphi_{ij} = \frac{\varphi_{ij}}{C_{ij}} dC_{ij} - \frac{\varepsilon}{\varepsilon - 1} \frac{\varphi_{ij}}{\delta_{ij}} d\delta_{ij} - \frac{\varepsilon}{\varepsilon - 1} \frac{\varphi_{ij}}{L_{Cj}} dL_{Cj}, \quad i, j = H, F, \tag{B-5}$$

which, using the symmetric condition of (B-4) to substitute out $d\delta_{ij}$, becomes:

$$d\varphi_{ij} = \frac{\varepsilon \varphi_{ij}}{L_{Cj}(\varepsilon - 1) \left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon - 1} \Phi_j \right)} dL_{Cj} - \frac{\varphi_{ij}}{C_{ij} \left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon - 1} \Phi_j \right)} dC_{ij}, \quad i, j = H, F \tag{B-6}$$

For future use, we substitute the symmetric condition of (B-6) into (B-4):

$$d\delta_{ji} = \frac{\delta_{ji}(\varepsilon - 1 + \Phi_i)}{C_{ji} \left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon - 1} \Phi_i \right)} dC_{ji} - \frac{\delta_{ji} \varepsilon (\varepsilon - 1 + \Phi_i)}{L_{Ci}(\varepsilon - 1) \left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon - 1} \Phi_i \right)} dL_{Ci}, \quad i, j = H, F \tag{B-7}$$

Finally, taking the total differential of (9), we have:

$$d\varphi_{ij} = \frac{\varphi_{ij}}{\varphi_{ii}} d\varphi_{ii} + \frac{\varepsilon}{\varepsilon - 1} \varphi_{ij} \left[\frac{d\tau_{Lj}}{\tau_{Lj}} - \frac{d\tau_{Li}}{\tau_{Li}} + \frac{dW_j}{W_j} - \frac{dW_i}{W_i} + \frac{d\tau_{Tij}}{\tau_{Tij}} \right], \quad i, j = H, F, \quad i \neq j \tag{B-8}$$

where $d\tau_{Tji} = 0$ if $i = j$ while $d\tau_{Tji} = \tau_{Xi} d\tau_{Ij} + \tau_{Ij} d\tau_{Xi}$ if $i \neq j$.

B.2.2 Total differentials of the two-sector model

In this section we restrict ourself to the case $\alpha < 1$. Hence we can use the simplification $W_i = 1$ for $i = H, F$. Under this restriction we combine the total differentials of the equilibrium equations to find 3 conditions that can be expressed as functions of dL_{Ci}, dC_{ii}, dC_{ij}, $d\tau_{Li}$, $d\tau_{Ii}$ and $d\tau_{Xi}$ only.[54]

(1) The first condition can be derived in the following way. Taking the symmetric condition of (B-8), using (B-3) to substitute out $d\varphi_{ji}$, solving for $d\varphi_{jj}$ and finally using (B-6) to substitute out $d\varphi_{ii}$, we obtain:

$$d\varphi_{jj} = -\frac{\varphi_{jj}}{\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i}\frac{\delta_{ii}}{1-\delta_{ii}}\left(\frac{\varepsilon}{\varepsilon-1}\frac{dL_{Ci}}{L_{Ci}} - \frac{dC_{ii}}{C_{ii}}\right) - \frac{\varepsilon}{\varepsilon-1}\varphi_{jj}\left(\frac{d\tau_{Li}}{\tau_{Li}} - \frac{d\tau_{Lj}}{\tau_{Lj}} + \frac{d\tau_{Ij}}{\tau_{Ij}} + \frac{d\tau_{Xi}}{\tau_{Xi}}\right) \tag{B-9}$$

Using (B-3) to substitute out $d\varphi_{jj}$ from (B-9) we find the following expression for $d\varphi_{ij}$:

$$d\varphi_{ij} = -\frac{\delta_{jj}\varphi_{ij}}{1-\delta_{jj}}\left[\frac{\varepsilon}{\varepsilon-1}\left(\frac{d\tau_{Lj}}{\tau_{Lj}} - \frac{d\tau_{Li}}{\tau_{Li}} - \frac{d\tau_{Tji}}{\tau_{Tji}}\right) - \frac{\delta_{ii}}{1-\delta_{ii}}\frac{1}{\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i}\left(\frac{\varepsilon}{\varepsilon-1}\frac{dL_{Ci}}{L_{Ci}} - \frac{dC_{ii}}{C_{ii}}\right)\right] \tag{B-10}$$

Moreover, we combine (B-6), (B-8) and (B-10) to obtain:

$$-\frac{d\tau_{Tij}}{\tau_{Tij}}(1-\delta_{jj}) + \frac{d\tau_{Tji}}{\tau_{Tji}}\delta_{jj} + \frac{d\tau_{Li}}{\tau_{Li}} - \frac{d\tau_{Lj}}{\tau_{Lj}} + \frac{1-\delta_{ii}-\delta_{jj}}{(1-\delta_{ii})\left(\varepsilon - 1 + \Phi_i\frac{\varepsilon}{\varepsilon-1}\right)}\left(\frac{\varepsilon-1}{\varepsilon}\frac{dC_{ii}}{C_{ii}} - \frac{dL_{Ci}}{L_{Ci}}\right) = 0$$

Finally we impose symmetry as well as $d\tau_{Lj} = d\tau_{Xj} = d\tau_{Ij} = 0$. This means that $d\tau_{Tji} = \tau_{Ij}d\tau_{Xi}$ and $d\tau_{Tij} = \tau_{Xj}d\tau_{Ii}$. Under these restrictions, we can rewrite the last equation as:

$$\frac{d\tau_{Li}}{\tau_{Li}} - (1-\delta_{ii})\frac{d\tau_{Ii}}{\tau_{Ii}} + \delta_{ii}\frac{d\tau_{Xi}}{\tau_{Xi}} + \frac{1-2\delta_{ii}}{(1-\delta_{ii})\left(\varepsilon - 1 + \Phi_i\frac{\varepsilon}{\varepsilon-1}\right)}\left(\frac{\varepsilon-1}{\varepsilon}\frac{dC_{ii}}{C_{ii}} - \frac{dL_{Ci}}{L_{Ci}}\right) = 0 \tag{B-11}$$

(2) The second condition can be found as follows. First, we combine (11) and (12):

$$P_{ij}C_{ij} = L_{Cj}\delta_{ij}\tau_{Tij}\tau_{Lj} \quad i,j = H,F \tag{B-12}$$

Second, we use (B-12) to rewrite (13) as follows:

$$L_{Cj} = \frac{\alpha L - L_{Ci}(\alpha + (1-\alpha)\delta_{ii}\tau_{Li} - \alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi})}{(1-\delta_{jj})\tau_{Lj}\tau_{Xj}(\alpha + (1-\alpha)\tau_{Ii})} \tag{B-13}$$

Third, using (B-4) to find an expression for $d\delta_{jj}$ and combining it with (B-9) we get:

$$d\delta_{jj} = \delta_{jj}(\varepsilon - 1 + \Phi_j)\left[\frac{\varepsilon}{\varepsilon-1}\left(\frac{d\tau_{Li}}{\tau_{Li}} - \frac{d\tau_{Lj}}{\tau_{Lj}} + \frac{d\tau_{Ij}}{\tau_{Ij}} + \frac{d\tau_{Xi}}{\tau_{Xi}}\right) - \frac{1}{\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)}\frac{\delta_{ii}}{1-\delta_{ii}}\left(\frac{dC_{ii}}{C_{ii}} - \frac{\varepsilon}{\varepsilon-1}\frac{dL_{Ci}}{L_{Ci}}\right)\right] \tag{B-14}$$

Taking the total differential of (B-13), using (B-7) and (B-14) to substitute out $d\delta_{ii}$ and $d\delta_{jj}$, imposing symmetry

[54]For the sake of brevity we omit to specify for which countries the equations hold.

and $d\tau_{Lj} = d\tau_{Xj} = d\tau_{Ij} = 0$, we obtain:

$$
\begin{aligned}
\frac{dL_{Cj}}{L_{Cj}} =& \left(\frac{\alpha}{(1-\alpha)\tau_{Ii}+\alpha} + \frac{\delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)}{(1-\delta_{ii})(\varepsilon-1)}\right)\frac{d\tau_{Xi}}{\tau_{Xi}} + \frac{\delta_{ii}(\varepsilon-1+\Phi_i)}{(1-\delta_{ii})\left(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1}\right)}\left(\frac{\delta_{ii}}{1-\delta_{ii}} + \frac{1-\alpha+\alpha\tau_{Xi}}{\tau_{Xi}((1-\alpha)\tau_{Ii}+\alpha)}\right)\frac{dC_{ii}}{C_{ii}} \\
&- \frac{\alpha+(1-\alpha)\delta_{ii}\tau_{Li}-\alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi}}{(1-\delta_{ii})\tau_{Li}\tau_{Xi}((1-\alpha)\tau_{Ii}+\alpha)}\frac{dL_{Ci}}{L_{Ci}} + \frac{\varepsilon\delta_{ii}(\varepsilon-1+\Phi_i)}{(1-\delta_{ii}(\varepsilon-1)(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1})}\left(\frac{\delta_{ii}}{1-\delta_{ii}} + \frac{1-\alpha+\alpha\tau_{Xi}}{\tau_{Xi}((1-\alpha)\tau_{Ii}+\alpha)}\right)\frac{dL_{Ci}}{L_{Ci}} \\
&- \frac{1-\alpha}{\alpha+(1-\alpha)\tau_{Ii}}d\tau_{Ii} - \left(\frac{(1-\alpha)\delta_{ii}-\alpha(1-\delta_{ii})\tau_{Xi}}{(1-\delta_{ii})\tau_{Xi}(\alpha+(1-\alpha)\tau_{Ii})} - \frac{\delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)}{(1-\delta_{ii})(\varepsilon-1)}\right)\frac{d\tau_{Li}}{\tau_{Li}} \qquad \text{(B-15)}
\end{aligned}
$$

In addition, we combine (B-12) with (14), we take its total differential, and then we substitute out $d\delta_{ii}$ and $d\delta_{jj}$ using (B-7) and (B-14), respectively. We then impose symmetry and $d\tau_{Lj} = d\tau_{Xj} = d\tau_{Ij} = 0$ to get:

$$
\begin{aligned}
&-(1-\alpha)L_{Ci}\left(\delta_{ii}+(1-\delta_{ii})\tau_{II}\tau_{Xi} + \frac{\delta_{ii}\varepsilon(1-\tau_{Ii}\tau_{XI})(\varepsilon-1+\Phi_j)}{\varepsilon-1}\right)d\tau_{Li} \\
&-(1-\alpha)L_{Ci}\left((1-\delta_{ii})\tau_{Li} + \frac{\delta_{ii}\varepsilon\tau_{Li}(1-\tau_{Ii}\tau_{Xi})(\varepsilon-1+\Phi_j)}{(\varepsilon-1)\tau_{Ii}\tau_{Xi}}\right)\tau_{Ii}d\tau_{Xi} - (1-\alpha)L_{Ci}(1-\delta_{ii})\tau_{Li}\tau_{Xi}d\tau_{Ii} \\
&+\frac{(1-\alpha)L_{Ci}\delta_{ii}}{\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1}}\left(\frac{\delta_{ii}\tau_{Li}(1-\tau_{Ii}\tau_{Xi})(\varepsilon-1+\Phi_j)}{1-\delta_{ii}} - \tau_{Li}(1-\tau_{Ij}\tau_{Xi})(\varepsilon-1+\Phi_i)\right)\left(\frac{dC_{ii}}{C_{ii}} - \frac{\varepsilon}{\varepsilon-1}\frac{dL_{Ci}}{L_{Ci}}\right) \\
&-(\alpha+(1-\alpha)\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}))dL_{Cj} - (\alpha+(1-\alpha)\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}))\,dL_{Ci} = 0 \qquad \text{(B-16)}
\end{aligned}
$$

We can then use condition (B-15) to substitute out dL_{Cj} from (B-16) and to rewrite (B-16) as follows:

$$
\begin{aligned}
&-\frac{(1-\alpha)(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}-\alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi})}{\alpha+(1-\alpha)\tau_{Ii}}d\tau_{Ii} \\
&-\frac{(1-\alpha)\delta_{ii}-\alpha(1-\delta_{ii})\tau_{Xi}}{(1-\delta_{ii})(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}+(1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii}\tau_{Xi})\frac{d\tau_{Li}}{\tau_{Li}} \\
&+\left((1-\alpha)(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}) + \frac{\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})(\varepsilon-1+\Phi_i)}{(\varepsilon-1)(1-\delta_{ii})\tau_{Li}}\right)d\tau_{Li} \\
&+\frac{\alpha(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}+(1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii}\tau_{Xi})}{(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}}d\tau_{Xi} \\
&+\left((1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii} + \frac{\delta_{ii}\varepsilon((1-\alpha)\tau_{Li}+\alpha)(\varepsilon-1+\Phi_i)}{(\varepsilon-1)(1-\delta_{ii})\tau_{Xi}}\right)d\tau_{Xi} \\
&+\frac{\delta_{ii}(\varepsilon-1+\Phi_i)}{(1-\delta_{ii})(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1})}\left(-\frac{\delta_{ii}(\alpha+(1-\alpha)\tau_{Li})}{1-\delta_{ii}} + (1-\alpha)\tau_{Li}(1-\tau_{Ii}\tau_{Xi})(1-\delta_{ii})\right. \\
&\left.-\frac{(1-\alpha+\alpha\tau_{Xi})(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}+(1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii}\tau_{Xi})}{(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}}\right)\frac{dC_{ii}}{C_{ii}} \\
&-\left[\left(\frac{\alpha+(1-\alpha)\delta_{ii}\tau_{Li}-\alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi}}{(1-\delta_{ii})(\alpha+(1-\alpha)\tau_{Ii})\tau_{Li}\tau_{Xi}} - 1\right)(\alpha+(1-\alpha)\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}))\right. \\
&-\frac{\delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)}{(1-\delta_{ii})(\varepsilon-1)(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1})}\left((\alpha+(1-\alpha)\delta_{ii}\tau_{Li}+(1-\alpha)(1-\delta_{ii})\tau_{Li}\tau_{Ii}\tau_{Xi})\frac{1-\alpha+\alpha\tau_{Xi}}{(\alpha+(1-\alpha)\tau_{Ii})\tau_{Xi}}\right. \\
&\left.\left.+\frac{\delta_{ii}(\alpha+(1-\alpha)\tau_{Li})}{1-\delta_{ii}} - (1-\delta_{ii})(1-\alpha)\tau_{Li}(1-\tau_{Ii}\tau_{Xi})\right)\right]\frac{dL_{Ci}}{L_{Ci}} = 0 \qquad \text{(B-17)}
\end{aligned}
$$

(3) The third condition can be retrieved as follows. First, we use (11) to solve for φ_{ii}. Second, we substitute the expression for φ_{ii} into (9) and solve for φ_{ij}. Finally, we employ this expression for φ_{ij} together with $\delta_{ij} = 1-\delta_{ii}$, and (B-13) to rewrite (11) as follows:

$$C_{ij} = C_{ii}\left(\frac{L_{Ci}\delta_{ii}\tau_{Ii}(L\alpha - L_{Ci}(\alpha + (1-\alpha)\delta_{ii}\tau_{Li} - \alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi}))}{\tau_{Li}(\alpha + (1-\alpha)\tau_{Ii})}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{B-18}$$

Taking the total differential of (B-18), using (B-7) to substitute out $d\delta_{ii}$ and (B-13) and (B-18) to define, respectively, L_{Cj} and C_{ij}, we have:

$$\begin{aligned} 0 = & \frac{\varepsilon-1}{\varepsilon}\frac{dC_{ij}}{C_{ij}} - \left(\frac{dC_{ii}}{C_{ii}}\frac{\varepsilon-1}{\varepsilon} - \frac{dL_{Ci}}{L_{Ci}}\right)\left(1 - \frac{\varepsilon(\varepsilon-1+\Phi_i)}{(\varepsilon-1)(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1})}\left(1 + \frac{L_{Ci}\delta_{ii}\tau_{Li}}{\Lambda_i}(1-\alpha+\alpha\tau_{Xi})\right)\right) \\ & + \frac{dL_{Ci}}{\Lambda_i}(\alpha + (1-\alpha)\delta_{ii}\tau_{Li} - \alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi}) - \frac{d\tau_{Ii}}{\tau_{Ii}}\frac{\alpha}{\alpha+(1-\alpha)\tau_{Ii}} - d\tau_{Xi}\alpha\frac{L_{Ci}(1-\delta_{ii})\tau_{Li}}{\Lambda_i} \\ & + d\tau_{Li}\left(\frac{L_{Ci}}{\Lambda_i}((1-\alpha)\delta_{ii} - (1-\delta_{ii})\alpha\tau_{Xi}) + \frac{1}{\tau_{Li}}\right) \end{aligned}$$

where $\Lambda_i \equiv \alpha L - L_{Ci}(\alpha + (1-\alpha)\delta_{ii}\tau_{Li} - \alpha(1-\delta_{ii})\tau_{Li}\tau_{Xi})$. Using (B-13), under symmetry we can rewrite the previous expression as follows:

$$\begin{aligned} 0 = & \frac{\varepsilon-1}{\varepsilon}\frac{dC_{ij}}{C_{ij}} - \frac{\alpha}{\alpha+(1-\alpha)\tau_{Ii}}\left(\frac{d\tau_{Ii}}{\tau_{Ii}} + \frac{d\tau_{Xi}}{\tau_{Xi}}\right) + \left(1 + \frac{(1-\alpha)\delta_{ii} - \alpha(1-\delta_{ii})\tau_{Xi}}{(1-\delta_{ii})\tau_{Xi}(\alpha+(1-\alpha)\tau_{Ii})}\right)\frac{d\tau_{Li}}{\tau_{Li}} \\ & - \left(1 - \frac{\varepsilon(\varepsilon-1+\Phi_i)}{(\varepsilon-1)(\varepsilon-1+\frac{\varepsilon\Phi_i}{\varepsilon-1})}\left(1 + \frac{\delta_{ii}(1-\alpha+\alpha\tau_{Xi})}{(1-\delta_{ii})\tau_{Xi}((1-\alpha)\tau_{Ii}+\alpha)}\right)\right)\left(\frac{\varepsilon-1}{\varepsilon}\frac{dC_{ii}}{C_{ii}} - \frac{dL_{Ci}}{L_{Ci}}\right) \\ & + \frac{\alpha + (1-\alpha)\delta_{ii}\tau_{Li} - \alpha(1-\delta_{ii})\tau_{Xi}\tau_{Li}}{(1-\delta_{ii})\tau_{Xi}\tau_{Li}((1-\alpha)\tau_{Ii}+\alpha)}\frac{dL_{Ci}}{L_{Ci}} \end{aligned} \tag{B-19}$$

Conditions (B-11), (B-17), and (B-19) can be used to find an explicit solution for either dL_{Ci}, dC_{ii} and dC_{ij} as linear functions of $d\tau_{Li}$, $d\tau_{Ii}$, and $d\tau_{Xi}$ (i.e., the solution for the unilateral deviations) or for $d\tau_{Li}$, $d\tau_{Ii}$, and $d\tau_{Xi}$ as linear functions of dL_{Ci}, dC_{ii} and dC_{ij}(i.e., the solution for the Nash problem with all policy instruments). Conditions (B-11), (B-17), and (B-19) also allow us to retrieve the solution for the Nash problem with only the production tax. All these expressions are available upon request.

B.2.3 Total differentials of the one-sector model

The total differentials computed in this section will be used only to study unilateral deviations. For this reason we can apply some simplifications to the total differentials defined in B.2.1. As explained in section A.4.2, when $\alpha = 1$ we get $Z_i = 0$, $L_{Ci} = L$ and $d\tau_{Li} = 0$ for $i = H, F$ so that $dZ_i = dL_{Ci} = 0$ for $i = H, F$. Also, $W_j = 1$ so that $dW_j = 0$ for $j = F$ and $d\tau_{Ij} = d\tau_{Xj} = 0$ for $j = F$. After taking the differentials, we evaluate them at the laissez-faire allocation ($\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$)

First, consider the case of heterogeneous firms. In this case, our objective is to retrieve 3 conditions as a function of $dW_i, dC_{ii}, dC_{ij}, d\tau_{Ii}$ and $d\tau_{Xi}$.

(1) To find the first condition, recall that (B-8) simplifies to:

$$d\varphi_{ij} = \frac{\varphi_{ij}}{\varphi_{ii}}d\varphi_{ii} - \frac{\varepsilon}{\varepsilon-1}\varphi_{ij}dW_i + \frac{\varepsilon}{\varepsilon-1}\varphi_{ij}d\tau_{Tij}, \quad i,j = H,F, \quad i \neq j \tag{B-20}$$

Second, from (B-6) we have:

$$d\varphi_{ii} = -\frac{\varphi_{ii}}{C_{ii}\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)}dC_{ii}, \quad i = H,F \tag{B-21}$$

Third, from (B-20) we have $d\varphi_{jj} = \frac{\varphi_{jj}}{\varphi_{ji}} d\varphi_{ji} - \frac{\varepsilon}{\varepsilon-1}\varphi_{jj} d\tau_{Tji}$ which, using (B-3), (B-21), and $d\tau_{Tji} = d\tau_{Xi}$ when $i = H, j = F$, can be written as:

$$d\varphi_{jj} = \frac{\varphi_{jj}}{C_{ii}\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)} \frac{\delta_{ii}}{1-\delta_{ii}} dC_{ii} - \frac{\varepsilon}{\varepsilon-1}\varphi_{jj} d\tau_{Xi}, \quad i = H, j = F \tag{B-22}$$

Finally, using (B-3) to express $d\varphi_{ij}$ together with (B-22) to substitute out $d\varphi_{jj}$ we have:

$$d\varphi_{ij} = \frac{\delta_{jj}}{1-\delta_{jj}}\varphi_{ij}\left(\frac{1}{C_{ii}\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)} \frac{\delta_{ii}}{1-\delta_{ii}} dC_{ii} + \frac{\varepsilon}{\varepsilon-1} d\tau_{Xi}\right), \quad i = H, j = F \tag{B-23}$$

Using (B-21), (B-23), $\delta_{jj} = \delta_{ii}$, and $d\tau_{Tij} = d\tau_{Ii}$ when $i = H, j = F$, we can rewrite (B-20) as follows:

$$(1-\delta_{ii})dW_i - (1-\delta_{ii})d\tau_{Ii} + \delta_{ii} d\tau_{Xi} + \frac{1-2\delta_{ii}}{(1-\delta_{ii})\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)} \frac{\varepsilon-1}{\varepsilon}\frac{dC_{ii}}{C_{ii}} = 0, \quad i = H, j = F \tag{B-24}$$

(2) To retrieve the second condition, recall that (13) simplifies to (A-20) which, using (11) and (12) together with $L_{Ci} = L$ for $i = H, F$, $\delta_{ji} = 1 - \delta_{ii}$ for $i, j = H, F$, $\tau_{Ij} = \tau_{Xj} = 1$ for $j = F$ can be rewritten as:

$$L(1-\delta_{ii})\tau_{Xi}\tau_{Li}W_i - L(1-\delta_{jj})\tau_{Lj} = 0, \quad i = H, j = F \tag{B-25}$$

Using (B-4) for $i = j$ together with (B-22) we can write:

$$d\delta_{jj} = \delta_{ii}(\varepsilon - 1 + \Phi_i)\left(\frac{\varepsilon}{\varepsilon-1} d\tau_{Xi} - \frac{\delta_{ii}}{1-\delta_{ii}} \frac{1}{\left(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i\right)} \frac{dC_{ii}}{C_{ii}}\right) \tag{B-26}$$

Taking the total differential of (B-25) and using (B-26) to substitute out $d\delta_{jj}$, and (B-4) to substitute out $d\delta_{ii}$, and evaluating it at the laissez-faire, we have:

$$dW_i + \left(1 + \frac{\delta_{ii}\varepsilon(\varepsilon - 1 + \Phi_i)}{(\varepsilon-1)(1-\delta_{ii})}\right) d\tau_{Xi} - \frac{\delta_{ii}(\varepsilon - 1 + \Phi_i)}{C_{ii}(1-\delta_{ii})^2(\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i)} dC_{ii} = 0, \quad i = H, j = F \tag{B-27}$$

(3) We can rewrite (B-5) by imposing $dL_{Cj} = 0$, using (B-23) to substitute out $d\varphi_{ij}$, and $\delta_{ij} = 1 - \delta_{jj}$ (implying $d\delta_{ij} = -d\delta_{jj}$) together with (B-26) to substitute out $d\delta_{ij}$ we obtain:

$$\frac{\varepsilon-1}{\varepsilon}\frac{dC_{ij}}{C_{ij}} + \frac{\delta_{ii}}{1-\delta_{ii}}\left(\frac{\varepsilon(\varepsilon - 1 + \Phi_i)}{\varepsilon - 1} - 1\right) d\tau_{Xi} + \frac{\delta_{ii}^2}{(1-\delta_{ii})^2}\frac{1}{\varepsilon - 1 + \frac{\varepsilon}{\varepsilon-1}\Phi_i}\left(\frac{\varepsilon-1}{\varepsilon} - \varepsilon + 1 - \Phi_i\right)\frac{dC_{ii}}{C_{ii}} = 0$$

$$i = H, j = F \tag{B-28}$$

Conditions (B-24), (B-27), and (B-28) can be used to derive an explicit solution for W_i, dC_{ii} and dC_{ij} as linear functions of $d\tau_{Ii}$ and $d\tau_{Xi}$. These expressions are available upon request.

