A Large Scale OLG Model for the Analysis of the Redistributive Effects of Policy Reforms

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Abstract

The paper presents a large scale overlapping generation model with heterogeneous agents, where the household is the decision unit. We calibrate the model for three European countries – France, Italy and Sweden – which show marked differences in the design of some public programmes. We examine the properties in terms of annual and life cycle redistribution of a number of tax-benefit programmes, by studying the impact of removing from our model economies some or all of them. We find that whether one considers a life cycle or an annual horizon, and whether behavioural responses are accounted for or not, has a large impact on the results. The model may provide useful insights for policy makers on which kind of reforms are more likely to achieve specific equity objectives.

Keywords: Redistribution, Fiscal policy, Computable OLG models.

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1 Introduction

Most of the public sector interventions in the economy are justified by redistributive goals. The set of tools employed to this end is typically large, including several tax and transfer programmes with complex interactions among them, and, in some cases, potentially offsetting effects. These interventions are typically motivated by inequality in income, based on annual measures. However inequality in living standards is more appropriately captured by the distribution of life cycle income and it is known that measures of annual and life cycle incomes are remarkably different (Aaberge and Mogstad, 2012).

Due to such complexities, a full understanding of the redistributive impact of public sector intervention is hard to obtain both theoretically and empirically. Theoretical contributions typically focus on one programme at the time, while the modelling of indirect effects on other programmes is usually stylised. From the empirical perspective, addressing these questions is highly demanding in terms of data availability. In particular, longitudinal data are rarely sufficiently rich to allow an estimate of life cycle income redistribution.[1]

We propose a large scale overlapping generation (OLG) model with intragenerational heterogeneity and with a comprehensive description of public sector intervention.[2] In particular, we consider personal income tax, consumption tax, capital income tax, social contributions, pension benefits, health care, child benefit, subsidy to daycare, and income support. We model these policies for three European countries: France, Italy and Sweden. Ideally, one would like to compare the size of redistribution across countries which are as similar as possible in terms of overall level of public intervention, but sufficiently different in terms of its composition. A comparison among France, Italy and Sweden seems to have these properties, to a reasonable extent. In 2013 the ratio of public expenditure over GDP ranged between 51% of Italy to 57% of France (Eurostat database[3]). On the other hand, the three

[1] Aaberge and Mogstad (2012) use longitudinal data to measure life cycle income inequality. However they do not assess how life cycle income inequality is affected by public policies.

[2] Another popular tool for the analysis of the redistributive impact of public policies is tax-benefit microsimulation (see, for example, Bourguignon and Spadaro 2006 for an overview). Computational constraints typically prevent OLG models to have the same level of heterogeneity as in microsimulation models. However, in OLG models behavioural responses are structural, while this is not necessarily the case in microsimulation models. Moreover, computable overlapping generations models also provide the opportunity to extend the analysis of inequality to the inter-generational dimension and to take general equilibrium effects into account.

[3] Available at http://ec.europa.eu/eurostat/data/database.

systems show some crucial differences. In the typical categorisation of welfare states (Barr, 2004) the three countries belong to three different groups: conservative/corporatist (France), Mediterranean (Italy), and social-democratic (Sweden). The composition of social expenditures is quite different across countries, with Italy spending much more on pensions and less on other transfers than the other two countries. On the revenues side, the tax unit for personal income taxation is the family in France, whereas it is the individual in Italy and Sweden. Given that the family is the decision unit in our model, the implications of this difference are particularly interesting to explore. Finally, the fact that social contributions in Italy fund almost exclusively pensions, whereas they also fund other programmes in France and Sweden, may also bear relevant redistributive implications.

In this paper we describe the characteristics of the model and we show that it performs well in matching some relevant real world data on some non-calibrated outcomes. We simulate the model to explore the annual and life cycle redistributive effects of the main tax and expenditure programmes. The results of this exercise may be relevant to policy-makers wishing to identify the most appropriate reform to target annual or life cycle inequality. We find that life cycle redistribution is generally much lower than annual redistribution, and that ignoring behavioural responses may severely bias the analysis of redistribution.