Finally, consider the case with homogeneous firms. In this case, we need to express dW_i as a function of $d\tau_{Ii}$ and $d\tau_{Xi}$. For this purpose, we can substitute conditions (16) and (17) into the trade balance (A-20). Taking the total differential of this condition and evaluating it at the free-trade allocation we get:

$$dW_i = \frac{\varepsilon\tau^\varepsilon}{\tau + (2\varepsilon-1)\tau^\varepsilon} d\tau_{Ii} - \frac{\tau + (\varepsilon-1)\tau^\varepsilon}{(\tau + (2\varepsilon-1)\tau^\varepsilon)} d\tau_{Xi} \tag{B-29}$$

C The Planner Allocation

In this appendix we first set up the planner problem and solve it using a three-stage approach. Next, we prove the Lemmata and Propositions of Section 3.

C.1 The Planner Problem

The full planner problem can be written as follows. The planner maximizes:

$$\sum_{i=H,F} U_i = \sum_{i=H,F} \left[\alpha \log \left(\sum_{j=H,F} C_{ij}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} + (1-\alpha) \log Z_i \right]$$

with respect to C_{ij},L_{Cij}, Z_i, N_i, $c_{ij}(\varphi)$, $l_{ij}(\varphi)$, φ_{ij}, for $i, j = H, F$ and subject to:

$$\begin{aligned}
C_{ij} &= \left[N_j \int_{\varphi_{ij}}^{\infty} c_{ij}(\varphi)^{\frac{\varepsilon-1}{\varepsilon}} dG(\varphi) \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad i, j = H, F \\
l_{ij}(\varphi) &= \frac{\tau_{ij} c_{ij}(\varphi)}{\varphi} + f_{ij}, \quad i, j = H, F \\
L_{Cij} &= N_j \int_{\varphi_{ij}}^{\infty} l_{ij}(\varphi) dG(\varphi), \quad i, j = H, F \\
L_{Ci} &= N_i f_E + \sum_{j=H,F} L_{Cji}, \quad i = H, F \\
\sum_{i=H,F} L_i &= \sum_{i=H,F} L_{Ci} + \sum_{i=H,F} Z_i
\end{aligned}$$

Notice that by combining L_{Cij} and $l_{ij}(\varphi)$ we get:

$$L_{Cij} = \tau_{ij} N_j \int_{\varphi_{ij}}^{\infty} \frac{c_{ij}(\varphi)}{\varphi} dG(\varphi) + N_j f_{ij} (1 - G(\varphi_{ij})), \quad i, j = H, F$$

This problem can be split into three separate stages. The proof that this approach is equivalent to solving the full planner problem in a single stage is available on request.

C.2 First stage

Here, we derive the results used in Section 3.1.

C.2.1 First-stage optimality conditions

At the first stage, the planner solves the problem stated in (18). Taking total differentials with respect to $c_{ij}(\varphi)$, $l_{ij}(\varphi)$ and φ_{ij}:

$$du_{ij} = \int_{\varphi_{ij}}^{\infty} \frac{\partial u_{ij}}{\partial c_{ij}(\varphi)} dc_{ij}(\varphi) dG(\varphi) + \frac{\partial u_{ij}}{\partial \varphi_{ij}} d\varphi_{ij} = 0 \tag{C-1}$$

$$dc_{ij}(\varphi) = \frac{\partial q_{ij}(\varphi)}{\partial l_{ij}(\varphi)} dl_{ij}(\varphi) \tag{C-2}$$

$$N_j \int_{\varphi_{ij}}^{\infty} dl_{ij}(\varphi) dG(\varphi) + \frac{\partial L_{Cij}}{\partial \varphi_{ij}} d\varphi_{ij} = 0 \tag{C-3}$$

for $i, j = H, F$. By using (C-2) and (C-3) to substitute out $d\varphi_{ij}$ from (C-1) we get:

$$\int_{\varphi_{ij}}^{\infty} \left(\frac{\partial u_{ij}}{\partial c_{ij}(\varphi)} - \frac{\frac{\partial u_{ij}}{\partial \varphi_{ij}}}{\frac{\partial L_{Cij}}{\partial \varphi_{ij}}} \frac{N_j}{\frac{\partial q_{ij}(\varphi)}{\partial l_{ij}(\varphi)}} \right) dc_{ij}(\varphi) dG(\varphi) = 0, \quad i, j = H, F \tag{C-4}$$

This condition holds for every $dc_{ij}(\varphi)$ and therefore:

$$\frac{\partial u_{ij}}{\partial c_{ij}(\varphi)} \frac{\partial q_{ij}(\varphi)}{\partial l_{ij}(\varphi)} = \frac{\partial u_{ij}}{\partial L_{Cij}} N_j, \quad i, j = H, F$$

for all $\varphi \in [\varphi_{ij}, \infty)$. As a consequence:

$$\frac{\partial u_{ij}}{\partial c_{ij}(\varphi_1)} \frac{\partial q_{ij}(\varphi_1)}{\partial l_{ij}(\varphi_1)} = \frac{\partial u_{ij}}{\partial c_{ij}(\varphi_2)} \frac{\partial q_{ij}(\varphi_2)}{\partial l_{ij}(\varphi_2)},$$

for any $\varphi_1 \in [\varphi_{ij}, \infty)$ and $\varphi_2 \in [\varphi_{ij}, \infty)$ which coincides with condition (19) in the main text.

C.2.2 First-stage aggregate production function

Using the functional forms, we get:

$$\begin{aligned}
\frac{\partial u_{ij}}{\partial c_{ij}(\varphi)} &= N_j C_{ij}^{\frac{1}{\varepsilon}} c_{ij}(\varphi)^{-\frac{1}{\varepsilon}} \\
\frac{\partial u_{ij}}{\partial \varphi_{ij}} &= \frac{\varepsilon}{\varepsilon - 1} N_j C_{ij}^{\frac{1}{\varepsilon}} c_{ij}(\varphi_{ij})^{\frac{\varepsilon-1}{\varepsilon}} \\
\frac{\partial q_{ij}(\varphi)}{\partial l_{ij}(\varphi)} &= \frac{\varphi}{\tau_{ij}} \\
\frac{\partial L_{Cij}}{\partial \varphi_{ij}} &= N_j l_{ij}(\varphi_{ij})
\end{aligned}$$

Plugging in these functional forms into (C-4) we obtain:

$$\int_{\varphi_{ij}}^{\infty} \left(c_{ij}(\varphi)^{-1/\varepsilon} - \frac{\varepsilon}{\varepsilon - 1} \frac{\tau_{ij}}{\varphi} \frac{c_{ij}(\varphi_{ij})^{\frac{\varepsilon-1}{\varepsilon}}}{l_{ij}(\varphi_{ij})} \right) dc_{ij}(\varphi) dG(\varphi) = 0, \quad i, j = H, F \tag{C-5}$$

This condition holds for every $dc_{ij}(\varphi)$ and therefore:

$$c_{ij}(\varphi) = \left(\frac{\varepsilon}{\varepsilon - 1}\right)^{-\varepsilon} \frac{c_{ij}(\varphi_{ij})^{1-\varepsilon}}{l_{ij}(\varphi_{ij})^{-\varepsilon}} \tau_{ij}^{-\varepsilon} \varphi^{\varepsilon}, \quad i, j = H, F \tag{C-6}$$

Substituting (C-6) into the definition of C_{ij}, using the definition of $\widetilde{\varphi}_{ij}$, and noting that $N_{ij} = [1 - G(\varphi_{ij})]N_j$, we get:

$$c_{ij}(\varphi_{ij})^{1-\varepsilon} = N_{ij}^{-\frac{\varepsilon}{\varepsilon-1}} C_{ij} \left(\frac{\varepsilon}{\varepsilon - 1}\right)^{\varepsilon} l_{ij}(\varphi_{ij})^{-\varepsilon} \tau_{ij}^{\varepsilon} \widetilde{\varphi}_{ij}^{-\varepsilon}, \quad i, j = H, F$$

If we substitute this back into (C-6) we obtain:

$$c_{ij}(\varphi) = N_{ij}^{-\frac{\varepsilon}{\varepsilon-1}} C_{ij} \left(\frac{\widetilde{\varphi}_{ij}}{\varphi}\right)^{-\varepsilon}, \quad i, j = H, F \tag{C-7}$$

Finally, we can aggregate the production function as follows:

$$\begin{aligned} L_{Cij} &= N_j \int_{\varphi_{ij}}^{\infty} l_{ij}(\varphi) dG(\varphi) = \tau_{ij} N_{ij} \int_{\varphi_{ij}}^{\infty} \frac{c_{ij}(\varphi)}{\varphi} \frac{dG(\varphi)}{1 - G(\varphi_{ij})} + f_{ij} N_{ij} \\ &= \tau_{ij} N_{ij}^{-\frac{1}{\varepsilon-1}} \frac{C_{ij}}{\widetilde{\varphi}_{ij}} + f_{ij} N_{ij}, \quad i, j = H, F \end{aligned} \tag{C-8}$$

This leads to the aggregate production function (20) in the main text:

$$Q_{Cij}(\tilde{\varphi}_{ij}, N_j, L_{Cij}) \equiv \frac{\tilde{\varphi}_{ij}}{\tau_{ij}} \left\{ [N_j(1 - G(\varphi_{ij})]^{\frac{1}{\varepsilon-1}} L_{Cij} - f_{ij}[N_j(1 - G(\varphi_{ij})]^{\frac{\varepsilon}{\varepsilon-1}} \right\}, \quad i, j = H, F,$$

where $Q_{Cij}(\tilde{\varphi}_{ij}, N_j, L_{Cij}) = C_{ij}$.

C.2.3 First-stage comparison between planner and market allocation

We want to verify that the consumption of individual varieties chosen by the planner coincides with the one of the market allocation conditional on C_{ij}, N_{ij} and $\tilde{\varphi}_{ij}$ being the same. Recall that the demand function is $c_{ij}(\varphi) = \left(\frac{p_{ij}(\varphi)}{P_{ij}}\right)^{-\varepsilon} C_{ij}$. Since the price index is given by $P_{ij} = N_{ij}^{\frac{1}{1-\varepsilon}} p_{ij}(\tilde{\varphi}_{ij})$, it follows that $\frac{p_{ij}(\varphi)}{p_{ij}(\tilde{\varphi}_{ij})} = \frac{\tilde{\varphi}_{ij}}{\varphi}$. Thus, we can conclude that condition (C-7) holds also in the market equilibrium.

C.2.4 First-stage optimality conditions with homogeneous firms

In this case, the problem stated in (18) simplifies to choosing $c_{ij}(\omega)$ and $l_{ij}(\omega)$ for $i, j = H, F$ under the assumptions that $G(\varphi)$ is a degenerate distribution and that $f_{ij} = 0$ for $i, j = H, F$.

Solving this problem gives the same condition as derived with heterogeneous firms:

$$\frac{\partial u_{ij}}{\partial c_{ij}(\omega_1)} \frac{\partial q_{ij}(\omega_1)}{\partial l_{ij}(\omega_1)} = \frac{\partial u_{ij}}{\partial c_{ij}(\omega_2)} \frac{\partial q_{ij}(\omega_2)}{\partial l_{ij}(\omega_2)}$$

This implies that all firms will employ the same quantity of labor and produce the same amount of each variety, i.e., $l_{ij}(\omega) = l_{ij}$ and $c_{ij}(\omega) = c_{ij} \quad \forall \omega \in [0, N_j]$.

C.2.5 First-stage aggregate production function with homogeneous firms

Following the same steps as with heterogeneous firms we can derive the aggregate level of consumption

$$C_{ij} = N_j^{\frac{\varepsilon}{\varepsilon-1}} c_{ij}, i, j = H, F, \tag{C-9}$$

Hence, the aggregate production now simplifies to:

$$Q_{Cij}(N_j, L_{Cij}) \equiv \frac{1}{\tau_{ij}} N_j^{\frac{1}{\varepsilon-1}} L_{Cij}, \quad i, j = H, F, \tag{C-10}$$

where $Q_{Cij}(N_j, L_{Cij}) = C_{ij}$.

C.2.6 First-stage comparison between planner and market allocation with homogeneous firms

As for the case of heterogeneous firms, it is sufficient to recall that when firms are homogeneous $c_{ij} = \left(\frac{p_{ij}}{P_{ij}}\right)^{-\varepsilon} C_{ij}$ and $P_{ij} = N_{ij}^{\frac{1}{1-\varepsilon}} p_{ij}$, which implies that condition (C-9) holds also in the market equilibrium.

C.3 Second Stage

Here we derive the results of Section 3.2.

C.3.1 Second-stage optimality conditions

At the second stage the planner solves the problem described in (21). Taking total differentials:

$$\begin{aligned}
&\sum_{i=H,F} \sum_{j=H,F} \frac{\partial u_i}{\partial C_{ij}} dC_{ij} = 0 \\
&dN_i = -\frac{1}{f_E} \sum_{j=H,F} dL_{Cji}, \quad i = H, F \\
&dC_{ij} = \frac{\partial Q_{Cij}}{\partial N_j} dN_j + \frac{\partial Q_{Cij}}{\partial \tilde{\varphi}_{ij}} d\tilde{\varphi}_{ij} + \frac{\partial Q_{Cij}}{\partial L_{Cij}} dL_{Cij}, \quad i, j = H, F
\end{aligned}$$

Substituting the differentials of the constraints into the objective, we obtain:

$$\sum_{i=H,F} \sum_{j=H,F} \frac{\partial u_i}{\partial C_{ij}} \left[\frac{\partial Q_{Cij}}{\partial \tilde{\varphi}_{ij}} d\tilde{\varphi}_{ij} + \frac{\partial Q_{Cij}}{\partial L_{Cij}} dL_{Cij} - \frac{\partial Q_{Cij}}{\partial N_j} \frac{1}{f_E} \sum_{k=H,F} dL_{Ckj} \right] = 0$$

Collecting terms:

$$\sum_{j=H,F} \sum_{i=H,F} \frac{\partial u_i}{\partial C_{ij}} \frac{\partial Q_{Cij}}{\partial \tilde{\varphi}_{ij}} d\tilde{\varphi}_{ij} + \sum_{j=H,F} \sum_{i=H,F} \left[\frac{\partial u_i}{\partial C_{ij}} \frac{\partial Q_{Cij}}{\partial L_{Cij}} - \sum_{k=H,F} \frac{\partial u_k}{\partial C_{kj}} \frac{\partial Q_{Ckj}}{\partial N_j} \frac{1}{f_E} \right] dL_{Cij} = 0 \tag{C-11}$$

Since (C-11) should hold for any $d\tilde{\varphi}_{ij}$ and dL_{Cij} it follows that:

$$\frac{\partial Q_{Cij}}{\partial \tilde{\varphi}_{ij}} = 0, \quad i,j = H,F \tag{C-12}$$
$$\sum_{k=H,F} \frac{\partial u_k}{\partial C_{kj}} \frac{\partial Q_{Ckj}}{\partial N_j} = f_E \frac{\partial u_i}{\partial C_{ij}} \frac{\partial Q_{Cij}}{\partial L_{Cij}}, \quad i,j = H,F,$$

which leads to conditions (22), (23) and (24) in the main text.

C.3.2 Second-stage aggregate production function

Using the functional forms, we obtain the following derivatives:

$$\frac{\partial u_i}{\partial C_{ij}} = \frac{C_{ij}^{\frac{-1}{\varepsilon}}}{\sum_{k=H,F} C_{ik}^{\frac{\varepsilon-1}{\varepsilon}}} = \left(\frac{C_{ij}}{C_i}\right)^{\frac{-1}{\varepsilon}} C_i^{-1}, \quad i,j = H,F \tag{C-13}$$

$$\frac{\partial Q_{Cji}}{\partial N_i} = \frac{\tilde{\varphi}_{ji}}{\tau_{ji}} [N_i(1-G(\varphi_{ji}))]^{\frac{2-\varepsilon}{\varepsilon-1}} \frac{L_{Cji}}{(\varepsilon-1)}(1-G(\varphi_{ji})) - \frac{\tilde{\varphi}_{ji}}{\tau_{ji}} f_{ji}[N_i(1-G(\varphi_{ji}))]^{\frac{1}{\varepsilon-1}} \left(\frac{\varepsilon}{\varepsilon-1}\right)(1-G(\varphi_{ji})), \quad i,j = H,F$$

$$\frac{\partial Q_{Cji}}{\partial \tilde{\varphi}_{ji}} = \frac{1}{\tau_{ji}} \left\{ [N_i(1-G(\varphi_{ji}))]^{\frac{1}{\varepsilon-1}} L_{Cji} - f_{ji}[N_i(1-G(\varphi_{ji}))]^{\frac{\varepsilon}{\varepsilon-1}} \right\} - \frac{[N_i(1-G(\varphi_{ji}))]^{\frac{2-\varepsilon}{\varepsilon-1}}}{\tau_{ji}(\tilde{\varphi}_{ji}^{\varepsilon-1} - \varphi_{ij}^{\varepsilon-1})} L_{Cji}(1-G(\varphi_{ji}))\tilde{\varphi}_{ji}^{\varepsilon-1} N_i$$
$$+ \frac{f_{ji}[N_i(1-G(\varphi_{ji}))]^{\frac{1}{\varepsilon-1}}}{\tau_{ji}(\tilde{\varphi}_{ji}^{\varepsilon-1} - \varphi_{ji}^{\varepsilon-1})} \varepsilon(1-G(\varphi_{ji}))\tilde{\varphi}_{ji}^{\varepsilon-1} N_i, \quad i,j = H,F$$

$$\frac{\partial Q_{Cji}}{\partial L_{Cji}} = \frac{\tilde{\varphi}_{ji}}{\tau_{ji}} [N_i(1-G(\varphi_{ji}))]^{\frac{1}{\varepsilon-1}}, \quad i,j = H,F$$

Then, these conditions can be substituted into (24) to obtain:

$$L_{Cji}\left(1 - \frac{\tilde{\varphi}_{ji}^{\varepsilon-1}}{(\tilde{\varphi}_{ji}^{\varepsilon-1} - \varphi_{ji}^{\varepsilon-1})}\right) = f_{ji}[N_i(1-G(\varphi_{ji}))]\left(1 - \frac{\varepsilon\tilde{\varphi}_{ji}^{\varepsilon-1}}{(\tilde{\varphi}_{ji}^{\varepsilon-1} - \varphi_{ji}^{\varepsilon-1})}\right), \quad i,j = H,F$$

It follows that:

$$L_{Cji} = f_{ji} N_i (1-G(\varphi_{ji})) \left(\frac{\varphi_{ji}^{\varepsilon-1} + (\varepsilon-1)\tilde{\varphi}_{ji}^{\varepsilon-1}}{\varphi_{ji}^{\varepsilon-1}}\right), \quad i,j = H,F \tag{C-14}$$

Moreover, combining the derivatives above with condition (23) we obtain:

$$f_E = \sum_{j=H,F} \left[\frac{L_{Cji}}{N_i(\varepsilon-1)} - \frac{\varepsilon}{(\varepsilon-1)} f_{ji}(1-G(\varphi_{ji}))\right] \tag{C-15}$$

This implies that

$$\varepsilon N_i f_E + \sum_{j=H,F} \varepsilon(1-G(\varphi_{ji}))N_i f_{ji} = f_E N_i + \sum_{j=H,F} L_{Cji}, \quad i = H,F. \tag{C-16}$$

Using (C-14) and $L_{Ci} = f_E N_i + \sum_{j=H,F} L_{Cji}$ to substitute out L_{Cji} and $f_E N_i$ in (C-16), we find:

$$L_{Ci} = \sum_{j=H,F} \varepsilon f_{ji} N_i (1-G(\varphi_{ji})) \left(\frac{\tilde{\varphi}_{ji}}{\varphi_{ji}}\right)^{\varepsilon-1}, \quad i = H,F \tag{C-17}$$

We use this last condition to solve for N_i:

$$N_i = \frac{L_{Ci}}{\varepsilon \sum_{j=H,F}\left[f_{ji}(1-G(\varphi_{ji}))\left(\frac{\tilde{\varphi}_{ji}}{\varphi_{ji}}\right)^{\varepsilon-1}\right]}, \quad i=H,F \tag{C-18}$$

We now substitute (C-14) and (C-18) into the definition (20) to obtain (25) in the main text.

C.3.3 Second-stage comparison between planner and market allocation

Next, we check if the optimality conditions of the second stage are satisfied in the market allocation. First, consider condition (22). Plugging the relevant derivatives in (C-13), we obtain:

$$\frac{1}{C_i}\left(\frac{C_{ii}}{C_i}\right)^{\frac{-1}{\varepsilon}}\frac{\tilde{\varphi}_{ii}}{\tau_{ii}}[N_i(1-G(\varphi_{ii})]^{\frac{1}{\varepsilon-1}} = \frac{1}{C_j}\left(\frac{C_{ji}}{C_j}\right)^{\frac{-1}{\varepsilon}}\frac{\tilde{\varphi}_{ji}}{\tau_{ji}}[N_i(1-G(\varphi_{ji})]^{\frac{1}{\varepsilon-1}}, \quad i=H,F, \quad j\neq i \tag{C-19}$$

Now consider the market allocation. Using (8) jointly with (9), (11) and (A-18) after some manipulations we get:

$$\frac{1}{C_i}\left(\frac{C_{ii}}{C_i}\right)^{\frac{-1}{\varepsilon}}\frac{\tilde{\varphi}_{ii}}{\tau_{ii}}[N_i(1-G(\varphi_{ii})]^{\frac{1}{\varepsilon-1}} = \frac{1}{C_j}\left(\frac{C_{ji}}{C_j}\right)^{\frac{-1}{\varepsilon}}\frac{\tilde{\varphi}_{ji}}{\tau_{ji}}[N_i(1-G(\varphi_{ji})]^{\frac{1}{\varepsilon-1}}\left(\frac{C_j}{C_i}\frac{\tau_{ji}}{\tau_{ii}}\frac{\varphi_{ii}}{\varphi_{ji}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{f_{ii}}{f_{ji}}\right)^{\frac{-1}{\varepsilon}} i=H,F, j\neq i$$

Thus, in the market allocation:

$$\frac{\partial u_i}{\partial C_{ii}}\frac{\partial Q_{Cii}}{\partial L_{Cii}} = \Omega_{P2ji}\frac{\partial u_j}{\partial C_{ji}}\frac{\partial Q_{Cji}}{\partial L_{Cji}}, \quad i=H,F, \quad j\neq i,$$

where Ω_{P2ji} is the wedge between the planner and the market allocation. Under symmetry:

$$\Omega_{P2ji} = \left(\frac{\tau_{ji}}{\tau_{ii}}\frac{\varphi_{ii}}{\varphi_{ji}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{f_{ii}}{f_{ji}}\right)^{\frac{-1}{\varepsilon}}, \quad i=H,F, \quad j\neq i$$

Using condition (9), this can be written as $\Omega_{P2ji} = \tau_{Tji}^{-1}$. This leads to condition (29).

Next, consider the planner's optimality condition (23). Using the functional forms from (C-13), this corresponds to (C-15).

We now want to check if this condition is also fulfilled by the market allocation. Recalling the labor market clearing condition in (A-17) and that $L_{Ci} = \sum_{j=H,F} L_{Cji} + N_i f_E$, we obtain condition (C-15) and this proves that (23) is satisfied in any market allocation.

Finally, consider the planner's optimality condition (24). As shown in Section C.3.2, this condition can be rewritten as (C-14). Now consider the market allocation. Appendix C.2.3 shows that condition (C-7) holds in the market equilibrium. As a consequence, also condition (C-8) holds in the market equilibrium. We can then use (C-8) and substitute it in equation (11) to obtain (C-14). This confirms that this condition and then (24) always holds both in the planner and in the market allocation.

C.3.4 Second-stage optimality conditions with homogeneous firms

In this case, the problem is stated in (21) where $C_{ij} = Q_{Cij}(N_j, L_{Cij})$ simplifies to (C-10) and the planner chooses C_{ij}, L_{Cij}, N_i for $i,j=H,F$ only, leading to conditions (22) and (23).

C.3.5 Second-stage aggregate production function with homogeneous firms

We can use the functional forms to find the aggregate production function. As a first step, we obtain the following derivatives:

$$
\begin{aligned}
\frac{\partial u_i}{\partial C_{ij}} &= \frac{C_{ij}^{\frac{-1}{\varepsilon}}}{\sum_{k=H,F} C_{ik}^{\frac{\varepsilon-1}{\varepsilon}}} = \left(\frac{C_{ij}}{C_i}\right)^{\frac{-1}{\varepsilon}} C_i^{-1}, \quad i,j=H,F \\
\frac{\partial Q_{Cji}}{\partial N_i} &= \frac{1}{\tau_{ji}} N_i^{\frac{2-\varepsilon}{\varepsilon-1}} \frac{L_{Cji}}{\varepsilon-1}, \quad i,j=H,F \\
\frac{\partial Q_{Cji}}{\partial L_{Cji}} &= \frac{1}{\tau_{ji}} N_i^{\frac{1}{\varepsilon-1}}, \quad i,j=H,F
\end{aligned}
$$

Substituting these functional forms into (22) and (23), we obtain:

$$
C_{ji} = \tau_{ij}^{-\varepsilon} \left(\frac{C_i}{C_j}\right)^{\varepsilon-1} C_{ii}, \quad i,j=H,F \tag{C-20}
$$

and

$$
f_E = \sum_{j=H,F} \frac{L_{Cji}}{N_i(\varepsilon-1)}, \quad i=H,F \tag{C-21}
$$

Using $L_{Ci} = f_E N_i + \sum_{j=H,F} L_{Cji}$ to substitute out the term $\sum_{j=H,F} L_{Cji}$ we obtain:

$$
N_i = \frac{L_{Ci}}{\varepsilon f_E}, \quad i=H,F \tag{C-22}
$$

We can then substitute the first-stage aggregate production function (C-10) into (C-20) to get:

$$
L_{Cji} = \tau_{ji}^{1-\varepsilon} \left(\frac{C_i}{C_j}\right)^{\varepsilon-1} L_{Cii}, \quad i=H,F, j\neq i \tag{C-23}
$$

Substituting this into the labor market clearing $L_{Ci} = f_E N_i + \sum_{j=H,F} L_{Cji}$ and using condition (C-22), we find that:

$$
L_{Cji} = \tau_{ji}^{1-\varepsilon} \left(\frac{C_i}{C_j}\right)^{\varepsilon-1} L_{Ci} \frac{\varepsilon-1}{\varepsilon} \left[\sum_{k=H,F} \tau_{ki}^{1-\varepsilon} \left(\frac{C_i}{C_k}\right)^{\varepsilon-1}\right]^{-1}, \quad i,j=H,F
$$

Using again the definition of the first-stage aggregate production function (C-10), we get

$$
Q_{ij}(L_{Ci}, L_{Cj}) = \frac{\varepsilon-1}{\varepsilon} \tau_{ij}^{-\varepsilon} (\varepsilon f_E)^{\frac{-1}{\varepsilon-1}} L_{Cj}^{\frac{\varepsilon}{\varepsilon-1}} \left(\frac{C_j}{C_i}\right)^{\varepsilon-1} \left[\sum_{k=H,F} \tau_{kj}^{1-\varepsilon} \left(\frac{C_j}{C_k}\right)^{\varepsilon-1}\right]^{-1}, \quad i,j=H,F \tag{C-24}
$$

where $Q_{ij}(L_{Ci}, L_{Cj}) = C_{ij}$.

C.3.6 Second-stage comparison between planner and market allocation with homogeneous firms

Next, we check if the optimality conditions of the second stage are satisfied in the market allocation.

First, consider condition (C-20), which can be written as:

$$\frac{1}{C_i}\left(\frac{C_{ii}}{C_i}\right)^{\frac{-1}{\varepsilon}} = \frac{1}{C_j}\left(\frac{C_{ji}}{C_j}\right)^{\frac{-1}{\varepsilon}}\frac{1}{\tau_{ji}}, \quad i = H, F, \quad j \neq i \tag{C-25}$$

Now consider the market allocation. From the demand functions we get

$$\frac{C_{ii}}{C_{ji}} = \left(\frac{P_{ii}}{P_{ji}}\right)^{-\varepsilon}\left(\frac{C_i}{C_j}\right)^{1-\varepsilon}\left(\frac{P_iC_i}{P_jC_j}\right)^{\varepsilon}, \quad i = H, F, \quad j \neq i$$

This can also be written as:

$$\frac{1}{C_i}\left(\frac{C_{ii}}{C_i}\right)^{\frac{-1}{\varepsilon}} = \frac{1}{C_j}\left(\frac{C_{ji}}{C_j}\right)^{\frac{-1}{\varepsilon}}\frac{1}{\tau_{ji}}\tau_{ji}\frac{P_{ii}}{P_{ji}}\frac{P_jC_j}{P_iC_i}, \quad i = H, F, \quad j \neq i$$

In other words, in the market allocation:

$$\frac{\partial u_i}{\partial C_{ii}}\frac{\partial Q_{Cii}}{\partial L_{Cii}} = \frac{\partial u_j}{\partial C_{ji}}\frac{\partial Q_{Cji}}{\partial L_{Cji}}\Omega_{P2ji}, \quad i = H, F, \quad j \neq i$$

where $\Omega_{P2ji} \equiv \tau_{Tji}^{-1}\frac{P_jC_j}{P_iC_i}$ is the wedge between the planner and the market allocation. Under symmetry $\Omega_{P2ji} = \tau_{Tji}^{-1}$.

Next, consider the planner's optimality condition (C-21). We now want to check if this condition is also fulfilled in the market allocation. Recalling the labor market clearing requires $L_{Ci} = \varepsilon f_E N_i$ and that $L_{Ci} = \sum_{j=H,F} L_{Cji} + N_i f_E$, we obtain condition (C-21) and this proves that this condition is satisfied in any market allocation.

C.4 Third stage

Here we derive the results of Section 3.3.

C.4.1 Third-stage optimality conditions

In the third stage, the planner chooses C_{ij}, Z_i and L_{Ci} for $i, j = H, F$ to solve the maximization problem in (26). Taking total differentials of the objective function and of the constraints, we get:

$$\begin{aligned}
\sum_{i=H,F} dU_i &= \sum_{i=H,F}\sum_{j=H,F}\frac{\partial U_i}{\partial C_{ij}}dC_{ij} + \sum_{i=H,F}\frac{\partial U_i}{\partial Z_i}dZ_i \\
dC_{ij} &= \frac{\partial Q_{Cij}}{\partial L_{Cj}}dL_{Cj}, \quad i, j = H, F \\
dQ_{Zi} &= \frac{\partial Q_{Zi}}{\partial L_{Ci}}dL_{Ci}, \quad i = H, F \\
\sum_{i=H,F} dQ_{Zi} &= \sum_{i=H,F} dZ_i
\end{aligned}$$

Substituting the total differentials of the constraints into the total differential of the objective and rearranging terms, we obtain:

$$\sum_{k=H,F} dU_k = \sum_{k=H,F} \left[\sum_{l=H,F} \frac{\partial U_l}{\partial C_{lk}} \frac{\partial Q_{Clk}}{\partial L_{Ck}} + \frac{\partial U_k}{\partial Z_k} \frac{\partial Q_{Zk}}{\partial L_{Ck}} \right] dL_{Ck} + \left[\frac{\partial U_i}{\partial Z_i} - \frac{\partial U_j}{\partial Z_j} \right] dZ_j, \quad i = H, \quad j = F$$

It follows that at the optimum each term needs to equal zero, which leads to conditions (27) and (28) in the main text.

C.4.2 Third-stage comparison between planner and market allocation

Here, we compare the market allocation with the allocation emerging from the third stage of the planner problem. Using the functional forms, we obtain:

$$\begin{aligned} \frac{\partial U_i}{\partial Z_i} &= \frac{1-\alpha}{Z_i}, \quad i = H, F \\ \frac{\partial U_j}{\partial C_{ji}} &= \alpha C_{ji}^{\frac{-1}{\varepsilon}} C_j^{-\frac{\varepsilon-1}{\varepsilon}} \quad i, j = H, F \\ \frac{\partial Q_{Cji}}{\partial L_{Ci}} &= \frac{\varepsilon}{\varepsilon - 1} \frac{C_{ji}}{L_{Ci}}, \quad i, j = H, F \\ \frac{\partial Q_{Zi}}{\partial L_{Ci}} &= -1, \quad i = H, F \end{aligned} \tag{C-26}$$

First consider condition (27). Using (C-26) we get that $(1-\alpha)Z_j = (1-\alpha)Z_i$. This condition is satisfied in any symmetric market allocation.

Next consider condition (28). Using (C-26) we obtain:

$$\sum_{j=H,F} \frac{\alpha}{1-\alpha} \frac{Z_i}{L_{Ci}} \frac{1}{C_j} \left(\frac{C_{ji}}{C_j} \right)^{-\frac{1}{\varepsilon}} \frac{\varepsilon}{\varepsilon - 1} C_{ji} = 1, \quad i = H, F \tag{C-27}$$

From (A-1) and (A-3) the price of the differentiated bundle in the market allocation is given by:

$$P_{ji} = \frac{\alpha}{1-\alpha} Z_j \left(\frac{C_{ji}}{C_j} \right)^{-\frac{1}{\varepsilon}} \frac{1}{C_j}$$

Substituting the price into (C-27) we have:

$$\sum_{j=H,F} \frac{\varepsilon}{\varepsilon - 1} \frac{P_{ji} C_{ji}}{L_{Ci}} \frac{Z_i}{Z_j} = 1, \quad i = H, F \tag{C-28}$$

Finally recall that from (11) and (12) in the two-sector model we have:

$$P_{ji} C_{ji} = \delta_{ji} L_{Ci} \tau_{Tji} \tau_{Li}$$

so that (C-28) can be further rewritten as:

$$\frac{\varepsilon}{\varepsilon - 1} \tau_{Li} \left[1 + \delta_{ji} \tau_{Tji} \frac{Z_i}{Z_j} \right] = 1, \quad i = H, F, \quad j \neq i$$

We can thus define the third stage wedge between the planner and the market allocation as follows:

$$\Omega_{3Pi} \equiv \frac{\varepsilon}{\varepsilon-1}\tau_{Li}\left[\delta_{ii}+\delta_{ji}\tau_{Tji}\frac{Z_i}{Z_j}\right]$$

In the symmetric allocation $\Omega_{3Pi}=1$ if $\tau_L=\frac{\varepsilon-1}{\varepsilon}$ and $\tau_{Tij}=1$.

C.4.3 Third-stage optimality conditions with homogeneous firms

In the third stage, the planner chooses C_{ij}, Z_i and L_{Ci} for $i,j=H,F$ to solve a problem akin to problem (26) with the only difference that $Q_{Cij}(L_{Ci},L_{Cj})$ is implicitly defined in (C-24). Taking total differentials of the objective function and of the constraints we obtain conditions (27) and (28), like in the heterogeneous-firm case.

C.4.4 Third-stage comparison between planner and market allocation with homogeneous firms

As a first step, we show that at the optimum the derivatives implied by the functional forms are identical to those of the case with heterogeneous firms. While this is obvious for the first, the second and the fourth condition in (C-26), it needs to be proven for $\partial Q_{Cji}/\partial L_{Ci}$.

Taking total differentials of condition (C-24):

$$dQ_{Cij}=\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{C_{ij}}{L_{Cj}}dL_{Cj}$$
$$+C_{ij}(\varepsilon-1)\left[\left(\frac{C_j}{C_i}\right)^{-1}d\left(\frac{C_j}{C_i}\right)-\left[\sum_{k=H,F}\tau_{kj}^{1-\varepsilon}\left(\frac{C_j}{C_k}\right)^{\varepsilon-1}\right]^{-1}\sum_{k=H,F}\tau_{kj}^{1-\varepsilon}\left(\frac{C_j}{C_k}\right)^{\varepsilon-2}d\left(\frac{C_j}{C_k}\right)\right],\quad i,j=H,F,$$

$$d\left(\frac{C_i}{C_j}\right)=\left(\frac{C_i}{C_j}\right)^{\frac{1}{\varepsilon}}C_j^{\frac{1-\varepsilon}{\varepsilon}}\left(C_{ii}^{\frac{-1}{\varepsilon}}dC_{ii}+C_{ij}^{\frac{-1}{\varepsilon}}dC_{ij}\right)-\left(\frac{C_i}{C_j}\right)^{\frac{2\varepsilon-1}{\varepsilon}}C_i^{\frac{1-\varepsilon}{\varepsilon}}\left(C_{jj}^{\frac{-1}{\varepsilon}}dC_{jj}+C_{ji}^{\frac{-1}{\varepsilon}}dC_{ij}\right),\quad i,j=H,F$$

Notice that at the planner optimum, where the allocation is symmetric, this last condition equals zero not only for $i=j$ but also for $i\neq j$.

It follows that under symmetry

$$\frac{\partial Q_{Cji}}{\partial L_{Ci}}=\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{C_{ji}}{L_{Ci}},\quad i,j=H,F,$$

while $\partial Q_{Cji}/\partial L_{Cj}=0$ as in the heterogeneous-firm case. We can now turn to the comparison between the planner and the market allocation.

Condition (27) is satisfied like in the case for heterogeneous firms. For condition (28) we have to compare the expression

$$Z_i\frac{\alpha}{1-\alpha}=L_{Ci}\frac{\varepsilon-1}{\varepsilon},\quad i=H,F$$

with the corresponding condition in the market allocation. We know that in the market allocation the following holds:

$$Z_i\frac{\alpha}{1-\alpha}=P_iC_i=\sum_{j=H,F}P_{ij}C_{ij},\quad i=H,F$$

Moreover, from (A-21) and (A-22), we get:

$$P_{ij}C_{ij} = L_{Cj}W_j\left(\tau_{ij}\tau_{Tij}\right)^{1-\varepsilon}\tau_{Lj}\frac{\left(\frac{W_k}{W_j}\frac{\tau_{Lk}}{\tau_{Lj}}\right)^{\varepsilon} - \left(\frac{W_i}{W_j}\frac{\tau_{Li}}{\tau_{Lj}}\right)^{\varepsilon}\tau_{ki}^{\varepsilon-1}\tau_{Tki}^{\varepsilon}}{\tau_{Tik}^{-\varepsilon}\tau_{ki}^{1-\varepsilon} - \tau_{Tki}^{\varepsilon}\tau_{ki}^{\varepsilon-1}}, \quad i,j = H,F, \quad k \neq i$$

Hence:

$$\begin{aligned} Z_i\frac{\alpha}{1-\alpha} &= L_{Ci}\frac{\varepsilon-1}{\varepsilon}\sum_{j=H,F}\frac{\varepsilon}{\varepsilon-1}\frac{L_{Cj}}{L_{Ci}}W_j\left(\tau_{ij}\tau_{Tij}\right)^{1-\varepsilon}\tau_{Lj}\frac{\left(\frac{W_k}{W_j}\frac{\tau_{Lk}}{\tau_{Lj}}\right)^{\varepsilon} - \left(\frac{W_i}{W_j}\frac{\tau_{Li}}{\tau_{Lj}}\right)^{\varepsilon}\tau_{ki}^{\varepsilon-1}\tau_{Tki}^{\varepsilon}}{\tau_{Tik}^{-\varepsilon}\tau_{ki}^{1-\varepsilon} - \tau_{Tki}^{\varepsilon}\tau_{ki}^{\varepsilon-1}} \\ &= L_{Ci}\frac{\varepsilon-1}{\varepsilon}\Omega_{3Pi}, \quad i = H,F, \quad k \neq i \end{aligned}$$

where Ω_{3Pi} is the wedge between the planner and the market allocation. In any symmetric allocation:

$$\Omega_{3Pi} = \frac{\varepsilon}{\varepsilon-1}\tau_L\sum_{j=H,F}\left(\tau_{ij}\tau_{Tij}\right)^{1-\varepsilon}\frac{1-\tau_{ki}^{\varepsilon-1}\tau_{Tki}^{\varepsilon}}{\tau_{Tik}^{-\varepsilon}\tau_{ki}^{1-\varepsilon} - \tau_{Tki}^{\varepsilon}\tau_{ki}^{\varepsilon-1}}, \quad i = H,F, \quad k \neq i$$

which implies that $\Omega_{3P} = 1$ if $\tau_L = \frac{\varepsilon-1}{\varepsilon}$ and $\tau_{Tij} = 1$ for $i,j = H,F$ since:

$$\Omega_{3Pi} = \frac{1-\tau^{\varepsilon-1}}{\tau^{1-\varepsilon} - \tau^{\varepsilon-1}}\sum_{j=H,F}\tau_{ij}^{1-\varepsilon} = \frac{1-\tau^{\varepsilon-1}}{\tau^{1-\varepsilon} - \tau^{\varepsilon-1}}(1+\tau^{1-\varepsilon}) = 1, \quad i = H,F$$

C.5 Proof of Proposition 1

Proof For future convenience note that the minimum set of conditions determining the Pareto efficient allocation for the multi-sector model consists of: i) the conditions that hold in both the homogeneous and the heterogeneous firm model, namely conditions (4), (27), (28), and the labor constraint, $Z_i + Z_j = 2L - L_{Ci} - L_{Cj}$; ii) the conditions which are model specific, namely conditions (C-24) and (C-25) in the case of homogeneous firms, and conditions (10) (obtained properly combining (C-16) and (C-17)), (7) and (8), and the following the zero cut-off condition:

$$\frac{\varphi_{ii}}{\varphi_{ji}} = \left(\frac{f_{ii}}{f_{ji}}\right)^{\frac{1}{\varepsilon-1}}\frac{C_i}{C_j}\frac{1}{\tau_{ji}} \quad i = H,F \quad j \neq F \tag{C-29}$$

recovered by first using the first constraint in (18) and condition (C-6) evaluated at the cut-offs to substitute out $c(\varphi_{ij})$ and $l(\varphi_{ij})$ in condition (C-7) and then combining this condition with (20) and (C-19). When there is only one sector we drop (27) and (28) while the labor constraint simplifies to $L_{Ci} = L_{Cj} = L$.

What we need to show is that the planner problem has a unique and symmetric solution. We do that for all model versions considered, i.e., homogeneous and heterogeneous firms models with either one or multiple sectors. It is easy to verify that the symmetric allocation is always a solution of the above conditions. Thus, we only need to prove uniqueness.

C.5.1 Homogenous firms - one-sector model

First, note that by substituting (C-22) into (C-10) the second-stage aggregate production function can be written as:

$$C_{ji} = \tau_{ji}^{-1}L_{Ci}^{\frac{1}{\varepsilon-1}}(\varepsilon f_E)^{\frac{1}{1-\varepsilon}}L_{Cji}, \quad i,j = H,F \tag{C-30}$$

Substituting this into (C-20), we obtain:

$$L_{Cji} = \tau_{ji}^{1-\varepsilon} L_{Cii} \left(\frac{C_i}{C_j} \right)^{\varepsilon - 1}, \quad i = H, F \quad j \neq i \tag{C-31}$$

Using (4) and substituting again (C-30), we find:

$$L_{Cji} = \tau_{ji}^{1-\varepsilon} L_{Cii} \left[\frac{L_{Ci}^{\frac{1}{\varepsilon}} L_{Cii}^{\frac{\varepsilon-1}{\varepsilon}} + \tau_{ij}^{\frac{1-\varepsilon}{\varepsilon}} L_{Cj}^{\frac{1}{\varepsilon}} L_{Cij}^{\frac{\varepsilon-1}{\varepsilon}}}{L_{Cj}^{\frac{1}{\varepsilon}} L_{Cjj}^{\frac{\varepsilon-1}{\varepsilon}} + \tau_{ji}^{\frac{1-\varepsilon}{\varepsilon}} L_{Ci}^{\frac{1}{\varepsilon}} L_{Cji}^{\frac{\varepsilon-1}{\varepsilon}}} \right]^{\varepsilon}, \quad i = H, F \quad j \neq i \tag{C-32}$$

Combining the labor resource constraint with (C-21) and recalling that with a single sector $L_{Ci} = L$, we have

$$L_{Cji} = \frac{\varepsilon - 1}{\varepsilon} L - L_{Cii}, \quad i = H, F, \quad j \neq i$$

This last equation can be used to substitute out L_{Cji} and L_{Cij} from (C-32) in order to obtain a system of two equations in two variables:

$$F_i(L_{Cii}, L_{Cjj}) \equiv \left(\frac{L_{Cii}^{\frac{\varepsilon-1}{\varepsilon}} + \tau_{ij}^{\frac{1-\varepsilon}{\varepsilon}} \left[\frac{\varepsilon-1}{\varepsilon} L - L_{Cjj}\right]^{\frac{\varepsilon-1}{\varepsilon}}}{L_{Cjj}^{\frac{\varepsilon-1}{\varepsilon}} + \tau_{ji}^{\frac{1-\varepsilon}{\varepsilon}} \left[\frac{\varepsilon-1}{\varepsilon} L - L_{Cii}\right]^{\frac{\varepsilon-1}{\varepsilon}}} \right)^{\varepsilon} - \frac{\frac{\varepsilon-1}{\varepsilon} L - L_{Cii}}{\tau_{ji}^{1-\varepsilon} L_{Cii}} = 0, \quad i = H, F \quad j \neq i \tag{C-33}$$

Note that $F_H()$ is monotonically increasing in L_{CHH} and monotonically decreasing in L_{CFF}, while exactly the opposite is true for $F_F()$. This implies that the functions $F_H()$ and $F_F()$ cross only once, i.e., there is a unique solution. More specifically, the unique solution is given by

$$L_{Cji} = \frac{\varepsilon - 1}{\varepsilon} \frac{\tau_{ji}^{1-\varepsilon}}{1 + \tau^{1-\varepsilon}} L, \quad i, j = H, F \tag{C-34}$$

The remaining variables and their symmetry follow immediately.