Heterogenous agents OLG models have already been used to assess the impact of policy reforms. For example, İmrohoroglu *et al.* (1995) and Imrohoroglu *et al.* (1999), Huggett and Ventura (1999), Nishiyama and Smetters (2007), Fehr and Habermann (2008), Fehr *et al.* (2013) study the effect of pension reforms. Ventura (1999), Conesa and Krueger (2006) and Erosa and Koreshkova (2007) analyse the effects of progressive taxation, Nishiyama and Smetters (2005) focus on consumption taxation, and Conesa *et al.* (2009) study capital income taxation. A recent, relevant trend in computabale overlapping generations models is the expansion of the heterogeneity dimensions, well beyond age and individual productivity, to include for example gender, marital status and the number of children. Models featuring these additional sources of heterogeneity have already been used to study both pension reforms (Hong and Rios-Rull, 2007; Fehr *et al.*, 2012) and tax reforms (Guner *et al.*, 2012a,b). Our framework also includes these modelling features, i.e. we explicitly introduce in the model gender, marital status and the number of children.

Our analysis has three distinctive features. The first feature is the large number of policies that are simultaneously included in the model. We think that the inclusion of a wide range of taxes and transfers is important not only to study the overall redistributive impact of public intervention; it also matters because the impact of a specific programme is likely to depend on the characteristics of the other programmes that are in place. In other terms, complementarities/ substitutabilities between different transfers and taxes might exist and ignoring them is a potential source of bias. The second feature is that we devote specific attention to the difference between annual and life cycle redistribution. The third feature is that we apply our model to three European countries: France, Italy and Sweden. To the best of our knowledge, heterogeneous agents computable overlapping generations models have not been used to study the impact of public policies in Italy and France; for Sweden, we are aware of the work by Domeij and Klein (2002), whose focus is on the pension system only.

The structure of the paper is as follows. Section 2 presents the model. Section 3 provides information on the setting of the model parameters. In Section 4 we report results of some numerical experiments on different policy reforms. Finally, we report some concluding remarks in Section 5. Additional information concerning solution methods and calibration are provided in the Appendix.

2 The model

We consider a small open economy populated by $\overline{J}$ overlapping generations. We denote by $j = \left\{1, 2, ..., J^R, ..., \overline{J}\right\}$ the age of an individual, where J^R is the exogenously fixed retirement age and $\overline{J}$ is the maximum age that can be reached. Individuals may die before age $\overline{J}$, according to a survival probability that will be later specified. We assume that the size of each newborn generation is $1 + n$ times the previous one, where n is assumed to be constant over time. Therefore, since the survival probability function is also assumed to be constant over time, the population structure is stationary and the growth rate of population is equal to n.

Within a generation individuals are heterogeneous along several dimensions: gender, marital status, presence of children, educational level. For tractability reasons we take these dimensions as exogenous and make some simplifying assumptions. First, we assume that single individuals never get married and married individuals never get divorced, although they can become single if their spouses die. Second, we assume that individuals in a couple are of the same age. Finally, it is also assumed that: the number of children is either 0 or 2; only persons living in couple can have children; the birth of children, if any, always occurs when the age of the couple is $j = 1$.

The decision unit is the household. A single makes choices maximising his or her inter-temporal utility. Individuals within a couple pool their resources and maximise the sum of their inter-temporal utilities. At any age $j < J^R$ (working period) the household chooses labour supply and consumption of its members; for $j \geq J^R$ (retirement period) only consumption is chosen and labour supply is exogenously set equal to zero. Workers earn a wage per hour of labour that depends on the wage per efficiency unit and on the individual number of efficiency units; in turn, efficiency units depend on age, education level, gender, marital status and number of children of the worker. Efficiency units are also subject to an idiosyncratic shock which is independently distributed across agents. The shock follows a Markov process with transition probabilities denoted by $p(\zeta_{j+1}^g|\zeta_j^g)$ where ζ_j^g is the value of the shock at age j for an agent of gender g $(= m, f)$ and ζ_{j+1}^g is the value of the shock at age $j + 1$.