C.5.2 Homogenous firms - multi-sector model

For the multi-sector model, we also need to consider the third-stage optimality conditions (27) and (28). Using (C-26) they can be written as follows:

$$Z_i = Z, \quad i = H, F \tag{C-35}$$

$$L_{Ci} = \frac{\alpha}{1 - \alpha} \frac{\varepsilon}{\varepsilon - 1} Z \sum_{j=H,F} \left(\frac{C_{ji}}{C_j} \right)^{\frac{\varepsilon-1}{\varepsilon}}, \quad i = H, F \tag{C-36}$$

The second-stage aggregate production function (C-30) can be substituted in order to express this equation as:

$$C_i^{\frac{\varepsilon-1}{\varepsilon}} = \frac{\alpha}{1 - \alpha} \frac{\varepsilon}{\varepsilon - 1} Z (\varepsilon f_E)^{-\frac{1}{\varepsilon}} L_{Ci}^{-\frac{\varepsilon-1}{\varepsilon}} \left[L_{Cii}^{\frac{\varepsilon-1}{\varepsilon}} + \left(\frac{C_i}{C_j} \right)^{\frac{\varepsilon-1}{\varepsilon}} \tau_{ji}^{\frac{1-\varepsilon}{\varepsilon}} L_{Cji}^{\frac{\varepsilon-1}{\varepsilon}} \right], \quad i = H, F \quad j \neq i \tag{C-37}$$

Taking the ratio of this expression for both countries:

$$\left(\frac{C_i}{C_j}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \left(\frac{L_{Cj}}{L_{Ci}}\right)^{\frac{\varepsilon-1}{\varepsilon}} \frac{L_{Cii}^{\frac{\varepsilon-1}{\varepsilon}} + \left(\frac{C_i}{C_j}\right)^{\frac{\varepsilon-1}{\varepsilon}} \tau_{ji}^{\frac{1-\varepsilon}{\varepsilon}} L_{Cji}^{\frac{\varepsilon-1}{\varepsilon}}}{L_{Cjj}^{\frac{\varepsilon-1}{\varepsilon}} + \left(\frac{C_j}{C_i}\right)^{\frac{\varepsilon-1}{\varepsilon}} \tau_{ij}^{\frac{1-\varepsilon}{\varepsilon}} L_{Cij}^{\frac{\varepsilon-1}{\varepsilon}}}, \quad i = H, F \quad j \neq i \tag{C-38}$$

From (C-31) we get:

$$\left(\frac{C_i}{C_j}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \left(\frac{L_{Cji}}{L_{Cii}}\right)^{\frac{1}{\varepsilon}} \tau_{ji}^{\frac{\varepsilon-1}{\varepsilon}}, \quad i = H, F \quad j \neq i \tag{C-39}$$

Substituting this into (C-38), and using the fact that from (C-21) and (C-22) we have $\sum_{j=H,F} L_{Cji} = \frac{\varepsilon-1}{\varepsilon} L_{Ci}$, we get:

$$L_{Cji} = \tau_{ij}^{1-\varepsilon} \frac{L_{Ci}}{L_{Cj}} L_{Cjj}, \quad i = H, F \quad j \neq i \tag{C-40}$$

Substituting this expression again into (C-39) to write this equation in terms of L_{Cii} and L_{Cjj}, we get:

$$\left(\frac{C_i}{C_j}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \left(\frac{L_{Ci}}{L_{Cj}} \frac{L_{Cjj}}{L_{Cii}}\right)^{\frac{1}{\varepsilon}}, \quad i = H, F \quad j \neq i \tag{C-41}$$

Combining instead (C-38) with (C-40), we obtain:

$$\left(\frac{C_i}{C_j}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \left(\frac{L_{Cj}}{L_{Ci}} \frac{L_{Cii}}{L_{Cjj}}\right)^{\frac{\varepsilon-1}{\varepsilon}}, \quad i = H, F \quad j \neq i \tag{C-42}$$

From (C-41) and (C-42) it follows that $\frac{L_{Cii}}{L_{Cjj}} = \frac{L_{Ci}}{L_{Cj}}$ for $i = H, F$, $j \neq i$. Substituting this back into (C-41) it follows that $C_i = C$ for $i = H, F$.

Then from (C-40), we find:

$$L_{Cji} = \tau_{ji}^{1-\varepsilon} L_{Cii}, \quad i = H, F \quad j \neq i \tag{C-43}$$

Using $L_{Cii} + L_{Cji} = \frac{\varepsilon-1}{\varepsilon} L_{Ci}$ together with (C-40) and $\frac{L_{Cii}}{L_{Cjj}} = \frac{L_{Ci}}{L_{Cj}}$, we get:

$$L_{Cji} = \frac{\varepsilon-1}{\varepsilon} \frac{\tau_{ji}^{1-\varepsilon}}{1+\tau^{1-\varepsilon}} L_{Ci}, \quad i, j = H, F \tag{C-44}$$

Using this with (C-30):

$$C_{ji} = \frac{\varepsilon-1}{\varepsilon} (\varepsilon f_E)^{\frac{1}{1-\varepsilon}} \frac{\tau_{ji}^{-\varepsilon}}{1+\tau^{1-\varepsilon}} L_{Ci}^{\frac{\varepsilon}{\varepsilon-1}}, \quad i, j = H, F \tag{C-45}$$

Substituting (C-45) into (4) and using the fact that $C_i = C$ for $i = H, F$, we have that $L_{Ci} = L_C$ for $i = H, F$. It then follows easily that $L_{Cii} = L_{Cjj}$, $L_{Cij} = L_{Cji}$, $C_{ii} = C_{jj}$, and $C_{ij} = C_{ji}$ for $i = H, F$ and $j \neq i$. From the aggregate resource constraint it then follows that $Z = L - L_{Ci}$. From (C-45) is also follows that $\frac{C_{ii}}{C_{ji}} = \tau_{ji}^{\varepsilon}$ for $j \neq i$.

Imposing $C_i = C$, $C_{ij} = C_{ji}$, and $\frac{C_{ii}}{C_{ji}} = \tau_{ij}^{\varepsilon}$ for $i = H, F$ and $j \neq i$ in (C-36) we have:

$$L_{Ci} = \frac{\alpha}{1-\alpha} \frac{\varepsilon}{\varepsilon-1} Z \left[\frac{1}{1+\tau^{1-\varepsilon}} + \frac{1}{1+\tau^{\varepsilon-1}}\right] = \frac{\alpha}{1-\alpha} \frac{\varepsilon}{\varepsilon-1} Z, \quad i = H, F$$

Combining this last equation with $Z = L - L_{Ci}$, we find that $L_{Ci} = \frac{\alpha\varepsilon}{\varepsilon+\alpha-1} L$ for $i = H, F$, i.e., there exists a unique symmetric solution for L_{Ci}. Uniqueness and symmetry of the remaining variables then follow immediately.

C.5.3 Heterogeneous firms - one-sector model

Using the first constraint in (18) and condition (C-6) evaluated at the cut-offs to substitute out $c(\varphi_{ij})$ and $l(\varphi_{ij})$ in condition (C-7), and then combining this condition with (20) and (C-19) we obtain the following 2 equations:

$$\frac{C_i}{C_j} = \tau_{ji}\frac{\varphi_{ii}}{\varphi_{ji}}\left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon-1}}, \quad i,j = H,F \quad j \neq i \tag{C-46}$$

Combining these two equations to eliminate C_i/C_j we obtain the following set of two equations in four variables:

$$\frac{\varphi_{ii}}{\varphi_{ji}}\frac{\varphi_{jj}}{\varphi_{ij}}\tau_{ji}^2\left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{2}{\varepsilon-1}} - 1 = 0, \quad i = H,F \quad j \neq i \tag{C-47}$$

Using the definition of C_i, (4), and the second-stage aggregate production function, (25), we obtain:

$$\left(\frac{C_i}{C_j}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \frac{f_{ii}^{\frac{-1}{\varepsilon}}\varphi_{ii}^{\frac{\varepsilon-1}{\varepsilon}}L_{Ci}\delta_{ii} + f_{ij}^{\frac{-1}{\varepsilon}}\tau_{ij}^{\frac{1-\varepsilon}{\varepsilon}}\varphi_{ij}^{\frac{\varepsilon-1}{\varepsilon}}L_{Cj}\delta_{ij}}{f_{jj}^{\frac{-1}{\varepsilon}}\varphi_{jj}^{\frac{\varepsilon-1}{\varepsilon}}L_{Cj}\delta_{jj} + f_{ji}^{\frac{-1}{\varepsilon}}\tau_{ji}^{\frac{1-\varepsilon}{\varepsilon}}\varphi_{ji}^{\frac{\varepsilon-1}{\varepsilon}}L_{Ci}\delta_{ji}}, \quad i,j = H,F \quad j \neq i \tag{C-48}$$

Combining (C-46) with (C-48) we obtain:

$$\tau_{ji}^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{\varphi_{ii}}{\varphi_{ji}}\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon}} = \frac{f_{ii}^{\frac{-1}{\varepsilon}}\varphi_{ii}^{\frac{\varepsilon-1}{\varepsilon}}L_{Ci}\delta_{ii} + f_{ij}^{\frac{-1}{\varepsilon}}\tau_{ij}^{\frac{1-\varepsilon}{\varepsilon}}\varphi_{ij}^{\frac{\varepsilon-1}{\varepsilon}}L_{Cj}\delta_{ij}}{f_{jj}^{\frac{-1}{\varepsilon}}\varphi_{jj}^{\frac{\varepsilon-1}{\varepsilon}}L_{Cj}\delta_{jj} + f_{ji}^{\frac{-1}{\varepsilon}}\tau_{ji}^{\frac{1-\varepsilon}{\varepsilon}}\varphi_{ji}^{\frac{\varepsilon-1}{\varepsilon}}L_{Ci}\delta_{ji}}, \quad i,j = H,F \quad j \neq i \tag{C-49}$$

Given that $\delta_{ji} = 1 - \delta_{ii}$, $\delta_{ij} = 1 - \delta_{jj}$, that from (C-47) $\varphi_{ij} = \frac{\varphi_{ii}}{\varphi_{ji}}\varphi_{jj}\tau_{ji}^2\left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{2}{\varepsilon-1}}$, and that in the one-sector model $L_{Ci} = L$ for $i = H,F$, (C-49) implies that:

$$\frac{2\delta_{ii}-1}{2\delta_{jj}-1} - \tau_{ji}^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{\varphi_{jj}}{\varphi_{ji}}\right)^{\frac{\varepsilon-1}{\varepsilon]}}\left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon}} = 0, \quad i,j = H,F \quad j \neq i \tag{C-50}$$

We now want to show that there is a unique symmetric solution to (C-47) and (C-50). We do this by expressing these equations as implicit functions of φ_{ii} and φ_{jj} and showing that one relationship has a positive slope and the other one a negative slope, so that there is a unique intersection. In order to do this we use equation (B-3) that relates $d\varphi_{ji}$ to $d\varphi_{ii}$ for $i = H,F$, $j \neq i$, and equation (B-4) that relates $d\delta_{ji}$ to $d\varphi_{ji}$, for $i,j = H,F$.

Taking the total differential of (C-47) and using (B-3) we obtain:

$$\frac{d\varphi_{ii}}{d\varphi_{jj}} = -(1-\delta_{ii})\left(\frac{\delta_{jj}}{1-\delta_{jj}}\frac{\varphi_{ij}\varphi_{ji}}{\varphi_{jj}^2\tau_{ji}^2}\left(\frac{f_{ji}}{f_{ii}}\right)^{-\frac{2}{\varepsilon-1}} + \frac{\varphi_{ii}}{\varphi_{jj}}\right) < 0, \quad i,j = H,F \quad j \neq i$$

Similarly, taking the total differential of (C-50) and using (B-3) and (B-4) we obtain:

$$\frac{d\varphi_{ii}}{d\varphi_{jj}} = \frac{\frac{\varphi_{ii}}{\varphi_{jj}} \frac{1-\delta_{ii}}{\delta_{ii}} \tau_{ji}^{\frac{\varepsilon-1}{\varepsilon}} \left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon}} \left(\frac{\varphi_{jj}}{\varphi_{ji}}\right)^{\frac{\varepsilon-1}{\varepsilon}} \left(\varepsilon - 1 + 2\delta_{jj}((\varepsilon-1)^2 + \varepsilon\Phi_j)\right)}{2(1-\delta_{ii})\varepsilon(\varepsilon - 1 + \Phi_i) + (\varepsilon-1)(2\delta_{jj}-1)\tau_{ji}^{\frac{\varepsilon-1}{\varepsilon}} \left(\frac{f_{ij}}{f_{jj}}\right)^{\frac{1}{\varepsilon}} \left(\frac{\varphi_{jj}}{\varphi_{ji}}\right)^{\frac{\varepsilon-1}{\varepsilon}}}, \quad i,j = H,F, \quad j \neq i$$

The numerator is unambiguously positive. As for the denominator, it is also positive, as becomes clear when further simplifying it using (C-50):

$$\frac{d\varphi_{ii}}{d\varphi_{jj}} = \frac{\frac{\varphi_{ii}}{\varphi_{jj}} \frac{1-\delta_{ii}}{\delta_{ii}} \tau_{ji}^{\frac{\varepsilon-1}{\varepsilon}} \left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon}} \left(\frac{\varphi_{jj}}{\varphi_{ji}}\right)^{\frac{\varepsilon-1}{\varepsilon}} \left(\varepsilon - 1 + 2\delta_{jj}((\varepsilon-1)^2 + \varepsilon\Phi_j)\right)}{(1-\delta_{ii})((\varepsilon-1)(2\varepsilon-1) + 2\varepsilon\Phi_i) + \delta_{jj}(\varepsilon-1)} > 0, \quad i,j = H,F \quad j \neq i$$

Hence, while (C-47) is continuous and monotonically decreasing in the $(\varphi_{ii}, \varphi_{jj})$ space, the opposite is true for (C-50), implying that there exists a unique intersection and thus a unique combination of φ_{ii} and φ_{jj} consistent with the planner solution. From (B-3) and (B-4) we know that there is a monotonic relationship between φ_{ji} and φ_{ii} and between δ_{ij} and φ_{ij}. Therefore, there is a unique and symmetric solution for φ_{ij} and δ_{ij} with $i,j = H,F$. From (C-48) it then follows that $C_i = C_j$ for $i = H, j = F$. Uniqueness and symmetry of the remaining variables follows immediately.

C.5.4 Heterogeneous firms - multi-sector model

Observe that (C-47) holds also in the case of multiple sectors. Instead, this is not the case for (C-50) which was derived under the assumption that $L_{Ci} = L$ for $i = H,F$. Thus, we need to consider the third-stage optimality conditions, (C-35) and (C-36), to derive a second relationship between φ_{ii} and φ_{jj}.

Combining them with the second-stage aggregate production function C_{ij} (25), we find:

$$C_i^{\frac{\varepsilon-1}{\varepsilon}} = \frac{\alpha}{1-\alpha}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{1}{\varepsilon}} Z\varepsilon^{-\frac{1}{\varepsilon}} \left[f_{ii}^{-\frac{1}{\varepsilon}} \varphi_{ii}^{\frac{\varepsilon-1}{\varepsilon}} \delta_{ii} + f_{ji}^{-\frac{1}{\varepsilon}} \tau_{ji}^{\frac{1-\varepsilon}{\varepsilon}} \varphi_{ji}^{\frac{\varepsilon-1}{\varepsilon}} \delta_{ji} \left(\frac{C_i}{C_j}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right], \quad i = H,F \quad j \neq i$$

Dividing this by the corresponding equation for the other country:

$$\left(\frac{C_i}{C_j}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \frac{f_{ii}^{-\frac{1}{\varepsilon}} \varphi_{ii}^{\frac{\varepsilon-1}{\varepsilon}} \delta_{ii} + f_{ji}^{-\frac{1}{\varepsilon}} \tau_{ji}^{\frac{1-\varepsilon}{\varepsilon}} \varphi_{ji}^{\frac{\varepsilon-1}{\varepsilon}} \delta_{ji} \left(\frac{C_i}{C_j}\right)^{\frac{\varepsilon-1}{\varepsilon}}}{f_{jj}^{-\frac{1}{\varepsilon}} \varphi_{jj}^{\frac{\varepsilon-1}{\varepsilon}} \delta_{jj} + f_{ij}^{-\frac{1}{\varepsilon}} \tau_{ij}^{\frac{1-\varepsilon}{\varepsilon}} \varphi_{ij}^{\frac{\varepsilon-1}{\varepsilon}} \delta_{ij} \left(\frac{C_j}{C_i}\right)^{\frac{\varepsilon-1}{\varepsilon}}}, \quad i = H,F \quad j \neq i \tag{C-51}$$

Substituting (C-46) into (C-51) and using the fact that $\delta_{ii} = 1 - \delta_{ji}$ for $i = H,F$, $j \neq i$ we find:

$$\frac{\varphi_{ji}}{\varphi_{jj}} = \tau_{ji}\left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon-1}}, \quad i = H,F \quad j \neq i \tag{C-52}$$

Taking the total differential of (C-52) and using (B-3) we have:

$$\frac{d\varphi_{ii}}{d\varphi_{jj}} = -\tau_{ji}\left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon-1}} \frac{1-\delta_{ii}}{\delta_{ii}} \frac{\varphi_{ii}}{\varphi_{jj}} < 0, \quad i,j = H,F \quad ,j \neq i \tag{C-53}$$

Similarly to the one-sector model, the planner solution needs to satisfy two equations, (C-47) and (C-52), both of which can be expressed as implicit functions of $(\varphi_{ii}, \varphi_{jj})$. We showed that both functions monotonically decrease in the $(\varphi_{ii}, \varphi_{jj})$ space, implying that they cross at most once, i.e., there is a unique solution. The remaining steps are the same as in the one-sector model. ■

C.6 Proof of Lemma 1

Proof We prove this proposition in two steps.

First, observe that conditions (22) and (27) and (28) (when $\alpha < 1$) are optimality conditions of the planner problem, and therefore are necessary conditions for the market equilibrium to coincide with the planner allocation.
Second, we prove that if (22) and (27) and (28) (when $\alpha < 1$) hold, then the market allocation coincides with the planner allocation. If (22) holds, then as shown in Appendices C.2.3 and C.3.3 for the heterogeneous-firm model and Appendices C.2.4 and C.3.6 for the homogeneous-firm model, all the optimality conditions of the first and second stage of the planner problem are satisfied in the market equilibrium. Moreover, if for the case $\alpha < 1$ also conditions (27) and (28) are satisfied, then – as shown in Appendices C.4.2 and C.4.4 – all the optimality conditions of the third stage hold. As a consequence, the market equilibrium coincides with the planner allocation. ■

C.7 Proof of Proposition 2

Proof We prove Proposition 2 in two steps.

First we show that conditions (33) and (34) and – for the case of the multi-sector model – condition (35) are sufficient conditions for (30), (31) and (32) to hold in the market equilibrium. It is evident that with log utility condition (33) ($I_i = I_j$, $j \neq i$) implies condition (31). Moreover, utility maximization implies

$$P_{ij} = \frac{\partial U_i}{\partial C_{ij}} / \frac{\partial U_i}{\partial I_i} = \frac{\partial U_i}{\partial C_{ij}} I_i, \qquad i, j = H, F \tag{C-54}$$

Using this result with (34), we get:

$$\frac{\partial U_i}{\partial C_{ij}} I_i = \frac{\varepsilon}{\varepsilon - 1} \tau_{Lj} W_j \frac{\partial L_{Cij}}{\partial Q_{Cij}} \quad i, j = H, F.$$

Taking ratios of this condition for $i \neq j$ and using condition (33), we obtain:

$$\frac{\partial u_j}{\partial C_{jj}} \frac{\partial Q_{Cjj}}{\partial L_{Cjj}} = \frac{\partial u_i}{\partial C_{ij}} \frac{\partial Q_{Cij}}{\partial L_{Cij}} \quad j = H, F, \quad i \neq j$$

which proves that (30) holds.

Finally, by condition (34), condition (35) can be rewritten as follows:

$$\sum_{j=H,F} \tau_{Tji}^{-1} P_{ji} \frac{\partial Q_{Cji}}{\partial L_{Ci}} = W_i, \Leftrightarrow \sum_{j=H,F} P_{ji} \frac{\partial Q_{Cji}}{\partial L_{Ci}} = 1 \Rightarrow \sum_{j=H,F} \frac{\partial U_j}{\partial C_{ji}} \frac{\partial Q_{Cji}}{\partial L_{Ci}} = -\frac{\partial U_i}{\partial Z_i} \frac{\partial Q_{Zi}}{\partial L_{Ci}}, \quad i = H, F$$

where the last implication follows from conditions (33) and (C-54). This proves that (32) holds.

Second, we show that (33) and (34) and – in the multi-sector model – condition (35) are necessary conditions for (30), (31) and (32).

First, we consider condition (33). For the multi-sector model, it is straightforward to see that this is a necessary condition for the market equilibrium to be efficient: if condition (33) is not satisfied, condition (31) cannot be satisfied either. In the one sector model, showing necessity of condition (33) is a bit more involved. Suppose the market allocation is efficient. Then, by proposition (1), this allocation must be symmetric. This implies that we can use the assumption for the one-sector model that $\frac{\tau_{Li}W_i}{\tau_{Lj}W_j} = 1$ for $i \neq j$. Consider first the heterogeneous-firm case: it must be that by condition (9) $\tau_{Tij}^{-1} = 1$ for $i = H, F$ and $j \neq i$ since only under these conditions the market cutoffs correspond to the efficient cutoffs determined by conditions (8), (7), (10), and (C-29) under symmetry. At the same time, it must be that $\delta_{ij} = \delta_{ji}$ for $i, j = H, F$. This allows us to conclude that:

$$
\begin{aligned}
I_i = \sum_{k=H,F} P_{ik}C_{ik} &= \tau_{Li}W_i\delta_{ii}L + \tau_{Lj}W_j\delta_{ij}L = \tau_{Li}W_iL = \\
&= \tau_{Lj}W_jL = \tau_{Li}W_i\delta_{ji}L + \tau_{Lj}W_j\delta_{jj}L = \sum_{k=H,F} P_{jk}C_{jk} = I_j
\end{aligned}
$$

Consider now the homogeneous firm case. By conditions (C-30) and (C-34) if the market allocation is efficient it must be that in equilibrium $C_{ji} = \frac{\varepsilon-1}{\varepsilon}(\varepsilon f_E)^{\frac{1}{1-\varepsilon}} \frac{\tau_{ji}^{-\varepsilon}}{1+\tau^{1-\varepsilon}} L^{\frac{\varepsilon}{\varepsilon-1}}$ for $i, j = H, F$. Then, by conditions (16) it must also be that:

$$
\frac{1}{1+\tau^{1-\varepsilon}} = \frac{\tau_{Tij}^{-\varepsilon}\left[1 - \tau_{Tki}^{\varepsilon}\tau^{\varepsilon-1}\right]}{\tau_{Tik}^{-\varepsilon}\tau^{1-\varepsilon} - \tau_{Tki}^{\varepsilon}\tau^{\varepsilon-1}}, \quad i, j = H, F \quad k \neq i
$$

As a consequence, $1 + \tau^{1-\varepsilon} = \tau_{Tik}^{-\varepsilon}\tau^{1-\varepsilon} + \tau_{Tki}^{\varepsilon}$ and $\tau_{Tik}^{-\varepsilon} = \tau_{Tki}^{\varepsilon}$ for $i = H, F$ and $k \neq i$ Hence we can conclude that the market equilibrium is efficient only if $\tau_{Tij} = 1$ for $i = H, F$ and $j \neq i$. Therefore, by condition (16) and (17) $P_{ij}C_{ij} = \tau_{Lj}W_j\frac{\tau_{ij}^{1-\varepsilon}}{1+\tau^{1-\varepsilon}}L$ for $i, j = H, F$ and thus $I_i = \sum_{k=H,F} P_{ik}C_{ik} = \sum_{k=H,F} P_{jk}C_{jk} = I_j$ for $i \neq j$.

We next prove that condition (34) is necessary for condition (30) to hold in the market equilibrium. Without loss of generality, at this point we can assume that $I_i = I_j$ in the market equilibrium. From (29), the following condition must hold in a symmetric market allocation:

$$
\frac{\partial u_j}{\partial C_{jj}}\frac{\partial Q_{Cjj}}{\partial L_{Cjj}} = \frac{\partial u_i}{\partial C_{ij}}\frac{\partial Q_{Cij}}{\partial L_{Cij}}\tau_{Tij}^{-1} \qquad j = H, F \quad i \neq j \tag{C-55}
$$

Using condition (C-54), this equation can be written as $\frac{P_{jj}}{P_{ij}} = \frac{\partial L_{Cij}/\partial Q_{Cij}}{\partial L_{Cjj}/\partial Q_{Cjj}}\,\tau_{Tij}^{-1}$. Imposing that (34) must hold, it follows that condition (30) is satisfied in the market equilibrium only if $\tau_{Tij}^{-1} = 1$ for both $j = H, F$ and $i \neq j$. Thus, condition (30) holds only if conditions (34) is satisfied in equilibrium. Finally, suppose that (33) and (34) hold in the market equilibrium. Then, in the multi-sector model it follows that:

$$
\sum_{j=H,F} P_{ji}\frac{\partial Q_{Cji}}{\partial L_{Ci}} = \frac{\varepsilon}{\varepsilon-1}\tau_{Li}W_i, \Leftrightarrow \sum_{j=H,F} P_{ji}\frac{\partial Q_{Cji}}{\partial L_{Ci}} = \frac{\varepsilon}{\varepsilon-1}\tau_{Li} \Leftrightarrow \sum_{j=H,F} \frac{\partial U_j}{\partial C_{ji}}\frac{\partial Q_{Cji}}{\partial L_{Ci}} = -\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\frac{\partial U_i}{\partial Z_i}\frac{\partial Q_{Zi}}{\partial L_{Ci}}
$$

with $i = H, F$. Hence, condition (31) holds in the market equilibrium only if $\frac{\varepsilon}{\varepsilon-1}\tau_{Li} = 1$ for both $j = H, F$. Put differently, condition (31) holds only if conditions (35) is satisfied in equilibrium. ■

C.8 Proof of Lemma 2

To prove (37) it suffices to add and subtract $\tau_{Ii}^{-1}P_{ij}$ and then use (36):

$$
\begin{aligned}
P_{ij} - \frac{\varepsilon}{\varepsilon-1}\tau_{Lj}W_j\frac{\partial L_{Cij}}{\partial Q_{Cij}} &= P_{ij} - \tau_{Ii}^{-1}P_{ij} + \tau_{Ii}^{-1}P_{ij} - \frac{\varepsilon}{\varepsilon-1}\tau_{Lj}W_j\frac{\partial L_{Cij}}{\partial Q_{Cij}} \quad i = H, F \quad j \neq i \\
&= (\tau_{Ii} - 1)\tau_{Ii}^{-1}P_{ij} + \tau_{Ii}^{-1}P_{ij} - \tau_{Tij}^{-1}P_{ij} \quad i = H, F \quad j \neq i \\
&= (\tau_{Ii} - 1)\tau_{Ii}^{-1}P_{ij} + (\tau_{Xj} - 1)\,\tau_{Tij}^{-1}P_{ij} \quad i = H, F \quad j \neq i \\
&= \left(1 - \tau_{Tij}^{-1}\right)P_{ij} \quad i = H, F \quad j \neq i
\end{aligned}
$$

Condition (38) follows directly from (36) and the fact that in the multi-sector model $W_i = 1$. Finally, to prove (39) first notice that from (36) we have:

$$\tau_{Tji}^{-1} P_{ji} \frac{\partial Q_{Cji}}{\partial L_{Ci}} = -P_{ii} \frac{\partial Q_{Cii}}{\partial L_{Ci}} + \frac{\varepsilon}{\varepsilon - 1} \tau_{Li} W_i \quad i = H, F \quad j \neq$$

If we multiply everything by $\tau_{Xi} - 1$ and recall that in the multi-sector model $W_i = 1$, we obtain (39).

C.9 Two Lemmata and the Proof of Proposition 3

We first introduce two lemmata that will be useful for several proofs below and then we prove Proposition 3.

C.9.1 Lemmata 5 and 6 and their proofs

Lemma 5 *In the market equilibrium:*

$$\frac{\tau_{Xi} P_{ii} C_{ii}}{L_{Ci}} + \frac{\tau_{Ij}^{-1} P_{ji} C_{ji}}{L_{Ci}} = \tau_{Xi} \tau_{Li} W_i, \quad i = H, F, \quad j \neq i \tag{C-56}$$

Proof In the case of heterogeneous firms, using (11) and (12), we obtain:

$$\frac{P_{ji} C_{ji}}{L_{Ci}} = \tau_{Tji} \tau_{Li} \delta_{ji} W_i, \quad i, j = H, F,$$

which leads to C-56 once you recall that $\delta_{ii} = 1 - \delta_{ji}$. Similarly, for the case of homogeneous firms, one can use (16) and (17) to compute $P_{ii} C_{ii}$ and $P_{ji} C_{ji}$ and recover (C-56). ■

Lemma 6 *In the market equilibrium the following condition holds:*

$$\tau_{Xi} P_{ii} dC_{ii} + \tau_{Ij}^{-1} P_{ji} dC_{ji} - \frac{\varepsilon}{\varepsilon - 1} \tau_{Li} \tau_{Xi} dL_{Ci} = 0, \quad i = H, F \quad j \neq i \tag{C-57}$$

Proof we show that in equilibrium condition (C-57) is always satisfied. We first consider the case of firm heterogeneity and then turn to the case of homogeneous firms.

With heterogeneous firms, first, notice that equation (11) implies:

$$dC_{ji} = \frac{\partial C_{ji}}{\partial L_{Ci}} dL_{Ci} + \frac{\partial C_{ji}}{\partial \delta_{ji}} d\delta_{ji} + \frac{\partial C_{ji}}{\partial \varphi_{ji}} d\varphi_{ji} \quad i, j = H, F$$

Therefore, we can write

$$\begin{aligned} \tau_{Xi} P_{ii} dC_{ii} + \tau_{Ij}^{-1} P_{ji} dC_{ji} = & \left(\tau_{Xi} P_{ii} \frac{\partial C_{ii}}{\partial L_{Ci}} + \tau_{Ij}^{-1} P_{ji} \frac{\partial C_{ji}}{\partial L_{Ci}} \right) dL_{Ci} + \tau_{Xi} P_{ii} \frac{\partial C_{ii}}{\partial \varphi_{ii}} d\varphi_{ii} + \tau_{Xi} P_{ii} \frac{\partial C_{ii}}{\partial \delta_{ii}} d\delta_{ii} + \\ & + \tau_{Ij}^{-1} P_{ji} \frac{\partial C_{ji}}{\partial \varphi_{ji}} d\varphi_{ji} + \tau_{Ij}^{-1} P_{ji} \frac{\partial C_{ji}}{\partial \delta_{ji}} d\delta_{ji} \quad i = H, F \quad j \neq i \end{aligned} \tag{C-58}$$

Notice that by condition (C-56) and the fact that by (11) $\frac{\partial C_{ji}}{\partial L_{Ci}} = \frac{\varepsilon}{\varepsilon - 1} \frac{C_{ji}}{L_{Ci}}$, we get:

$$\left(\tau_{Xi} P_{ii} \frac{\partial C_{ii}}{\partial L_{Ci}} + \tau_{Ij}^{-1} P_{ji} \frac{\partial C_{ji}}{\partial L_{Ci}} \right) dL_{Ci} = \frac{\varepsilon}{\varepsilon - 1} \tau_{Li} \tau_{Xi} dL_{Ci}, \quad i = H, F \quad j \neq i \tag{C-59}$$

Therefore, in order for (C-57) to hold for the case of heterogeneous firms, it must be that in equilibrium:

$$\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \delta_{ii}}d\delta_{ii}+\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \varphi_{ii}}d\varphi_{ii}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \delta_{ji}}d\delta_{ji}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \varphi_{ji}}d\varphi_{ji}=0, \quad i=H,F \quad j\neq i$$

To prove this result, first consider that by (B-4):

$$\frac{\partial C_{ji}}{\partial \delta_{ji}}d\delta_{ji}+\frac{\partial C_{ji}}{\partial \varphi_{ji}}d\varphi_{ji}=\frac{C_{ji}}{\varphi_{ji}}\left[1-\frac{\varepsilon}{\varepsilon-1}\left(\Phi_i+(\varepsilon-1)\right)\right]d\varphi_{ji}, \quad i,j=H,F$$

Hence:

$$\begin{aligned}&\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \delta_{ii}}d\delta_{ii}+\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \varphi_{ii}}d\varphi_{ii}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \delta_{ji}}d\delta_{ji}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \varphi_{ji}}d\varphi_{ji}\\&=\left[1-\frac{\varepsilon}{\varepsilon-1}\left(\Phi_i+(\varepsilon-1)\right)\right]\left(\tau_{Xi}P_{ii}\frac{C_{ii}}{\varphi_{ii}}d\varphi_{ii}+\tau_{Ij}^{-1}P_{ji}\frac{C_{ji}}{\varphi_{ji}}d\varphi_{ji}\right), \quad i=H,F \quad j\neq i,\end{aligned}$$

which by (11) and (12) can be rewritten as:

$$\begin{aligned}&\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \delta_{ii}}d\delta_{ii}+\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \varphi_{ii}}d\varphi_{ii}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \delta_{ji}}d\delta_{ji}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \varphi_{ji}}d\varphi_{ji}\\&=\tau_{Xi}\tau_{Li}W_iL_{Ci}\left[1-\frac{\varepsilon}{\varepsilon-1}\left(\Phi_i+(\varepsilon-1)\right)\right]\left(\frac{\delta_{ii}}{\varphi_{ii}}d\varphi_{ii}+\frac{1-\delta_{ii}}{\varphi_{ji}}d\varphi_{ji}\right) \quad i=H,F \quad j\neq i\end{aligned}$$

Finally, recalling (B-3), we can conclude that, as postulated, this last condition is equal to zero in equilibrium for all $0\leq\alpha\leq 1$.

Similarly, in the presence of homogeneous firms first condition (16) leads to:

$$\begin{aligned}dC_{ji}=&\frac{\partial C_{ji}}{\partial L_{Ci}}dL_{Ci}+\frac{\partial C_{ji}}{\partial W_j}dW_j+\frac{\partial C_{ji}}{\partial \tau_{Li}}d\tau_{Li}+\frac{\partial C_{ji}}{\partial \tau_{Lj}}d\tau_{Lj}\\&+\frac{\partial C_{ji}}{\partial \tau_{Ii}}d\tau_{Ii}+\frac{\partial C_{ji}}{\partial \tau_{Xj}}d\tau_{Xj}+\frac{\partial C_{ji}}{\partial \tau_{Ij}}d\tau_{Ij}+\frac{\partial C_{ji}}{\partial \tau_{Xi}}d\tau_{Xi} \quad i,j=H,F\end{aligned}$$

where we already used the normalization $W_i=1$. Hence, in this case

$$\begin{aligned}\tau_{Xi}P_{ii}dC_{ii}+\tau_{Ij}^{-1}P_{ji}dC_{ji}=&\left(\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial L_{Ci}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial L_{Ci}}\right)dL_{Ci}+\left(\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial W_j}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial W_j}\right)dW_j\\&+\left(\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Li}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Li}}\right)d\tau_{Li}+\left(\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Lj}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Lj}}\right)d\tau_{Lj}\\&+\left(\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Ii}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Ii}}\right)d\tau_{Ii}+\left(\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Ij}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Ij}}\right)d\tau_{Ij}\\&+\left(\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Xi}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Xi}}\right)d\tau_{Xi}+\left(\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Xj}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Xj}}\right)d\tau_{Xj} \quad i=H,F \quad j\neq i\end{aligned}$$

Note that condition (C-56) and by (16) $\frac{\partial C_{ji}}{\partial L_{Ci}}=\frac{\varepsilon}{\varepsilon-1}\frac{C_{ji}}{L_{Ci}}$ hold in equilibrium also in the case of homogeneous firms, implying that (C-59) is valid too. Thus, (C-57) hold since by (16) and (17):

$$\begin{aligned}&\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial W_j}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial W_j}=\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Li}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Li}}=\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Lj}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Lj}}=\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Ii}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Ii}}\\&=\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Ij}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Ij}}=\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Xi}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Xi}}=\tau_{Xi}P_{ii}\frac{\partial C_{ii}}{\partial \tau_{Xj}}+\tau_{Ij}^{-1}P_{ji}\frac{\partial C_{ji}}{\partial \tau_{Xj}}=0 \quad i=H,F \quad j\neq i\end{aligned}$$

■

C.9.2 Proof of Proposition 3

Proof We prove Proposition 3 point by point.

(a) To show why condition (40) holds first consider that by conditions (37) and (38) (39):

$$(P_{ij} - \frac{\varepsilon}{\varepsilon-1}\tau_{Lj}W_j\frac{\partial L_{Cij}}{Q_{Cij}})dC_{ij} = (\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}dC_{ij} + (\tau_{Xj}-1)\tau_{Tij}^{-1}P_{ij}dC_{ij} \quad i=H,F \quad j\neq i \tag{C-60}$$

$$\begin{aligned}\sum_{i=H,F}(\tau_{Tij}^{-1}P_{ij}\frac{\partial Q_{Cij}}{L_{Cij}}-1)dL_{Cj} &= (\frac{\varepsilon}{\varepsilon-1}\tau_{Lj}-1)dL_{Cj}\\ &= (\frac{\varepsilon}{\varepsilon-1}\tau_{Xj}\tau_{Lj}-1)dL_{Cj} + (1-\tau_{Xj})\frac{\varepsilon}{\varepsilon-1}\tau_{Lj}dL_{Cj}\\ &= (\frac{\varepsilon}{\varepsilon-1}\tau_{Xj}\tau_{Lj}-1)dL_{Cj} - (\tau_{Xj}-1)\tau_{Tij}^{-1}P_{ij}dC_{ij} + (1-\tau_{Xj})P_{jj}dC_{jj}, \end{aligned} \tag{C-61}$$

where last equality follows from condition (C-57). Summing (C-60) and (C-61) we obtain:

$$\begin{aligned}&(P_{ij} - \frac{\varepsilon}{\varepsilon-1}\tau_{Lj}W_j\frac{\partial L_{Cij}}{Q_{Cij}})dC_{ij} + \sum_{i=H,F}(\tau_{Tij}^{-1}P_{ij}\frac{\partial Q_{Cij}}{L_{Cij}}-1)dL_{Cj}\\ &= (\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}dC_{ij} + (\frac{\varepsilon}{\varepsilon-1}\tau_{Xj}\tau_{Lj}-1)dL_{Cj} + (1-\tau_{Xj})P_{jj}dC_{jj}, \quad j=H,F \quad i\neq j\end{aligned} \tag{C-62}$$

Finally, we can sum the two conditions in (C-62) to obtain condition (40).
To show that condition (41) holds, recall that by condition (C-57) it follows that

$$(1-\tau_{Xi})P_{ii}dC_{ii} = (1-\tau_{Xi})\frac{\varepsilon}{\varepsilon-1}\tau_{Li}dL_{Ci} - (1-\tau_{Xi})\tau_{Tji}^{-1}P_{ji}dC_{ji}, \quad i=H,F \quad j\neq i \tag{C-63}$$

Substituting this condition into condition (40) we get:

$$\sum_{i=H,F}dE_i = \sum_{\substack{i=H,F\\ j\neq i}}(\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}dC_{ij} - \sum_{\substack{i=H,F\\ j\neq i}}(1-\tau_{Xj})\tau_{Tij}^{-1}P_{ij}dC_{ij} + \sum_{i=H,F}(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}-1)dL_{Ci}$$

which then leads to condition (41).

(b)(i) If all wedges in (40) are zero, i.e., $\tau_{Ii} = \tau_{Xi} = 1$ – and in the multi-sector model also $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ – for $i = H, F$.[55] the allocation determined by the conditions listed at the beginning of Appendix C.5 is also the solution of the set of equilibrium conditions listed in Section 2.4. As a consequence, if $\tau_{Ii} = \tau_{Xi} = 1$ and $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ for $i = H, F$ the market allocation is efficient.

(b)(ii) Conditions (36) state the equations that correspond to (34) and (35) in the market equilibrium. It is obvious from these equations that (34) and (35) are satisfied in the market equilibrium if and only if $\tau_{Tij} = 1$ and - in the multiple sector model - $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ for $i, j = H, F$, namely if and only if all wedges in (41) are zero. Then by Proposition 2, it must be that the market allocation is efficient if and only if $I_i = I_j$ and all the wedges in (41) are zero. ■

[55] Note that if $\alpha = 1$ then $dL_C = 0$ for $= H, F$ and only the wedges in dC_{ij} are present in (40).

D The Policy Maker's Problem and the Welfare Decomposition

Here we prove the Propositions of Section 4. For these proofs it is useful to recall that $\varepsilon > 1$, $0 < \alpha \leq 1$, $0 < \delta_{ii} < 1$, $\Phi_i > 0$, and $L_{Ci} > 0$.

D.1 Proof of Proposition 4

Proof The proof is organized in two steps. First, we derive the total differential of individual-country welfare by using the total differential of the trade-balance condition (13) and we show that this total differential leads to condition (43) given Lemma 6 Second, we show that if $I_i = I_j$ for $i \neq j$, condition (43) leads to condition (44).

(1) Substituting the definition of the consumption aggregator (4) into the utility function (3), we get:

$$U_i = \alpha \frac{\varepsilon}{\varepsilon - 1} \log \left[\sum_{j=H,F} C_{ij}^{\frac{\varepsilon-1}{\varepsilon}} \right] + (1-\alpha) \log Z_i, \quad i = H, F$$

Taking the total differential of this objective function, we obtain:

$$dU_i = \alpha \sum_{j=H,F} \frac{C_{ij}^{-\frac{1}{\varepsilon}}}{C_i^{\frac{\varepsilon-1}{\varepsilon}}} dC_{ij} + \frac{1-\alpha}{Z_i} dZ_i, \quad i = H, F \tag{D-1}$$

Note that $\frac{1-\alpha}{Z_i} = \frac{1}{I_i}$ and $\alpha \frac{C_{ij}^{-\frac{1}{\varepsilon}}}{C_i^{\frac{\varepsilon-1}{\varepsilon}}} = \left(\frac{C_i}{C_{ij}}\right)^{1/\varepsilon} \frac{P_i}{I_i} = \frac{P_{ij}}{I_i}$ since $\left(\frac{C_i}{C_{ij}}\right)^{1/\varepsilon} = \frac{P_{ij}}{P_i}$ for $i, j = H, F$. As a result, condition (D-1) can be rewritten as:

$$dU_i = \frac{1}{I_i} \sum_{j=H,F} P_{ij} dC_{ij} + \frac{1}{I_i} dZ_i, \quad i = H, F \tag{D-2}$$

Then, we can take the total differential of condition (13) and of its foreign counterpart[56] and use the fact that $Z_i = \frac{1-\alpha}{\alpha} \sum_{j=H,F} P_{ij} C_{ij}$ to get:

$$-dZ_i - dL_{Ci} + C_{ji} d(\tau_{Ij}^{-1} P_{ji}) + \tau_{Ij}^{-1} P_{ji} dC_{ji} - C_{ij} d(\tau_{Ii}^{-1} P_{ij}) - (\tau_{Ii}^{-1} P_{ij}) dC_{ij} = 0, \quad i = H, F \quad j \neq i$$

Dividing this condition by I_i and adding it to (D-2), we obtain:

$$\begin{aligned}
dU_i &= \frac{P_{ii}}{I_i} dC_{ii} + \frac{P_{ij}}{I_i} dC_{ij} + \frac{1}{I_i} dZ_i - \frac{1}{I_i} dZ_i - \frac{1}{I_i} dL_{Ci} + \frac{C_{ji}}{I_i} d(\tau_{Ij}^{-1} P_{ji}) + \frac{\tau_{Ij}^{-1} P_{ji}}{I_i} dC_{ji} - \frac{C_{ij}}{I_i} d(\tau_{Ii}^{-1} P_{ij}) - \frac{\tau_{Ii}^{-1} P_{ij}}{I_i} dC_{ij} \\
&= \frac{P_{ii}}{I_i} dC_{ii} + (\tau_{Ii} - 1) \frac{\tau_{Ii}^{-1} P_{ij}}{I_i} dC_{ij} - \frac{1}{I_i} dL_{Ci} + \frac{C_{ji}}{I_i} d(\tau_{Ij}^{-1} P_{ji}) - \frac{C_{ij}}{I_i} d(\tau_{Ii}^{-1} P_{ij}) + \frac{\tau_{Ij}^{-1} P_{ji}}{I_i} dC_{ji}, \quad i = H, F \quad j \neq i
\end{aligned}$$

[56] This condition can be recovered by combining (13) with (14).

Adding and subtracting terms, this can be rewritten as:

$$dU_i = (1-\tau_{Xi})\frac{P_{ii}}{I_i}dC_{ii} + (\tau_{Ii}-1)\tau_{Ii}^{-1}\frac{P_{ij}}{I_i}dC_{ij} + \left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi}-1\right)\frac{dL_{Ci}}{I_i} + \frac{C_{ji}}{I_i}d(\tau_{Ij}^{-1}P_{ji}) - \frac{C_{ij}}{I_i}d(\tau_{Ii}^{-1}P_{ij})$$
$$+\tau_{Xi}\frac{P_{ii}}{I_i}dC_{ii} + \tau_{Ij}^{-1}P_{ji}\frac{dC_{ji}}{I_i} - \frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi}\frac{dL_{Ci}}{I_i}, \quad i=H,F \quad j\neq i$$

Recall that by Lemma 6 in equilibrium the following condition holds:

$$\tau_{Xi}P_{ii}dC_{ii} + \tau_{Ij}^{-1}P_{ji}dC_{ji} - \frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi}dL_{Ci} = 0, \quad i=H,F \quad j\neq i$$

If this is true, then:

$$dU_i = (1-\tau_{Xi})\frac{P_{ii}}{I_i}dC_{ii} + (\tau_{Ii}-1)\tau_{Ii}^{-1}\frac{P_{ij}}{I_i}dC_{ij} + \left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi}-1\right)\frac{dL_{Ci}}{I_i} + \frac{C_{ji}}{I_i}d(\tau_{Ij}^{-1}P_{ji}) - \frac{C_{ij}}{I_i}d(\tau_{Ii}^{-1}P_{ij})$$
$$= \frac{dE_i}{I_i} + \frac{C_{ji}}{I_i}d(\tau_{Ij}^{-1}P_{ji}) - \frac{C_{ij}}{I_i}d(\tau_{Ii}^{-1}P_{ij}), \quad i=H,F \quad j\neq i \tag{D-3}$$

where dE_i is defined in Proposition 3. Summing the total differentials for both countries condition (D-3) leads to condition (43). It also leads to the decomposition of individual-country welfare in (46). Notice that if condition (C-57) holds, condition (D-3) holds even with homogeneous firms and when considering the one-sector model in which $\alpha = 1$ and $dL_{Ci} = 0$.

(2) Finally, if $I_i = I$ for $i = H, F$ so that (43) leads to:

$$I\sum_{i=H,F} dU_i = \sum_{i=H,F} dE_i + \sum_{\substack{i=H,F\\ j\neq i}}\Big(C_{ji}d(\tau_{Ij}^{-1}P_{ji}) - C_{ij}d(\tau_{Ii}^{-1}P_{ij})\Big)$$
$$= \sum_{i=H,F} dE_i$$

which by Proposition 3 point (a) corresponds to condition (44) and where the last equality follows from the fact that terms of trade effects exactly cancel out. ∎

D.2 Proof of Proposition 5

Proof We prove Proposition 5 point by point.

(a) In appendix B.1.2 we explained how to apply the total differential approach to solve a constrained optimization problem. In this case we have 28 variables (22 endogenous variables plus 6 policy instruments) and 22 constraints (conditions (7) -(14)).[57]. To show point (a) we the proceed as follows: (i) we show how to express the total differential in (42) in terms of 6 differentials and then 6 wedges. Setting these wedges to zero gives us 6 additional conditions to determine the optimal policies; (ii) we make clear that these conditions correspond to setting $I_i = I_j$ and the wedges in (44) individually equal to zero.

[57] When $\alpha = 1$ there are 26 endogenous variables and 22 constraints. Indeed, as made clear in Appendix A.4.2, in the one-sector model we assume that policymakers abstain from using labor subsidies. In the homogeneous firm model, there are 16 variables and 10 constraints if $\alpha < 1$ and 14 variables and 10 constraints if $\alpha = 1$.

(i) In order to rewrite the differential in (42), we combine it with condition (40) to obtain

$$\sum_{i=H,F} dU_i = \sum_{\substack{i=H,F\\ j\neq i}} \frac{(\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}dC_{ij} + (1-\tau_{Xi})P_{ii}dC_{ii} + \left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi}-1\right)dL_{Ci} + C_{ji}d(\tau_{Ij}^{-1}P_{ji}) - C_{ij}d(\tau_{Ii}^{-1}P_{ij})}{I_i}$$

Then, we use this condition and condition (C-63) to get

$$\sum_{i=H,F} dU_i = \sum_{\substack{i=H,F\\ j\neq i}} \frac{(\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}dC_{ij} - (1-\tau_{Xi})\tau_{Tji}^{-1}P_{ji}dC_{ji} + \left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}-1\right)dL_{Ci} + C_{ji}d(\tau_{Ij}^{-1}P_{ji}) - C_{ij}d(\tau_{Ii}^{-1}P_{ij})}{I_i}$$

$$= \sum_{\substack{i=H,F\\ j\neq i}} \left(\frac{1-\tau_{Ij}^{-1}}{I_j} - \frac{\tau_{Tji}^{-1}-\tau_{Ij}^{-1}}{I_i}\right) P_{ji}dC_{ji} + \sum_{i=H,F} \frac{\left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}-1\right)}{I_i} dL_{Ci} + \sum_{\substack{i=H,F\\ j\neq i}} \frac{(I_i-I_j)}{I_iI_j} C_{ij}d(\tau_{Ii}^{-1}P_{ij}) \quad \text{(D-4)}$$

which confirms that the differential in (42) can be expressed as a function of dC_{ij}, dL_{Ci}, $d(\tau_{Ij}^{-1}P_{ji})$, $d(\tau_{Ii}^{-1}P_{ij})$ for $i = H, F$ and $j \neq i$ only.
(ii) Setting the wedges in (D-4) individually equal to zero leads to:

$$\tau_{Li} = \frac{\varepsilon-1}{\varepsilon} \quad i = H, F \qquad I_i = I_j \quad i = H \quad j = F \qquad \tau_{Tji} = 1 \quad i = H, F \quad j \neq i$$

which, as claimed, is equivalent to imposing $I_i = I_j$ and to setting to zero the wedges in (44). Finally, notice how by (D-4) we can impose only 5 restrictions, and we are thus left with 1 degree of freedom in the choice of the 6 policy instruments, consistently with point (b).