We have uncertainty at the individual level, due to the stochastic processes that define survival and efficiency units of labour, while there is no uncertainty at the aggregate level. Households are assumed to have perfect foresight on the future values of the return on assets and of the wage rate per efficiency unit. We assume perfectly competitive markets.

As to the production side, there is a sector where a physical good is produced by a representative firm using capital and labour in efficiency units. This good can be used for consumption, investment, purchase of daycare services and it is chosen in each period as the numeraire, i.e., its price is normalised to 1.

The government is empowered with a large set of policy tools: personal income tax, consumption tax, capital income tax, social contributions, pension system, health care system, child benefits, daycare subsidies and low-income support programmes.

In what follows we specify in more details the features of our model economy. We focus on a steady state equilibrium and therefore we omit time subscripts and we only use the age subscript j. For a summary of the definition of the key variables, see Table A1 in Appendix A.

2.1 Firms

The physical good Y is produced by a representative firm according to a Cobb-Douglas technology:

$$Y = AK^{\nu}L^{1-\nu} \tag{1}$$

where A is total factor productivity (assumed constant over time), K is aggregate capital stock, L is aggregate labour supply in efficiency units and $0 < \nu < 1$ is the share of capital income on output.

Profit maximisation implies the standard conditions:

$$w = (1 - \nu)Ak^{\nu} \tag{2}$$

$$r + \delta = \nu A k^{1-\nu} \tag{3}$$

where w is the wage rate per efficiency unit, r is the return on assets, δ is the depreciation rate of capital and $k \equiv K/L$.

2.2 Households

We assume that each individual has an additively time separable utility function with a momentary utility $u(c_j, z_j)$ defined over consumption c_j and leisure z_j. Individual consumption c_j is equal to the aggregate consumption of the household q_j divided by an equivalence scale θ_j, which depends on the number of adults and children in the household (see Section 3). The momentary utility function of each individual takes the following form:

$$u\left(c_j, z_j\right) = \frac{1}{\left(1 - \frac{1}{\gamma}\right)} \left(c_j^{\left(1-\frac{1}{\omega}\right)} + \alpha z_j^{\left(1-\frac{1}{\omega}\right)}\right)^{\left(\frac{1-\frac{1}{\gamma}}{1-\frac{1}{\omega}}\right)} \tag{4}$$

with:

$$c_j = \frac{q_j}{\theta_j} \tag{5}$$

where γ denotes the inter-temporal elasticity of substitution between consumption at different ages, ω defines the intra-temporal elasticity of substitution between consumption and leisure at each age j and α is an age-independent leisure preference parameter.

2.2.1 Constraints

For each individual the following time constraint holds:

$$l_j + z_j = 1 \tag{6}$$

where l_j is labour supply and the time endowment is normalised to 1. Equation (6) states that, in each period of life, the time endowment can be used for labour or for leisure time.

Moreover, we assume that a child, in his/her first period of life (corresponding to age $j = 1$ of parents), requires to be cared for all the time; accordingly for every unit of time when both parents work, daycare services must be purchased. Hence, the household demand level for daycare services, d_j, is

$$d_j = \begin{cases} \kappa_1 \min\left\{l_j^m, l_j^f\right\} & \text{for} \quad j = 1 \\ 0 & \text{for} \quad j > 1, \end{cases} \tag{7}$$

where κ_1 $(= 0, 2)$ is the number of children at age j. The number of children κ_j evolves according to the following exogenous transition equation:

$$\kappa_{j+1} = \kappa(\kappa_j, j) = \begin{cases} \kappa_j & \text{for} \quad j < J^\kappa \\ 0 & \text{for} \quad j \geq J^\kappa \end{cases} \tag{8}$$

where J^κ is the exogenously fixed age of parents at the time when children become independent.