(b) By point (a) above, the global policy $\{\tau_{Li}, \tau_{Ii}, \tau_{Xi}\}_{i=H,F}$ solves the world-policy-maker problem *if and only if* all the following conditions hold for $i = H, F$, $j \neq i$: (1) $I_i = I_j$; (2) $\tau_{Tij} = 1$; (3) $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ when $\alpha < 1$. At the same time, by Proposition 3 point (b) (ii) the market allocation is efficient *if and only if* all conditions (1) to (3) hold. Thus, at the optimum the global policy maker implements the planner allocation.

What remains to prove is that the global policy is optimal if and only if $\tau_{Ii} = \tau_{Ij}$ (or equivalently if and only if $\tau_{Xi} = \tau_{Xj}$). Put differently, we need to show that condition $I_i = I_j$ can be substituted away with condition $\tau_{Ii} = \tau_{Ij}$ (or equivalently condition $\tau_{Xi} = \tau_{Xj}$). More specifically, we need to prove that:

(b1) If $I_i = I_j$, $\tau_{Tij} = 1$, and when $\alpha < 1$ $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$, then $\tau_{Ii} = \tau_{Ij}$;

(b2) If $\tau_{Ii} = \tau_{Ij}$, $\tau_{Tij} = 1$, and when $\alpha < 1$ $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$, then $I_i = I_j$.

We start from (b1). If $I_i = I_j$, $\tau_{Tij} = 1$, and, when $\alpha < 1$, $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$, then by Proposition 3 point (b) (ii) the market allocation is efficient and by Proposition (1) also symmetric. Under these restrictions condition 12 (and condition(17) for the case of homogeneous firms) implies that $P_{ij} = P_{ji}$ both in the one sector[58] and in the multiple sector model. As a consequence, condition (14) can be simplified as $L - L_{Ci} = \frac{1-\alpha}{\alpha}\sum_{j=H,F} P_{ij}C_{ij}$. Thus, since $P_{ij} = P_{ji}$ and $C_{ij} = C_{ji}$ the trade balance condition (13) can hold in equilibrium only if $\tau_{Ii} = \tau_{Ij}$.

We now move to (b2). First, recall that from the second stage of the Planner's problem we know that when $\tau_{Tij} = 1$ and $\tau_{Li} = \frac{\varepsilon-1}{\varepsilon}$ when $\alpha < 1$ (or $\frac{\tau_{Li}W_i}{\tau_{Lj}W_j} = 1$ when $\alpha = 1$) then in equilibrium the cutoffs φ_{ij} for $i, j = H, F$ are efficient. When this is the case, conditions (7) to (10) can be used to find the efficient allocation for φ_{ij}, $\tilde{\varphi}_{ij}$, δ_{ij} for $i, j = H, F$. Recall again from Proposition 1 that the efficient allocation is unique and symmetric. For the case of homogeneous firms we simply have $\varphi_{ij} = 1$ for $i, j = H, F$. When $\alpha = 1$ we have $L_{Ci} = L$ for $i = H, F$ and it thus follows from (11) and (12) (and from (16) and (17) for the case of homogeneous firms) that the solution for P_{ij} and C_{ij} is also symmetric. This implies that $I_i = P_iC_i = P_jC_j = I_j$. When $\alpha < 1$

[58]Recall that in A.4.2 we assumed $\frac{\tau_{Li}W_i}{\tau_{Lj}W_j} = 1$ in any symmetric allocation.

instead, we can use (11) and (12) to get $P_{ij}C_{ij} = \delta_{ij}L_{Cj}$ when firms are heterogeneous, and (16) and (17) to get $P_{ij}C_{ij} = L_{Cj}\tau_{ij}^{1-\varepsilon}\frac{1-\tau_{ki}^{\varepsilon-1}}{\tau_{ki}^{1-\varepsilon}-\tau_{ki}^{\varepsilon-1}}$ with $i, j = H, F$ and $k \neq i$ when firms are homogeneous. In both case we can think of $P_{ij}C_{ij}$ as being a linear function of L_{Cj}. We can thus use the two equations (13) and (14) to solve for L_{Ci} and L_{Cj}. Note that this is a linear system in the two variables, and thus has a unique solution. It thus suffices to recall that the symmetric allocation is a possible solution when $\tau_{Tij} = 1$ and $\tau_{Ii} = \tau_{Ij}$. Therefore, the unique solution is symmetric and $L_{Ci} = L_{Cj}$. It then follows symmetry of P_{ij}, C_{ij}, P_i, C_i, Z_i and thus $I_i = P_iC_i + Z_i$. ■

D.3 Proof of Proposition 6

Proof We derived the total differential of the individual-country policy maker (condition (46)) in the proof of Proposition 4. More specifically see point 1 of Proof D.1. ■

E How Policy Instruments affect the Terms of Trade and Production Efficiency

First we state and prove two Lemmata. The first one identifies conditions for $\delta_{ij} \geq 1/2$ with $i, j = H, F$. The second one signs the contribution of each component to the terms-of-trade effect of condition (46).

E.1 Lemma 7 and its proof

Lemma 7 *Let $f_{ji} > f_{ii}\tau_{ij}^{1-\varepsilon}$ for $i \neq j$ and $i = H, F$. Then at any symmetric allocation:*

(i) $\delta_{ii} \geq 1/2$ if trade taxes are not used, namely such that $\tau_{Tij} = 1$ for $i, j = H, F$.

(ii) $\delta_{ii} < 1/2$ only if there are export or import subsidies such that $\tau_{Tij} < 1$ for $i, j = H, F$.

Proof We prove this lemma point by point.

(i) Using equations (7), (8) and (9), imposing symmetry of the allocation and of taxes and $\tau_{Tij} = 1$, we obtain $\delta_{ii} = \left[1 + \tau_{ij}^{1-\varepsilon}\frac{\int_{\varphi_{ji}}^{\infty}\varphi^{\varepsilon-1}dG(\varphi)}{\int_{\varphi_{ii}}^{\infty}\varphi^{\varepsilon-1}dG(\varphi)}\right]^{-1}$ and $\left(\frac{\varphi_{ji}}{\varphi_{ii}}\right) = \left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon-1}}\tau_{ij}\tau_{Tij}^{\frac{\varepsilon}{\varepsilon-1}}$. Since $\left(\frac{\varphi_{ji}}{\varphi_{ii}}\right) = \left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon-1}}\tau_{ij} > 1$ by assumption it follows that $\frac{\int_{\varphi_{ji}}^{\infty}\varphi^{\varepsilon-1}dG(\varphi)}{\int_{\varphi_{ii}}^{\infty}\varphi^{\varepsilon-1}dG(\varphi)} < 1$. Thus $1 + \tau_{ij}^{1-\varepsilon}\frac{\int_{\varphi_{ji}}^{\infty}\varphi^{\varepsilon-1}dG(\varphi)}{\int_{\varphi_{ii}}^{\infty}\varphi^{\varepsilon-1}dG(\varphi)} < 2$ and $\delta_{ii} > 1/2$.

(ii) We prove this point by contradiction. Suppose that $\tau_{Tij} \geq 1$. Combining again conditions (7), (8) and (9) and imposing symmetry we get $\delta_{ii} = \left[1 + \tau_{ij}^{1-\varepsilon}\tau_{Tij}^{-\varepsilon}\frac{\int_{\varphi_{ji}}^{\infty}\varphi^{\varepsilon-1}dG(\varphi)}{\int_{\varphi_{ii}}^{\infty}\varphi^{\varepsilon-1}dG(\varphi)}\right]^{-1}$ and $\frac{\varphi_{ji}}{\varphi_{ii}} = \left(\frac{f_{ji}}{f_{ii}}\right)^{\frac{1}{\varepsilon-1}}\tau_{ij}\tau_{Tij}^{\frac{\varepsilon}{\varepsilon-1}}$. Then if $\tau_{Tij} \geq 1$, $\varphi_{ji} > \varphi_{ii}$ since $f_{ji} > f_{ii}\tau_{ij}^{1-\varepsilon}$. As a result, $\int_{\varphi_{ji}}^{\infty}\varphi^{\varepsilon-1}dG(\varphi) < \int_{\varphi_{ii}}^{\infty}\varphi^{\varepsilon-1}dG(\varphi)$ and thus $1 + \tau_{ij}^{1-\varepsilon}\frac{\int_{\varphi_{ji}}^{\infty}\varphi^{\varepsilon-1}dG(\varphi)}{\int_{\varphi_{ii}}^{\infty}\varphi^{\varepsilon-1}dG(\varphi)} < 2$ and hence $\delta_{ii} > 1/2$. Therefore, $\delta_{ii} < 1/2$ only if $\tau_{Tij} < 1$. ■

E.2 Lemma 8 and its proof

Lemma 8 *Consider a marginal unilateral increase in each trade policy instrument at a time, starting from the laissez-faire equilibrium, i.e., with $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$. Then:*

(a) In the one-sector model deviating from the laissez-faire equilibrium induces:

(i) $\frac{dW_i}{W_i} - \frac{dW_j}{W_j} > 0$ *when* $d\tau_{Ii} > 0$ *and* $d\tau_{Xi} = 0$*;*
$\frac{dW_i}{W_i} - \frac{dW_j}{W_j} < 0$ *when* $d\tau_{Ii} = 0$ *and* $d\tau_{Xi} > 0$*;*

(ii) $\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}} > 0$ *when* $d\tau_{Ii} > 0$ *and* $d\tau_{Xi} = 0$*;*
$\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}} = 0$ *when* $d\tau_{Ii} = 0$ *and* $d\tau_{Xi} > 0$*;*

(iii) $\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}} < 0$ *when* $d\tau_{Ii} > 0$ *and* $d\tau_{Xi} = 0$*;*
$\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}} = 0$ *when* $d\tau_{Ii} = 0$ *and* $d\tau_{Xi} > 0$*.*

(b) In the multi-sector model deviating from the laissez-faire equilibrium induces:

(i) $\frac{dL_{Cj}}{L_{Cj}} - \frac{dL_{Ci}}{L_{Ci}} < 0$ *when* $d\tau_{Ii} > 0$ *and* $d\tau_{Xi} = 0$*;*
$\frac{dL_{Cj}}{L_{Cj}} - \frac{dL_{Ci}}{L_{Ci}} > 0$ *when* $d\tau_{Ii} = 0$ *and* $d\tau_{Xi} > 0$*;*

(ii) $\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}} < 0$ *iff* $\delta_{ii} > 1/2$ *when* $d\tau_{Ii} > 0$ *and* $d\tau_{Xi} = 0$*;*
$\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}} > 0$ *iff* $\delta_{ii} > 1/2$ *when* $d\tau_{Ii} = 0$ *and* $d\tau_{Xi} > 0$*;*

(iii) $\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}} > 0$ *iff* $\delta_{ii} > 1/2$ *when* $d\tau_{Ii} > 0$ *and* $d\tau_{Xi} = 0$*;*
$\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}} < 0$ *iff* $\delta_{ii} > 1/2$ *when* $d\tau_{Ii} = 0$ *and* $d\tau_{Xi} > 0$*.*

Proof We prove Lemma 8 point by point.

(a) In the case of the one-sector model we can prove points (i), (ii) and (iii) as follows:

(i) Combining conditions (B-28), (B-27) and (B-24) we find at the laissez-faire equilibrium:

$$dW_i = A_{\tau Ii} d\tau_{Ii} + A_{\tau Xi} d\tau_{Xi} \tag{E-1}$$

where $A_{\tau Ii} = \frac{\delta_{ii} d\tau_{Ii} \varepsilon(\Phi_i + \varepsilon - 1)}{\delta_{ii}\Phi_i\varepsilon + (\varepsilon-1)(1-\delta_{ii}+\delta_{ii}(\varepsilon-1))} > 0$ and $A_{\tau Xi} = -1 < 0$;

(ii) Recall that $d\delta_{ji} = -d\delta_{ii}$ for $i, j = H, F$ and $j \neq i$. Then we can use (B-26) and its symmetric counterpart to obtain:

$$\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}} = B_{\tau Ii} d\tau_{Ii}, \tag{E-2}$$

where $B_{\tau Ii} = -\frac{\delta_{ii}\varepsilon\phi_{ij}(\varepsilon - 1 + \Phi_i)}{\delta_{ii}\Phi_i\varepsilon + (\varepsilon-1)(1-\delta_{ii}+\delta_{ii}(\varepsilon-1))} < 0$;

(iii) Using the solution for dC_{ii} found in point (a), condition (B-23) and their symmetric counterparts we obtain:

$$\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}} = \Gamma_{\tau Ii} d\tau_{Ii} \tag{E-3}$$

where $\Gamma_{\tau Ii} = -\frac{\delta_{ii}\varepsilon\phi_{ij}}{\delta_{ii}\Phi_i\varepsilon + (\varepsilon-1)(1-\delta_{ii}+\delta_{ii}(\varepsilon-1))} < 0$

(b) In the case of the two-sector model we can show points (i), (ii) and (iii) in the following way.

(i) Combining (B-15), (B-11), (B-17) and (B-19) together with the restrictions $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ and $d\tau_{Li} = d\tau_{Lj} = d\tau_{Ij} = d\tau_{Xj} = 0$ we obtain:

$$\frac{dL_{Cj}}{L_{Cj}} - \frac{dL_{Ci}}{L_{Ci}} = \Delta_{\tau Ii} d\tau_{Ii} + \Delta_{\tau Xi} d\tau_{Xi} \tag{E-4}$$

where $\Delta_{\tau Ii} \equiv -\frac{(1-\delta_{ii})[(\varepsilon-1)(1-\alpha+2\delta_{ii}(\varepsilon-1+\alpha))+2\delta_{ii}\varepsilon\Phi_i]}{(1-2\delta_{ii})^2(\varepsilon-1)} < 0$ and $\Delta_{\tau Xi} \equiv \frac{(1-\delta_{ii})[(1-\alpha\delta_{ii}+\alpha(1-\delta_{ii})+2\delta_{ii}(\varepsilon-1))(\varepsilon-1)+2\delta_{ii}\varepsilon\Phi_i]}{(1-2\delta_{ii})^2(\varepsilon-1)} > 0$.

(ii) Recall that $\delta_{ji} = 1 - \delta_{ii}$, implying that $d\delta_{ji} = -d\delta_{ii}$ and $d\delta_{ij} = -d\delta_{jj}$. Using (B-7) and (B-14) to compute $d\delta_{ii}$ and $d\delta_{jj}$, and combing them with (B-11), (B-17) and (B-19) together with the restrictions $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ and $d\tau_{Li} = d\tau_{Lj} = d\tau_{Ij} = d\tau_{Xj} = 0$ we obtain:

$$\frac{d\delta_{ij}}{\delta_{ij}} - \frac{d\delta_{ji}}{\delta_{ji}} = Z_{\tau Ii} d\tau_{Ii} + Z_{\tau Xi} d\tau_{Xi} \tag{E-5}$$

where $Z_{\tau Ii} = -Z_{\tau Xi} \equiv -\frac{(1-\delta_{ii})\delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)}{\delta_{ij}(\varepsilon-1)(2\delta_{ii}-1)} < 0$ iff $\delta_{ii} > 1/2$;

(iii) First, we use (B-3) and (B-6) to compute $d\varphi_{ji}$ and (B-10) to compute $d\varphi_{ij}$ and we impose symmetry. Second, combining these conditions with (B-11), (B-17) and (B-19) together with the restrictions $\tau_{Li} = \tau_{Ii} = \tau_{Xi} = 1$ and $d\tau_{Li} = d\tau_{Lj} = d\tau_{Ij} = d\tau_{Xj} = 0$ we get:

$$\frac{d\varphi_{ij}}{\varphi_{ij}} - \frac{d\varphi_{ji}}{\varphi_{ji}} = H_{\tau Ii} d\tau_{Ii} + H_{\tau Xi} d\tau_{Xi} \tag{E-6}$$

where $H_{\tau Ii} = -H_{\tau Xi} \equiv \frac{\delta_{ii}\varepsilon}{(2\delta_{ii}-1)(\varepsilon-1)} > 0$ iff $\delta_{ii} > 1/2$. ■

E.3 Proof of Lemma 3

Proof We prove Lemma 3 point by point.

(a) In the one-sector model $dL_{Ci} = 0$ in (46) so that the production-efficiency effect is zero for all policy instruments.

(b) When $\tau_{Ii} = \tau_{Xi} = 1$, the consumption-efficiency effect in (46) is zero for any dC_{ii} and dC_{ij}.

(c) In the case of heterogeneous firms, we can substitute conditions (E-1), (E-2) and (E-3) into (47) and impose $dL_{Ci} = 0$ and $\tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$ and $dW_j = 0$ for $j \neq i$ to obtain:

$$C_{ij}[d(\tau_{Ij}^{-1} P_{ji}) - d(\tau_{Ii}^{-1} P_{ij})] = \Theta_{\tau Ii}\ d\tau_{Ii},$$

where $\Theta_{\tau Ii} = \frac{L(1-\delta_{ii})\delta_{ii}\varepsilon((\varepsilon-1)^2+\varepsilon\Phi_i)}{(\varepsilon-1)[(\varepsilon-1)(1-\delta_{ii}+\delta_{ii}(\varepsilon-2))+\varepsilon\delta_{ii}\Phi_i]} > 0$.

Similarly, in the case of homogeneous firms, condition (47) can be simplified by setting $dL_{Ci} = d\delta_{ij} = d\varphi_{ij} = 0$ and $\tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$ and $dW_j = 0$. Then, we can use condition (B-29) to get:

$$C_{ij}[d(\tau_{Ij}^{-1} P_{ji}) - d(\tau_{Ii}^{-1} P_{ij})] = I_{\tau Ii} d\tau_{Ii} + I_{\tau Xi}\ d\tau_{Xi},$$

where $I_{\tau Ii} = \frac{\varepsilon\tau^{\varepsilon}}{\tau+(2\varepsilon-1)\tau^{\varepsilon}} > 0$ and $I_{\tau Ii} = \frac{\varepsilon\tau^{\varepsilon}}{\tau+(2\varepsilon-1)\tau^{\varepsilon}} > 0$.

(d) This follows from the previous points. ■

E.4 Proof of Lemma 4

Proof We prove Lemma 4 point by point.

(a) Using conditions (B-11), (B-17) and (B-19) we can rewrite the production-efficiency effect in (46) as

$$\left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}-1\right)dL_{Ci}=E_{\tau Ii}d\tau_{Ii}+E_{\tau Xi}d\tau_{Xi}+E_{\tau Li}d\tau_{Li},$$

where $E_{\tau Ii}\equiv\frac{L_{Ci}(1-\delta_{ii})\delta_{ii}}{(1-2\delta_{ii})^2(\varepsilon-1)^2}\left[(\varepsilon-1)((1-\alpha)(1-2\delta_{ii})+\varepsilon)+\varepsilon\Phi_i\right]$, $E_{\tau Xi}\equiv-\frac{L_{Ci}(1-\delta_{ii})}{(1-2\delta_{ii})^2(\varepsilon-1)^2}[(\varepsilon-1)(1+\delta_{ii}(\alpha+2\delta_{ii}(1-\alpha)+\varepsilon-3))+\delta_{ii}\varepsilon\Phi_i]$ and $E_{\tau Li}\equiv L_{Ci}\frac{(\varepsilon-1)[2\delta_{ii}^2(\varepsilon+\alpha-2)-1-\delta_{ii}(2\varepsilon+\alpha-4)]-2(1-\delta_{ii})\delta_{ii}\varepsilon\Phi_i}{(1-2\delta_{ii})^2(\varepsilon-1)^2}$. To see why $E_{\tau Ii}>0$ it is sufficient to notice that $(1-\alpha)(1-2\delta_{ii})+\varepsilon=(1-\alpha)(1-\delta_{ii})+\varepsilon-(1-\alpha)\delta_{ii}$. What remains to show is that: (i) $E_{\tau Xi}<0$ and (ii) $E_{\tau Li}<0$.

(i) A sufficient condition for $E_{\tau Xi}<0$ is $\overline{E}_{\tau Xi}(\delta_{ii})\equiv 1+\delta_{ii}(\alpha+\varepsilon-3)+2\delta_{ii}^2(1-\alpha)>0$ for all $0\leq\delta_{ii}\leq 1$. In what follows we show that this is the case.

First, consider that $\overline{E}_{\tau Xi}(\delta_{ii})$ is quadratic in δ_{ii} with $\overline{E}''_{\tau Xi}(\delta_{ii})=4(1-\alpha)>0$ (i.e., the function has a minimum) and the minimum is equal to $\min\overline{E}_{\tau Xi}(\delta_{ii})\equiv\overline{E}^M_{\tau Xi}(\varepsilon,\alpha)=-\frac{(1+\alpha)^2-2(3-\alpha)\varepsilon+\varepsilon^2}{8(1-\alpha)}$. Second, note that $\overline{E}_{\tau Xi}(\delta_{ii})>0$ for both $\delta_{ii}=0$ and $\delta_{ii}=1$ since $\overline{E}_{\tau Xi}(0)=1$ and $\overline{E}_{\tau Xi}(1)=\varepsilon-\alpha>0$. This implies that if $\overline{E}'_{\tau Xi}(0)\geq 0$ ($\overline{E}'_{\tau Xi}(1)\leq 0$), then $\overline{E}_{\tau Xi}(\delta_{ii})$ is monotonically increasing (decreasing) and always positive for $0\leq\delta_{ii}\leq 1$. Therefore, the two necessary conditions for $\overline{E}_{\tau Xi}(\delta_{ii})<0$ for $0<\delta_{ii}<1$ are $\overline{E}'_{\tau Xi}(0)=\varepsilon+\alpha-3<0$ and $\overline{E}'_{\tau Xi}(1)=\varepsilon+1-3\alpha>0$, i.e., $\max\{1,3\alpha-1\}<\varepsilon<3-\alpha$. Hence, the last step to demonstrate that $\overline{E}_{\tau Xi}(\delta_{ii})>0$ for all $0\leq\delta_{ii}\leq 1$ is to show that $\overline{E}^M_{\tau Xi}(\varepsilon,\alpha)>0$ always when $\max\{1,3\alpha-1\}<\varepsilon<3-\alpha$. Let's call $\overline{E}^{M\prime}_{\tau Xi}(\varepsilon)$ the partial derivative of $\overline{E}^M_{\tau Xi}(\varepsilon)$ w.r.t. ε. Note that $\overline{E}^{M\prime}_{\tau Xi}(\varepsilon)=\frac{3-\varepsilon-\alpha}{4(1-\alpha)}$ decreases in ε and is greater than zero as long as $\varepsilon<3-\alpha$. This implies that $\overline{E}^M_{\tau Xi}(\varepsilon,\alpha)$ increases in ε in the admissible parameter range. What remains to do is then to evaluate the sign of $\overline{E}^M_{\tau Xi}(\varepsilon,\alpha)$ at the minimum admissible range for ε. There are two cases. If $\alpha>\frac{2}{3}$, then $\varepsilon=\max\{1,3\alpha-1\}=3\alpha-1$. Instead if $\alpha<\frac{2}{3}$, then $\varepsilon=\max\{1,3\alpha-1\}=1$. In the first case, $\overline{E}^M_{\tau Xi}(3\alpha-1,\alpha)=2\alpha-1>0$ always for $\alpha>\frac{2}{3}$. In the second case, $\overline{E}^M_{\tau Xi}(1,\alpha)=\frac{4-\alpha(4+\alpha)}{8(1-\alpha)}$. Note that $\overline{E}^M_{\tau Xi}(1,\alpha)>0$ for $\alpha^1<\alpha<\alpha^2$ where $\alpha^1=-2-\sqrt{2}<0$ and $\alpha^2=-2+2\sqrt{2}>\frac{2}{3}$. As a consequence, $\overline{E}^M_{\tau Xi}(1,\alpha)>0$ in the relevant parameter range $0<\alpha<\frac{2}{3}$. We can thus conclude that if $\overline{E}_{\tau Xi}(\delta_{ii})$ has a minimum for $0<\delta_{ii}<1$, such a minimum is always positive.

(ii) A sufficient condition for $E_{\tau Li}<0$ is $\overline{E}_{\tau Li}(\delta_{ii})\equiv-1-\delta_{ii}(2\varepsilon+\alpha-4)+2\delta_{ii}^2(\varepsilon+\alpha-2)<0$ for all $0\leq\delta_{ii}\leq 1$. In what follows we show that this is always the case.

First, note that $\overline{E}_{\tau Li}(\delta_{ii})$ is quadratic in δ_{ii} with $\overline{E}''_{\tau Li}(\delta_{ii})=4(\varepsilon-2+\alpha)$ and its critical point is equal to $\overline{E}^M_{\tau Li}(\varepsilon,\alpha)=-\frac{\varepsilon}{2}-\frac{\alpha^2}{8(-2+\alpha+\varepsilon)}$. Second, observe that $\overline{E}_{\tau Li}(0)=-1<\overline{E}_{\tau Li}(1)=-(1-\alpha)<0$. As a consequence, if $\overline{E}''_{\tau Li}(\delta_{ii})>0$, $\overline{E}_{\tau Li}(\delta_{ii})$ has a minimum for $0\leq\delta_{ii}<1$ and it is always negative in this range. Thus, what remains to show is that $\overline{E}_{\tau Li}(\delta_{ii})<0$ even when $\overline{E}''_{\tau Li}(\delta_{ii})<0$ i.e., when $\varepsilon<2-\alpha$ and $\overline{E}_{\tau Li}(\delta_{ii})$ has a maximum. Two scenarios are possible. If $\varepsilon\geq 2-\frac{3}{2}\alpha$, then $\overline{E}'_{\tau Li}(1)=-4+3\alpha+2\varepsilon\geq 0$. As a result, $\overline{E}_{\tau Li}(\delta_{ii})$ is monotonically increasing and thus always negative for $0<\delta_{ii}\leq 1$. Instead, when $1<\varepsilon<2-\frac{3}{2}\alpha$, $\overline{E}_{\tau Li}(\delta_{ii})$ has a maximum for $0<\delta_{ii}<1$. Hence, the last step is to show that such a maximum is always negative. Notice that $\overline{E}^M_{\tau Li}(\varepsilon,\alpha)=0$ for $\varepsilon^1=1-\sqrt{1-\alpha}-\frac{\alpha}{2}$ and $\varepsilon^2=1+\sqrt{1-\alpha}-\frac{\alpha}{2}$. It is easy to see that $\varepsilon^1<1$ and that $\varepsilon^2>2-\frac{3}{2}\alpha$, i.e., $\overline{E}^M_{\tau Li}(\varepsilon,\alpha)$ never changes sign in $1<\varepsilon<2-\frac{3}{2}\alpha$ and $0<\alpha<1$. To complete the proof it is then enough to show that $\overline{E}^M_{\tau Li}(\varepsilon,\alpha)<0$ at one point in our interval. For example, if $\alpha=0.5$, $\varepsilon=1.2<2-\frac{3}{2}\alpha$ and $\overline{E}^M_{\tau Li}(1.2,0.5)=-0.49<0$.

(b) It is easy to see that the consumption-efficiency effect in (46) is zero for all policy instruments when $\tau_{Ii}=\tau_{Xi}=1$.

(c) At the laissez-faire allocation we can use conditions (E-4), (E-5) and (E-6) and impose $dW_i = 0$ and $\tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$ to rewrite terms of trade effects in (47) as:

$$C_{ij}[d(\tau_{Ij}^{-1}P_{ji}) - d(\tau_{Ii}^{-1}P_{ij})] = \Sigma_{\tau Ii}d\tau_{Ii} + \Sigma_{\tau Xi}d\tau_{Xi} + \Sigma_{\tau Li}d\tau_{Li}, \tag{E-7}$$

where $\Sigma_{\tau Ii} \equiv -\frac{L_{Ci}(1-\delta_{ii})[(1-\delta_{ii})(\varepsilon-1)(1-\alpha+(\varepsilon-1+\alpha)2\delta_{ii})+\delta_{ii}\varepsilon\Phi_i]}{(1-2\delta_{ii})^2(\varepsilon-1)^2}$, $\Sigma_{\tau Li} \equiv \frac{L_{Ci}(1-\delta_{ii})[(\varepsilon-\alpha(2\delta_{ii}-1))(\varepsilon-1)+\varepsilon 2\delta_{ii}\Phi_{ii}]}{(1-2\delta_{ii})^2(\varepsilon-1)^2}$ and $\Sigma_{\tau Xi} \equiv \frac{L_{Ci}(1-\delta_{ii})[(\varepsilon-1)(\delta_{ii}+2\delta_{ii}^2(\varepsilon-1)+(\varepsilon+\alpha(1-\delta_{ii}))(1-2\delta_{ii}))+\delta_{ii}\varepsilon\Phi_i]}{(1-2\delta_{ii})^2(\varepsilon-1)^2}$. If is easy to show that $\Sigma_{\tau Ii} < 0$ in the relevant parameter range. To see why $\Sigma_{\tau Li} > 0$ it is sufficient to observe that $\varepsilon - \alpha(2\delta_{ii} - 1) > 0$ for all $0 < \delta_{ii} < 1$. Therefore, what remains to demonstrate is that $\Sigma_{\tau Xi} > 0$.

A sufficient condition for $\Sigma_{\tau Xi} > 0$ is $\overline{\Sigma}_{\tau Xi}(\delta_{ii}) \equiv \delta_{ii} + 2\delta_{ii}^2(\varepsilon-1) + (\varepsilon + \alpha(1-\delta_{ii}))(1-2\delta_{ii}) > 0$ for $0 \leq \delta_{ii} \leq 1$. First, consider that $\overline{\Sigma}_{\tau Xi}(\delta_{ii})$ is quadratic in δ_{ii} with $\overline{\Sigma}''_{\tau Xi}(\delta_{ii}) = 4(\varepsilon - 1 + \alpha) > 0$ i.e., the function has a minimum and this minimum is equal to $\min \overline{\Sigma}_{\tau Xi}(\delta_{ii}) \equiv \overline{\Sigma}^M_{\tau Xi}(\delta_{ii}) = \frac{4\varepsilon(\alpha+\varepsilon-1)-(1+\alpha)^2}{8(\varepsilon-1+\alpha)}$. Second, observe that $\overline{\Sigma}_{\tau Xi}(0) = \varepsilon + \alpha > \varepsilon - 1 = \overline{\Sigma}_{\tau Xi}(1) > 0$ i.e., $\overline{\Sigma}_{\tau Xi}(\delta_{ii})$ is positive at both ends of the relevant interval. Then, there are two cases. If $\varepsilon \leq \frac{3-\alpha}{2}$, $\overline{\Sigma}'_{\tau Xi}(1) = 2\varepsilon + \alpha - 3 < 0$ implying $\overline{\Sigma}_{\tau Xi}(\delta_{ii})$ is monotonically decreasing and always positive for $0 \leq \delta_{ii} \leq 1$. By contrast, if $\varepsilon > \frac{3-\alpha}{2}$, then $\overline{\Sigma}'_{\tau Xi}(1) > 0$ implying $\overline{\Sigma}_{\tau Xi}(\delta_{ii})$ reaches a minimum for $0 \leq \delta_{ii} \leq 1$. However, when $\varepsilon > \frac{3-\alpha}{2}$ then $\overline{\Sigma}^M_{\tau Xi}(\delta_{ii}) > 0$. Indeed, in this case $4\varepsilon(\alpha + \varepsilon - 1) - (1+\alpha)^2 > 4\frac{3-\alpha}{2}(\alpha + \frac{3-\alpha}{2} - 1) - (1+\alpha)^2 = 2(1-\alpha^2) > 0$.

(d) Combining the effects found at point (a), (b) and (c) we find that (46) can be rewritten as:

$$\begin{aligned} dU_i =& \frac{1}{I_i}\left[\left(\frac{\varepsilon}{\varepsilon-1} - 1\right) dL_{Ci} + C_{ji}\left(d(\tau_{Ij}^{-1}P_{ji}) - d(\tau_{Ii}^{-1}P_{ij})\right)\right] \\ =& \frac{1}{I_i}\left[E_{\tau Ii}d\tau_{Ii} + E_{\tau Xi}d\tau_{Xi} + E_{\tau Li}d\tau_{Li} + \Sigma_{\tau Ii}d\tau_{Ii} + \Sigma_{\tau Xi}d\tau_{Xi} + \Sigma_{\tau Li}d\tau_{Li}\right] \\ =& \frac{1}{I_i}\left[\Omega_{\tau Ii}d\tau_{Ii} + \Omega_{\tau Xi}d\tau_{Xi} + \Omega_{\tau Li}d\tau_{Li}\right], \end{aligned} \tag{E-8}$$

where $\Omega_{\tau Ii} \equiv \frac{L_{Ci}(1-\delta_{ii})[\delta_{ii}\varepsilon-(2\delta_{ii}-1)(1-\alpha)]}{(2\delta_{ii}-1)(\varepsilon-1)} d\tau_{Ii} > 0$, $\Omega_{\tau Xi} \equiv \frac{L_{Ci}(1-\delta_{ii})[(1-2\delta_{ii})(1-\alpha)-(1-\delta_{ii})\varepsilon]}{(2\delta_{ii}-1)(\varepsilon-1)} < 0$ and $\Omega_{\tau Li} \equiv \frac{L_{Ci}[(1-2\delta_{ii})(1-\alpha)-(1-\delta_{ii})\varepsilon]}{(2\delta_{ii}-1)(\varepsilon-1)} < 0$ iff $\delta_{ii} > 1/2$. To see why this is the case first note that the denominators of all these coefficients are positive iff $\delta_{ii} > 1/2$. Moreover, the numerator of $\Omega_{\tau Ii}$ is always positive since $\delta_{ii} > 2\delta_{ii} - 1$ for $\delta_{ii} < 1$, while the numerators of $\Omega_{\tau Xi}$ and $\Omega_{\tau Li}$ are always negative since $1 - 2\delta_{ii} < 1 - \delta_{ii}$ and $1 - \alpha < \varepsilon$ and as a consequence $(1 - 2\delta_{ii})(1-\alpha) - (1-\delta_{ii})\varepsilon < 0$. ■

F The Design of Trade Agreements in the Presence of Domestic Policies

In this section we prove Propositions 7, 8 and 9, which state the main results on strategic policies when all policy instruments (Proposition 7) or only production taxes (Propositions 8 and 9) are available. In both cases, we solve the Nash problems using the total-differential approach described in Appendix B. We focus on symmetric Nash equilibria in the two-sector model for which $\alpha < 1$ and $W_i = W_j = 1$ for $i, j = H, F$.

F.1 Proof of Proposition 7

Proof We prove Proposition 7 point by point.

(a) First, we write the differential of the terms-of trade effect in (46) in terms of dL_{Ci}, dC_{ii}, dC_{ij}. For this purpose, we use the differentials of the equilibrium conditions derived in Appendix B.2.2 – imposing symmetry and the restrictions $d\tau_{Lj} = d\tau_{Ij} = d\tau_{Xj} = 0$ – to evaluate each component of the terms-of-trade effects as decomposed in (47). In particular, we use: conditions (B-15) and (B-16) for term (ii) (differential of the amount of labor in both countries allocated to the differentiated sectors); conditions (B-7) and (B-14) jointly with the fact that $d\delta_{ji} = -d\delta_{ii}$ for term (iii) (differential of the average-profit shares in the export markets) and conditions (B-3), (B-6) and (B-10) for term (iv) (differentials of the export productivity cut-offs). Finally, we employ (B-11), (B-17) and (B-19) to substitute out $d\tau_{Li}$, $d\tau_{Ii}$ and $d\tau_{Xi}$ to obtain:

$$C_{ji}d(\tau_{Ij}^{-1}P_{ji}) - C_{ij}d(\tau_{Ii}^{-1}P_{ij}) = \Sigma_{Cii}dC_{ii} + \Sigma_{Cij}dC_{ij} + \Sigma_{LCi}dL_{Ci} \tag{F-1}$$

where:

$$\Sigma_{Cii} = -\frac{(\varepsilon f_{ij})^{\frac{1}{\varepsilon-1}}\tau_{Li}\tau_{Xi}}{(L_{Ci}\delta_{ii})^{\frac{1}{\varepsilon-1}}\delta_{ii}(\varepsilon-1)^2} \frac{(\varepsilon-1)[(1-\alpha)(\varepsilon-\delta_{ii})\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}) + \alpha\delta_{ii}(\varepsilon-1) + \alpha\varepsilon(1-\delta_{ii})\tau_{Li}\tau_{Xi}] + \delta_{ii}\varepsilon[\alpha+(1-\alpha)\tau_{Li}]\Phi_i}{\delta_{ii}[\alpha+(1-\alpha)\delta_{ii}\tau_{Li}] - (1-\delta_{ii})\tau_{Li}\tau_{Xi}[\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii}] - \frac{(\varepsilon-1+\Phi_i)\varepsilon}{\varepsilon-1}\delta_{ii}[\alpha+(1-\alpha)\tau_{Li}]}$$

$$\Sigma_{Cij} = \frac{(\varepsilon f_{ij})^{\frac{1}{\varepsilon-1}}\tau_{ij}\tau_{Li}\tau_{Xi}}{(L_{Ci}(1-\delta_{ii}))^{\frac{1}{\varepsilon-1}}\varphi_{ij}} \frac{[(\varepsilon-1+\delta_{ii})(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}) - (1-\delta_{ii})(\alpha\varepsilon+(1-\alpha)(1-\delta_{ii})\tau_{Ii})\tau_{Li}\tau_{Xi} - \frac{\delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)}{\varepsilon-1}((1-\alpha)\tau_{Li}+\alpha)]}{(\delta_{ii}H-\Pi)(\varepsilon-1) - \delta_{ii}\varepsilon[(1-\alpha)\tau_{Li}+\alpha](\varepsilon-1+\Phi_i)}$$

$$\Sigma_{LCi} = \frac{\tau_{Li}\tau_{Xi}\left[(\varepsilon-\delta_{ii})\frac{1-\alpha}{\varepsilon-1}\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi}) + \alpha\delta_{ii} + \alpha\frac{\varepsilon}{\varepsilon-1}(1-\delta_{ii})\tau_{Li}\tau_{Xi} + \delta_{ii}\frac{\varepsilon}{(\varepsilon-1)^2}(\alpha+(1-\alpha)\tau_{Li})\Phi_i\right]}{\delta_{ii}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li}) - (1-\delta_{ii})\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii}) - \frac{\delta_{ii}\varepsilon}{\varepsilon-1}(\alpha+(1-\alpha)\tau_{Li})(\varepsilon-1+\Phi_i)}$$

where Σ_{Cii}, Σ_{Cij}, and Σ_{LCi} have been simplified using equations (8)-(14). Moreover, $\Pi = (1-\delta_{ii})(\alpha+(1-\alpha)\tau_{Ii})\tau_{Li}\tau_{Xi}$ and $H = \alpha + (1-\alpha)\tau_{Li}[\delta_{ii} + (1-\delta_{ii})\tau_{Ii}\tau_{Xi}]$. Condition (F-1) allows us to write (46) as follows:

$$\begin{aligned} dV_i =& (1-\tau_{Xi})P_{ii}dC_{ii} + (\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}dC_{ij} + \left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi} - 1\right)dL_{Ci} + C_{ji}d(\tau_{Ij}^{-1}P_{ji}) - C_{ij}d(\tau_{Ii}^{-1}P_{ij}) \\ =& E_{Cii}dC_{ii} + E_{Cij}dC_{ij} + E_{LCi}dLC_i + \Sigma_{Cii}dC_{ii} + \Sigma_{Cij}dC_{ij} + \Sigma_{LCi}dL_{Ci} \\ =& \Omega_{Cii}dC_{ii} + \Omega_{Cij}dC_{ij} + \Omega_{LCi}dL_{Ci} \end{aligned} \tag{F-2}$$

where $E_{Cii} \equiv (1-\tau_{Xi})P_{ii}$, $E_{Cij} \equiv (\tau_{Ii}-1)\tau_{Ii}^{-1}P_{ij}$, $E_{LCi} \equiv \frac{\varepsilon}{\varepsilon-1}\tau_{Li}\tau_{Xi} - 1$, $\Omega_{Cii} \equiv E_{Cii} + \Sigma_{Cii}$, $\Omega_{Cij} \equiv E_{Cij} + \Sigma_{Cij}$, and $\Omega_{LCi} \equiv E_{LCi} + \Sigma_{LCi}$. Condition (F-2) corresponds to condition (50) in the main text.

(b) In appendix B.1.2 we explained how to apply the total differential approach to solve a constrained optimization problem in n variables with m constraints. In this case we have 25 variables (22 endogenous variables plus 3 policy instruments) and 22 constraints i.e., exactly 3 degrees of freedom to choose the policy instruments so has to maximize world welfare.[59]. In point (a) we show how to rewrite the total differential of (50) as function of 3 total differentials ($dC_{ii}, dC_{ij}, dL_{Ci}$ with $i = H, F$ and $i \neq j$). As explained in B.1.2, at the optimum the wedges multiplying each differential needs to be individually equal to zero, i.e., $\Omega_{Cii} = \Omega_{Cij} = \Omega_{LCi} = 0$. This gives a set of 3 additional equations which can be used to solve for the optimal policy instruments. Once we have the solution for the instruments we can use the 22 constraints to determine the solution of the remaining 22 variables.

Before moving to point (c) we simplify each of these wedges to make them tractable.

First, consider $\Omega_{Cij} \equiv E_{Cij} + \Sigma_{Cij}$. Using (12) and imposing symmetry, the consumption-efficiency wedge E_{Cij}

[59]When $\alpha = 1$ we have 24 variables and 21 constraints while for the model with homogeneous firms we have 13 variables and 10 constraints.

in (F-2) can be written as:

$$E_{Cij} = \frac{(\tau_{Ii}-1)(\varepsilon f_{ij})^{\frac{1}{\varepsilon-1}}\varepsilon\tau_{ij}\tau_{Li}\tau_{Xi}}{(L_{Ci}(1-\delta_{ii}))^{\frac{1}{\varepsilon-1}}(\varepsilon-1)\varphi_{ij}}$$

Then, recalling condition (F-1) we obtain

$$\Omega_{Cij} = \frac{\overline{\Omega}_{Cij}\tau_{ij}\tau_{Li}\tau_{Xi}(\varepsilon f_{ij})^{\frac{1}{\varepsilon-1}}}{\varphi_{ij}(\varepsilon-1)(L_{Ci}(1-\delta_{ii}))^{\frac{1}{\varepsilon-1}}\left[(\delta_{ii}H-\Pi)(\varepsilon-1)-\delta_{ii}\varepsilon((1-\alpha)\tau_{Li}+\alpha)(\varepsilon-1+\Phi_i)\right]},$$

where

$$\overline{\Omega}_{Cij} = (\varepsilon-1)((\varepsilon-1)(1-\delta_{ii})H+\varepsilon\tau_{Ii}(\delta_{ii}H-\Pi))-\delta_{ii}\varepsilon(\varepsilon-1+\Phi_i)((1-\alpha)\tau_{Li}+\alpha)(\varepsilon\tau_{Ii}-\varepsilon+1). \tag{F-3}$$

Second, consider $\Omega_{Cii} \equiv E_{Cii}+\Sigma_{Cii}$. Again using (12), the consumption-efficiency wedge E_{Cii} in (F-2) can be simplified as:

$$E_{Cii} = \frac{(\tau_{Xi}-1)(\varepsilon f_{ii})^{\frac{1}{\varepsilon-1}}\varepsilon\tau_{Li}}{(L_{Ci}\delta_{ii})^{\frac{1}{\varepsilon-1}}(\varepsilon-1)\varphi_{ii}}$$

Therefore, by (F-1)

$$\Omega_{Cii} = \frac{\overline{\Omega}_{Cii}(\varepsilon f_{ii})^{\frac{1}{\varepsilon-1}}\tau_{Li}(L_{Ci}\delta_{ii})^{-\frac{1}{\varepsilon-1}}(\varepsilon-1)^{-2}\varphi_{ii}^{-1}}{\delta_{ii}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li})-(1-\delta_{ii})\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii})-\frac{(\varepsilon-1+\Phi_i)\varepsilon}{\varepsilon-1}(\alpha+(1-\alpha)\tau_{Li})},$$

where

$$\begin{aligned}\overline{\Omega}_{Cii} \equiv{}& (1-\tau_{Xi})[\varepsilon(\varepsilon-1)(\delta_{ii}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li})-(1-\delta_{ii})\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii}))\\ &-(\varepsilon-1+\Phi_i)\varepsilon^2\delta_{ii}(\alpha+(1-\alpha)\tau_{Li})]\\ &-\tau_{Xi}[(\varepsilon-1)(\varepsilon(1-\alpha)\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi})-(1-\alpha)\delta_{ii}\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi})\\ &+\alpha\delta_{ii}(\varepsilon-1)+\alpha\varepsilon(1-\delta_{ii})\tau_{Li}\tau_{Xi})+\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})\Phi_i]\end{aligned} \tag{F-4}$$

Finally, consider $\Omega_{LCi} \equiv E_{LCi}+\Sigma_{LCi}$. Combining the production-efficiency wedge in (F-2) and condition (F-1) we obtain:

$$\Omega_{LCi} = \frac{\overline{\Omega}_{LCi}(\varepsilon-1)^{-1}}{\delta_{ii}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li})-(1-\delta_{ii})\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii})-\frac{\delta_{ii}\varepsilon}{\varepsilon-1}(\alpha+(1-\alpha)\tau_{Li})(\varepsilon-1+\Phi_i)}$$

where

$$\begin{aligned}\overline{\Omega}_{LCi} \equiv{}& \delta_{ii}(\varepsilon-1)\tau_{Li}\tau_{Xi}[\alpha+(1-\alpha)\tau_{Li}(\delta_{ii}+(1-\delta_{ii})\tau_{Ii}\tau_{Xi})-\varepsilon(\alpha+(1-\alpha)\tau_{Li})]\\ &-(\varepsilon-1)[\delta_{ii}(\alpha+(1-\alpha)\delta_{ii}\tau_{Li})-(1-\delta_{ii})\tau_{Li}\tau_{Xi}(\alpha+(1-\alpha)(1-\delta_{ii})\tau_{Ii})-\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})]\\ &-(\tau_{Li}\tau_{Xi}-1)\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})\Phi_i\end{aligned} \tag{F-5}$$

Notice that from (F-3), (F-4) and (F-5) we can conclude that $\Omega_{LCi} = \Omega_{Cii} = \Omega_{Cij} = 0$ iff $\overline{\Omega}_{LCi} = \overline{\Omega}_{Cii} = \overline{\Omega}_{Cij} = 0$.

(c) First recall that from point (b) in the Nash equilibrium

$$\overline{\Omega}_{LCi} = \overline{\Omega}_{Cii} = \overline{\Omega}_{Cij} = 0, \tag{F-6}$$

where $\overline{\Omega}_{LCi}$, $\overline{\Omega}_{Cii}$, and $\overline{\Omega}_{Cij}$ are defined in (F-3), (F-4), and (F-5). These wedges are functions of 8 variables only: τ_{Li}, τ_{Ii}, τ_{Xi}, φ_{ii}, φ_{ij}, $\widetilde{\varphi}_{ii}$, $\widetilde{\varphi}_{ij}$, and δ_{ii}. Observe that once we impose symmetry and we take into account that $\delta_{ji} = 1-\delta_{ii}$ also conditions (7) -(10) are functions of these variables only. Therefore, we can fully characterize the symmetric Nash equilibrium using the 3 conditions in (F-6) jointly with the 5 equilibrium equations (7)-(10). In what follows we use the superscript N to indicate that a variable is evaluated at the Nash equilibrium.

To prove point (c), we proceed in 3 steps. First, we show that in the Nash equilibrium it must be the case that $\tau_L^N = \frac{\varepsilon-1}{\varepsilon}$. Second, we show that $\overline{\Omega}_{LCi} > 0$ always when $\tau_X < 1$ and $\tau_L = \tau_L^N$. Therefore, when a Nash equilibrium exists it must be such that $\tau_X^N > 1$. Finally, we show that $\overline{\Omega}_{Cij} < 0$ always when $\tau_I > 1$, $\tau_X > 1$ and $\tau_L = \tau_L^N$. Hence, when a Nash equilibrium exists it must be such that $\tau_I^N < 1$.