The budget constraint of the household is given by:

$$\begin{aligned} a_{j+1} = & \underbrace{ra_j}_{\text{capital income}} + \underbrace{I_m(g)\left[(y_j^m - sc_j^m(y_j^m)\right] + I_f(g)\left[y_j^f - sc_j^f(y_j^f)\right]}_{\text{labour income net of social contributions}} + \\ & - \underbrace{t_{y,j}(y_j^m, y_j^f, \kappa_j) - \tau_r ra_j - \tau_q q}_{\text{taxes}} + \underbrace{tr_{\kappa,j}(y_j^m, y_j^f, \kappa_j)}_{\text{child subsidy}} + \underbrace{tr_{y,j}(y_j^m, y_j^f)}_{\text{income support transfer}} + \\ & - \underbrace{I_m(g)\left[\overline{hs}_j^m - hs_j^m\right] - I_f(g)\left[\overline{hs}_j^f - hs_j^f\right]}_{\text{private health expenditure}} - \underbrace{p_d(1-\tau_d)d_j}_{\text{net daycare expenditure}} - \underbrace{q_j}_{\text{consumption}} + \\ & \underbrace{\hat{B}_{j+1}}_{\text{accidental bequests}} \end{aligned} \tag{9}$$

where a_j denotes assets available at the beginning of period j; $g = m, f$; $I_m(g)$ and $I_f(g)$ are indicator functions; y_j^g denotes labor income and pension benefits; $sc_j^g(y_j^g)$ are social contributions, that are a function of income y_j^g; $t_{y,j}(y_j^m, y_j^f, \kappa_j)$ is the personal income tax paid by the household, which depends on income and may also depend on the number of children; τ_r is the tax rate on capital income; τ_q is the tax rate on consumption; $tr_{\kappa,j}(y_j^m, y_j^f, \kappa_j)$ is a child subsidy, which, in general, may depend on income and on the number of children; $tr_{y,j}(y_j^m, y_j^f)$ is a low-income support transfer, which typically depends on income; $\overline{hs}_j^g$ is the overall (private plus public) per-capita expenditure on health, which is assumed to be exogenous, and hs_j^g denotes the subsidy implicitly provided by the public sector through public health expenditure (therefore the difference between $\overline{hs}_j^g$ and hs_j^g is the exogenous private expenditure on health); p_d is the resource cost per hour of daycare services, τ_d is the subsidy to the purchase of daycare services; $\hat{B}_j$ denotes accidental bequests received by the household. In particular we assume that assets of deceased households are confiscated by the government and then redistributed in a lump sum way to working age households, i.e. $\hat{B}_j = 0$ for $j \geq J^R$, and $\hat{B}_j = \hat{B} > 0$ (constant across ages and agents) otherwise.[4]

A crucial variable is clearly income, y_j^g, which is given by:

$$y_j^g = \begin{cases} w l_j^g e_{j,h,ms,\kappa,\zeta}^g & \text{for} \quad j < J^R \\ p_j^g(sb_{J^R}^g) & \text{for} \quad j \geq J^R \end{cases} \tag{10}$$

In equation (10), $e_{j,h,ms,\kappa,\zeta}^g$ stands for efficiency units at age j of an individual of gender g, education level h, marital status ms, number of children κ, receiving an idiosyncratic shock ζ; $p_j^g(sb_{J^R}^g)$ is the pension transfer received by the household and $sb_{J^R}^g$ stands for the amount of pension rights accumulated over the working life.

Note that the specific functional forms of taxes and transfers appearing in equation (9) and (10) depend on the legislation in place in the countries we consider (see Section 3.4 for a description of the institutional features of public policies).

We assume that households face a liquidity constraint given by:

$$a_j \geq \bar{a} \quad \forall j \tag{11}$$

and we further assume $\bar{a} = 0$.