(1) We use $\overline{\Omega}_{LCi} = \overline{\Omega}_{Cii} = 0$ to solve for τ_L and τ_I and we obtain two sets of solutions, (τ_L^1, τ_I^1) and (τ_L^2, τ_I^2):

$$
\begin{aligned}
\tau_L^1 &= \frac{\varepsilon - 1}{\varepsilon} \\
\tau_I^1 &= \frac{(1-\alpha)\delta_{ii}^2(\varepsilon(1-\tau_X)+\tau_X) - \alpha\varepsilon\tau_X + \delta_{ii}\varepsilon((\varepsilon-1+\alpha)\tau_X - \varepsilon)}{(1-\alpha)(1-\delta_{ii})\tau_X[\varepsilon(1-\delta_{ii}) + \delta_{ii}\tau_X(\varepsilon-1)]} \\
&+ \frac{\delta_{ii}\varepsilon(\varepsilon-1+\alpha)(\varepsilon(\tau_X-1)-\tau_X)\Phi_i}{(1-\alpha)(1-\delta_{ii})(\varepsilon-1)^2\tau_X[\varepsilon(1-\delta_{ii})+\delta_{ii}\tau_X(\varepsilon-1)]} \\
\tau_L^2 &= -\alpha\frac{1+\varepsilon(\varepsilon-2+\Phi_i)}{(\varepsilon-1)[(1-\alpha)(\varepsilon-\delta_{ii}) + \alpha(1-\delta_{ii})\tau_X] + (1-\alpha)\varepsilon\Phi_i} \\
\tau_I^2 &= -\frac{\alpha}{1-\alpha}
\end{aligned}
$$

Note that $\tau_I^2 < 0$, which is outside the admissible range for τ_I. Thus, the only possible solution is (τ_L^1, τ_I^1), implying that when a Nash equilibrium exists, it must be that $\tau_L^N = \frac{\varepsilon-1}{\varepsilon}$. We can thus substitute τ_L^N into $\overline{\Omega}_{LCi}$, $\overline{\Omega}_{Cii}$, and $\overline{\Omega}_{Cij}$ (labeling these expressions $\overline{\Omega}_{LCi}^N$, $\overline{\Omega}_{Cii}^N$, and $\overline{\Omega}_{Cij}^N$ respectively) to obtain:

$$
\begin{aligned}
\overline{\Omega}_{LCi}^N &= \overline{\Omega}_{LCi}^{\mathcal{N}} + \overline{\Omega}_{LCi}^{\Phi} \\
\overline{\Omega}_{Cii}^N &= -\frac{\overline{\Omega}_{LCi}^N}{\varepsilon} \\
\overline{\Omega}_{Cij}^N &= \overline{\Omega}_{Cij}^{\mathcal{N}} + \overline{\Omega}_{Cij}^{\Phi}
\end{aligned}
$$

where

$$
\begin{aligned}
\overline{\Omega}_{LCi}^{\mathcal{N}} &\equiv (\varepsilon-1)^2[\delta_{ii}(\varepsilon-(\varepsilon-1)\tau_X)(\varepsilon-(1-\alpha)\delta_{ii}) + \delta_{ii}(\varepsilon-1)\tau_X((1-\alpha)(1-\delta_{ii})\tau_I\tau_X) \\
&\quad + \varepsilon((1-\delta_{ii})(\alpha+(1-\alpha)(1-\delta_{ii})\tau_I)\tau_X)] \\
\overline{\Omega}_{LCi}^{\Phi} &\equiv \delta_{ii}\varepsilon(\varepsilon-1+\alpha)(\varepsilon-(\varepsilon-1)\tau_X)\Phi_i \\
\overline{\Omega}_{Cij}^{\mathcal{N}} &\equiv (\varepsilon-1)\left[\delta_{ii}(\varepsilon-1+\alpha)(\varepsilon(1-\tau_I)-1) + \delta_{ii}\tau_I(\alpha\varepsilon + \delta_{ii}(\varepsilon-1)(1-\alpha)) + (1-\delta_{ii})(\varepsilon-1)\left(\alpha + \varepsilon^{-1}\delta_{ii}(1-\alpha)(\varepsilon-1)\right)\right. \\
&\quad \left. +(1-\delta_{ii})(\varepsilon-1)\tau_I\tau_X\left(\varepsilon^{-1}(1-\alpha)(\varepsilon-1)(1-\delta_{ii}) - \alpha - (1-\alpha)(1-\delta_{ii})\tau_I\right)\right] \\
\overline{\Omega}_{Cij}^{\Phi} &\equiv \delta_{ii}(\varepsilon-1+\alpha)(\varepsilon(1-\tau_I)-1)\Phi_i
\end{aligned}
$$

Note that $\overline{\Omega}_{Cii}^N$ and $\overline{\Omega}_{LCi}^N$ are collinear. In the next steps we thus use only $\overline{\Omega}_{LCi}^N$ and $\overline{\Omega}_{Cij}^N$ to characterize the Nash equilibrium for the remaining two instruments, τ_X^N and τ_I^N.

(2) First, observe that $\varepsilon - (\varepsilon-1)\tau_X > 0$ iff $\tau_X < \frac{\varepsilon}{\varepsilon-1}$. This implies that when $\tau_X < \frac{\varepsilon}{\varepsilon-1}$ then both $\overline{\Omega}_{LCi}^{\mathcal{N}} > 0$ and $\overline{\Omega}_{LCi}^{\Phi} > 0$. Therefore, $\overline{\Omega}_{LCi}^N > 0$ for all $\tau_X < \frac{\varepsilon}{\varepsilon-1}$, implying that there cannot be a Nash equilibrium in this region as it will never be the case that $\overline{\Omega}_{LCi}^N = 0$. Thus, in the Nash equilibrium it must be the case that $\tau_X^N > \frac{\varepsilon}{\varepsilon-1} > 1$.

(3) What remains to show is that $\tau_I^N < 1$. We prove this by contradiction. Assume $\tau_I^N > 1$. In the previous point, we already showed that $\tau_X^N > 1$, thus if $\tau_I^N > 1$ also $\tau_I^N\tau_X^N > 1$. First, consider that $\overline{\Omega}_{Cij}^{\Phi} < 0$ when $\tau_I^N > 1$. As a consequence, a necessary condition for the Nash equilibrium to exist in the region $\tau_I > 1$ is that there exist a $\tau_I > 1$ such that $\overline{\Omega}_{Cij}^{\mathcal{N}} > 0$. To see whether this is the case, observe that $\overline{\Omega}_{Cij}^{\mathcal{N}}$ is linear in α since δ_{ii} (as implicitly determined by conditions (7)-(10)) is independent of α. Moreover, when $\alpha = 0$ $\overline{\Omega}^{\mathcal{N}}_{Cij} = (\varepsilon-1)^2\left[-\delta_{ii}(1-\delta_{ii}+\varepsilon(\tau_I-1)(\varepsilon-\delta_{ii})) - (1-\delta_{ii})^2(1+\varepsilon(\tau_I-1))\tau_I\tau_X\right] < 0$ while when $\alpha = 1$, $\overline{\Omega}^{\mathcal{N}}_{Cij} = -(\varepsilon-1)^2[(\tau_I\tau_X-1)(1-\delta_{ii}) + \delta_{ii}\varepsilon(\tau_I-1)] < 0$. This implies that $\overline{\Omega}^{\mathcal{N}}_{Cij} < 0$ for all $\tau_I > 1$. Therefore, $\overline{\Omega}^{N}_{Cij} < 0$ for all $\tau_I > 1$ which contradicts our original hypothesis of a Nash equilibrium with $\tau_I^N > 1$. Thus, if a Nash equilibrium exists it must be such that $\tau_I^N < 1$. ■

F.2 Proof of Proposition 8

Proof We prove Proposiiton 8 point by point.

(a) When only production taxes are available $\tau_{Ii} = \tau_{Xi} = 1$ for $i = H, F$. Therefore, the consumption-efficiency wedges in (46) are absent. Hence, to prove this point it is sufficient to rewrite the term-of-trade effect as a function of dL_{Ci} only, and then add it to the production-efficiency term.

For this purpose, we follow the same approach used in point (a) of Proof F.1. We use the differentials of the equilibrium conditions derived in Appendix B.2.2 to evaluate each component of the terms-of-trade effects as decomposed in (47) with the difference that in this case we do not only impose symmetry and $d\tau_{Lj} = d\tau_{Ij} = d\tau_{Xj} = 0$ but also the restrictions $d\tau_{Ii} = d\tau_{Xi} = 0$. Moreover, given the system of 3 equations ((B-11), (B-17), and (B-19)) in 6 variables ($d\tau_{Li}$, $d\tau_{Ii}$, $d\tau_{Xi}$, dL_{Ci}, dC_{ii}, dC_{ij}) and given that here we are imposing $d\tau_{Ii} = d\tau_{Xi} = 0$, we are able to express $d\tau_{Li}$, dC_{ii}, dC_{ij} as a function of dL_{Ci} only. This allows us to obtain:

$$C_{ji}dP_{ji} - C_{ij}dP_{ij} = \Sigma_i dL_{Ci}$$

with:

$$\Sigma_i \equiv \frac{(1-\delta_{ii})(\alpha+(1-\alpha)\tau_{Li})[(\alpha(2\delta_{ii}-1)(1+\varepsilon(\tau_{Li}-1)) - \varepsilon\tau_{Li}) - 2\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})\Phi_i]}{(\varepsilon-1)\Sigma_{di}} \tag{F-7}$$

$$\begin{aligned}\Sigma_{di} \equiv\ & (\varepsilon-1)\left[(1-\delta_{ii})(1+2\delta_{ii}(\varepsilon-1))(\alpha+(1-\alpha)\tau_{Li}) + (1-\alpha)(1-2\delta_{ii})(\alpha(\tau_{Li}-1)-\delta_{ii}\tau_{Li})\right] \\ & + 2(1-\delta_{ii})\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})\Phi_i\end{aligned} \tag{F-8}$$

Then, in this case condition (46) can be simplified as:

$$\begin{aligned} dV_i =& \left(\frac{\varepsilon}{\varepsilon-1}\tau_{Li} - 1\right)dL_{Ci} + C_{ji}dP_{ji} - C_{ij}dP_{ij} \\ =& E_i dL_{Ci} + \Sigma_i dL_{Ci} \\ =& \Omega_i dL_{Ci} \end{aligned} \tag{F-9}$$

where $\Omega_i \equiv E_i + \Sigma_i$ and $E_i \equiv \frac{\varepsilon}{\varepsilon-1}\tau_{Li} - 1$. Condition (F-9) corresponds to condition (51) in the main text.

(b) Characterizing the Nash problem when only production taxes are available means solving the constrained problem in (45) imposing $\tau_{Ii} = \tau_{Xi} = 1$. We follow the same steps explained in general terms in Appendix B.1.2. The problem can be reduced to a maximization problem in 23 variables (22 endogenous variables plus 1 policy instrument) subject to the equilibrium conditions (7)-(14). In the previous point we showed how to rewrite the total differential of (45) as in (51) namely as a function of one total differential only, dL_{Ci}. The number of policy instruments available to the individual-country policy maker is also one. This implies that at the optimum condition (51) must be equal to zero, i.e., $\Omega_i = 0$. Note how we can rewrite Ω_i as:

$$\Omega_i = \frac{\overline{\Omega}_i}{(\varepsilon-1)\Sigma_{di}}$$

where

$$
\begin{aligned}
\overline{\Omega}_i \equiv\ & (\varepsilon-1)\left[(1+\varepsilon(\tau_{Li}-1))((1-\delta_{ii})(1-\alpha+2\delta_{ii}(\varepsilon-(1-\alpha)))(\alpha+(1-\alpha)\tau_{Li})\right. \\
& \left.+(1-\alpha)(1-2\delta_{ii})(\alpha(\tau_{Li}-1)-\delta_{ii}\tau_{Li}))-(1-\delta_{ii})(\alpha+(1-\alpha)\tau_{Li})\varepsilon\tau_{Li}\right] \\
& +2(1-\delta_{ii})\delta_{ii}\varepsilon(\alpha+(1-\alpha)\tau_{Li})(\varepsilon-(1-\alpha))(\tau_{Li}-1)\Phi_i
\end{aligned} \tag{F-10}
$$

Given this last condition we can conclude that $\Omega_i = 0$ iff $\overline{\Omega}_i = 0$.

(c) First, note that $\overline{\Omega}_i$ is a function of 6 variables: τ_{Li}, φ_{ii}, φ_{ij}, $\widetilde{\varphi}_{ii}$, $\widetilde{\varphi}_{ij}$, and δ_{ii}. Second, under symmetry and when $\tau_{Ii} = \tau_{Xi} = 1$, the equilibrium equations (7)-(10) give us 5 conditions, which provide a solution for φ_{ii}, φ_{ji}, $\widetilde{\varphi}_{ii}$, $\widetilde{\varphi}_{ji}$, and δ_{ii} independently from τ_{Li}. Hence, condition

$$
\overline{\Omega}_i = 0 \tag{F-11}
$$

jointly with conditions (7)-(10) allows us to fully characterize the Nash equilibrium when only the production tax is available.

For what follows, note that $\overline{\Omega}_i$ can be conceived as a quadratic polynomial in τ_{Li} (called $\overline{\Omega}_i(\tau_{Li})$). Differently from the Nash problem with all instruments, the symmetric Nash-equilibrium policy will not affect the profit-share from sales in the domestic market and thus δ_{ii} can be determined independently of τ_{Li}. Moreover, $\overline{\Omega}_i(0) < 0$ for $0 < \delta_{ii} \leq 1$ and $\overline{\Omega}_i(0) = 0$ when $\delta_{ii} = 0$ since $\overline{\Omega}_i(0) = -(\varepsilon-1)^2\alpha\left[(1-\delta_{ii})(1-\alpha+2\delta_{ii}(\alpha+\varepsilon-1))\right.$ $\left.-(1-2\delta_{ii})(1-\alpha)\right]-2\alpha(1-\delta_{ii})\delta_{ii}\varepsilon(\alpha+\varepsilon-1)\Phi_i$ and both $1-\delta_{ii} > 1-2\delta_{ii}$ and $1-\alpha+2\delta_{ii}(\alpha+\varepsilon-1) > 1-\alpha$. In addition, $\overline{\Omega}_i(\frac{\varepsilon-1}{\varepsilon}) = -(1-\delta_{ii})(\alpha+\varepsilon-1)\left[(\varepsilon-1)^2+2\delta_{ii}(\alpha+\varepsilon-1)\Phi_i\right]\varepsilon^{-1}$. Hence, $\overline{\Omega}_i(\frac{\varepsilon-1}{\varepsilon}) < 0$ for $0 \leq \delta_{ii} < 1$ and $\overline{\Omega}_i(\frac{\varepsilon-1}{\varepsilon}) = 0$ when $\delta_{ii} = 1$. Moreover, observe that $\overline{\Omega}_i(1) = (2\delta_{ii}-1)(\varepsilon-1)\left[(1-\delta_{ii})(\varepsilon-1+\alpha)+\delta_{ii}(1-\alpha)\right]$. As a consequence, $\overline{\Omega}_i(1) \geq 0$ iff $\delta_{ii} \geq \frac{1}{2}$. Finally, take into account that $\overline{\Omega}_i''(\tau_{Li}) = 2(1-\alpha)\delta_{ii}\varepsilon[(\varepsilon-1)\varpi_i(\delta_{ii})$ $+2(1-\delta_{ii})(\alpha+\varepsilon-1)\Phi_i]$ where $\varpi_i(\delta_{ii}) \equiv 2\delta_{ii}(2-\alpha-\varepsilon)+2\varepsilon+\alpha-3$ is linear in δ_{ii} and can be characterized as follows: $\varpi_i(0) = 2\varepsilon+\alpha-3 \geq 0$ iff $\varepsilon \geq \frac{3-\alpha}{2}$, $\varpi_i(1) = 1-\alpha > 0$ and $\varpi_i(\delta_{ii}) \geq 0$ iff $\delta_{ii} \geq \frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)}$. Now, we are ready to prove points (i) and (ii) point by point.

(i) Consider the case $\delta_{ii} \geq \frac{1}{2}$. This implies that $\overline{\Omega}_i(1) \geq 0$. Recall that $\overline{\Omega}_i(\tau_{Li})$ is quadratic, implying that it has at most two zeros. Note that $\overline{\Omega}_i(0) < 0$ and $\overline{\Omega}_i(\frac{\varepsilon-1}{\varepsilon}) < 0$. If $\overline{\Omega}_i''(\tau_{Li}) \geq 0$ then $\overline{\Omega}_i(\tau_{Li})$ is convex, and the zeros must be such that $\tau_L^1 < 0$ and $\frac{\varepsilon-1}{\varepsilon} \leq \tau_L^2 \leq 1$. However, $\tau_{Li} \geq 0$ by assumption. Hence, as long as $\delta_{ii} \geq \frac{1}{2}$ and $\overline{\Omega}_i''(\tau_{Li}) \geq 0$, there exist a unique symmetric Nash equilibrium, namely $\frac{\varepsilon-1}{\varepsilon} \leq \tau_L^N = \tau_L^2 \leq 1$. Therefore, what remains to show in order to prove point (c) (i) is that $\overline{\Omega}_i''(\tau_{Li}) \leq 0$ when $\delta_{ii} \geq \frac{1}{2}$. The second derivative is given by $\overline{\Omega}_i''(\tau_{Li}) = 2(1-\alpha)\delta_{ii}\varepsilon\left[(\varepsilon-1)\varpi_i(\delta_{ii})+2(1-\delta_{ii})(\alpha+\varepsilon-1)\Phi_i\right]$ where $\varpi_i(\delta_{ii}) \equiv 2\delta_{ii}(2-\alpha-\varepsilon)+2\varepsilon+\alpha-3$.

Note that if $\varepsilon \geq \frac{3-\alpha}{2}$, then by linearity $\varpi_i(\delta_{ii}) \geq 0$ for all $0 \leq \delta_{ii} \leq 1$. Instead, if $\varepsilon < \frac{3-\alpha}{2}$, then $\varpi_i(\delta_{ii}) \geq 0$ for all $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} \leq \delta_{ii} \leq 1$. However, we can show that $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} < \frac{1}{2}$ when $\varepsilon < \frac{3-\alpha}{2}$. Indeed, $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} < \frac{1}{2}$ iff $\frac{2\varepsilon+\alpha-3}{\varepsilon+\alpha-2} < 1$ and $\varepsilon+\alpha-2 < 0$ when $\varepsilon < \frac{3-\alpha}{2}$. Therefore, in this case $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} < \frac{1}{2}$ iff $2\varepsilon+\alpha-3 > \varepsilon+\alpha-2$. This inequality holds since $\varepsilon > 1$. As a consequence, $\varpi_i(\delta_{ii}) \geq 0$ for all $\frac{1}{2} \leq \delta_{ii} \leq 1$, which implies that $\overline{\Omega}_i(\tau_{Li})$ is convex in this parameter range.

(ii) Now consider the case $\delta_{ii} < \frac{1}{2}$. In this case $\overline{\Omega}_i(1) < 0$. In the previous point we have already argued that $\overline{\Omega}_i(\tau_{Li})$ is convex when either $\varepsilon \geq \frac{3-\alpha}{2}$ or when $\frac{2\varepsilon+\alpha-3}{2(\varepsilon+\alpha-2)} \leq \delta_{ii} < \frac{1}{2}$ and $\varepsilon < \frac{3-\alpha}{2}$. Since $\overline{\Omega}_i(\tau_{Li})$ is quadratic $\overline{\Omega}_i(0) \leq 0$ and $\overline{\Omega}_i(\frac{\varepsilon-1}{\varepsilon}) < 0$, there exist two zeros of $\overline{\Omega}_i(\tau_{Li})$ such that $\tau_L^1 \leq 0$ and $\tau_L^2 > 1$. Again, we can exclude $\tau_L^1 \leq 0$ since $\tau_{Li} > 0$ by assumption. As a consequence, there exists a unique symmetric Nash equilibrium with $\tau_L^N = \tau_L^2 \geq 1$.

■

F.3 Proof of Proposition 9

Proof I We prove Proposition 9 point by point.

(a) First, consider the case of heterogeneous firms. According to Proposition 8, when $\delta_{ii} \geq \frac{1}{2}$ and only domestic policies are available any symmetric Nash equilibrium is such that $\frac{\varepsilon-1}{\varepsilon} \leq \tau_L^N \leq 1$. Hence, a sufficient condition for the Nash allocation to entail higher welfare than the free-trade allocation is that in a symmetric equilibrium individual-country welfare is monotonically decreasing in τ_{Li}. In other words, we need to demonstrate that in a symmetric equilibrium $\frac{dU_i}{d\tau_{Li}} \leq 0$ as long as $\tau_{Li} \geq \frac{\varepsilon-1}{\varepsilon}$. To show this result, first observe that $\frac{dU_i}{d\tau_{Li}} = \frac{dU_i}{dL_{Ci}} \frac{dL_{Ci}}{d\tau_{Li}}$. Second, consider that the total differential of the utility in (3) can be written as in condition (D-2). Then, if we combine this total differential with the total differential of (13) and (14) departing from a symmetric allocation we get:

$$dU_i = -\frac{P_{ii}}{I_i} dC_{ii} - \frac{P_{ij}}{I_i} dC_{ij} + -\frac{1}{I_i} dL_{Ci}$$

Moreover, it can be shown[60] that under symmetry $dC_{ij} = \frac{C_{ij}}{L_{Ci}} \frac{\varepsilon}{\varepsilon-1} dL_{Ci}$ for $i, j = H, F$. By substituting these conditions into the differential above and taking into account conditions (11) and (12) we obtain:

$$dU_i = \frac{1}{I_i} \left(\frac{\varepsilon}{\varepsilon - 1} \tau_{Li} - 1 \right) dL_{Ci} \tag{F-12}$$

This last result follows directly from the fact that symmetric deviations of the production subsidy from a symmetric allocation do not have an impact on the cut offs φ_{ij} and on the market shares δ_{ij}, implying that terms-of-trade effects are zero. Moreover, consumption-efficiency wedges are also zero since import tariffs and export taxes are absent. Hence, changes in welfare in condition (46) are equal to the production-efficiency effects only. Finally, it can be shown that:

$$\frac{dL_{Ci}}{d\tau_{Li}} = -\frac{(1-\alpha)L_{Ci}}{\alpha + \tau_{Li}(1-\alpha)} < 0$$

This allows us to conclude that $\frac{dU_i}{d\tau_{Li}} = -\frac{L_{Ci}}{I_i} \left(\frac{\varepsilon}{\varepsilon-1} \tau_{Li} - 1 \right) \frac{1-\alpha}{\alpha+\tau_{Li}(1-\alpha)} \leq 0$ if and only if $\tau_{Li} \geq \frac{\varepsilon-1}{\varepsilon}$. Moving to the homogeneous-firm set up, it is easy to show that when starting from a symmetric allocation condition (F-12) still holds. Moreover, Campolmi et al. (2014) have already proved that also in this case $\frac{dL_{Ci}}{d\tau_{Li}} = \frac{dN_i}{d\tau_{Li}} < 0$. Therefore with both homogeneous and heterogeneous firms, $\frac{dU_i}{d\tau_{Li}} \leq 0 \iff \tau_{Li} \geq \frac{\varepsilon-1}{\varepsilon}$ and independently of the value of δ_{ii}. We know from Proposition 8 that $\frac{\varepsilon-1}{\varepsilon} \leq \tau_L^N \leq 1$ when $\delta_{ii} \geq \frac{1}{2}$. As a consequence, when $\delta_{ii} < \frac{1}{2}$ the symmetric Nash equilibrium is welfare dominated by the free-trade allocation.

(b) By taking the the differential of conditions (7), (8) and (9) with respect to f_{ij} and τ_{ij}, it can be shown that:

$$d\delta_{ii} = \frac{(\varepsilon - 1 + \Phi_i)\delta_{ii}(1-\delta_{ii})}{\tau_{ij}} d\tau_{ij} + \frac{\Phi_i \delta_{ii}(1-\delta_{ii}) \tilde{\varphi}_{ij}^{1-\varepsilon} \varphi_{ij}^{\varepsilon-1}}{(\varepsilon-1) f_{ij}} df_{ij},$$

which confirms that δ_{ii} is monotonically increasing in both τ_{ij} and f_{ij}. ■

[60] The proof is available on request.