2.2.2 Household optimisation problem

The optimisation problem of singles and couples can be represented using dynamic programming. The state vector of an age j household is given by:

$$x_j = (h^m, h^f, \kappa_j, \zeta_j^m, \zeta_j^f, a_j, sb_j^m, sb_j^f). \tag{12}$$

We denote by h^g the level of education of an individual of gender g and we use the convention that $h^g \leq 0$ if and only if an individual of gender g is

[4] Assuming that accidental bequest are confiscated by the government and then redistributed in a lump sum way to (a subset of) survival households is common in large scale OLG models (see Fehr *et al.* 2013, Nishiyama 2011, Nishiyama 2003, Ventura 1999 among others). This is obviously not a real world institutional feature, but it is a tractable assumption that allows to deal in a simple way with accidental bequests due to lifespan uncertainty and the absence of a perfect annuity market. Note that, taking explicitly into account the children-parent link when allocating accidental bequests would require to further increase the number of state variables; this is done for example in De Nardi (2004) who however abstracts from many other features that our model takes into account.

not present in the household; in particular, $h^g = 0$ if an individual of gender g has never been in the household and $h^g < 0$ if an individual of gender g was in the household when the household enters the economy but then died (i.e. $h^g < 0$ identifies the case of a household with a widowed person). Note that, if $h^g = 0$ we obviously have $sb_j^g = 0$, i.e., the amount of pension rights accumulated up to age j by an individual of gender g in that household is zero; moreover, given our assumption that only couples have children, we also have $\kappa_j = 0$, i.e., the number of children is zero. However, if $h^g < 0$, $sb_j^g \geq 0$ (the spouse who died may have accumulated pension rights, that depending on the specific rules of the pension system may give rise to a survivor pension) and $\kappa_j \geq 0$ (children could be born before one of the two spouses passed away). We denote the state vector at age j of a single agent of gender g as x_j^g and of a couple as x_j^{co}.

Note that the educational levels h^g are constant along the life cycle; the other state variables however do change with age. The number of children κ_j evolves according to an exogenous transition rule (8). As already mentioned, the productivity shock evolves according to a Markov process with transition probabilities $p(\zeta_{j+1}|\zeta_j)$. Transition equations for the remaining state variables are endogenous. Assets a_j change over time according to equation (9). The dynamics of pension rights sb_j^g depends on the legal rules of pension system.

A single with state vector $x_j = x_j^g$ $(g = m, f)$ solves the following maximisation problem to determine optimal consumption and leisure decisions:

$$\max_{c_j^g, z_j^g} u\left(c_j^g, z_j^g\right) + \psi_{j+1}\left(g\right)\beta V^g(x_{j+1}^g) \tag{13}$$

where $V^g(x_{j+1}^g)$ is the value function of an agent of gender g at age $j+1$; β is the discount factor (pure time preference), and $\psi_{j+1}\left(g\right)$ is a gender-specific probability of surviving up to age $j+1$, conditional on having reached age j.

Maximisation in (13) is carried out subject to the transition equations for the state variables, the liquidity constraint (11) and the time constraint (6).

A married couple with state vector $x_j = x_j^{co}$ determines consumption and leisure from the following maximisation problem:

$$\begin{aligned}\max_{q_j, z_j^m, z_j^f} & u\left(c_j^m, z_j^m\right) + u\left(c_j^f, z_j^f\right) + \\ & \psi_{j+1}\left(m\right)\beta\left(\psi_{j+1}\left(f\right)V^m(x_{j+1}^{co}) + \left(1-\psi_{j+1}\left(f\right)\right)V^m(x_{j+1}^m)\right) + \\ & \psi_{j+1}\left(f\right)\beta\left(\psi_{j+1}\left(m\right)V^f(x_{j+1}^{co}) + \left(1-\psi_{j+1}\left(m\right)\right)V^f(x_{j+1}^f)\right)\end{aligned} \tag{14}$$

maximisation in (14) is carried out subject to all the constraints relevant for the individual, plus the demand for daycare services constraint (7) and the relationship between household consumption and individual consumption given by equation (5).

The solution to the household optimisation problem yields decision rules for consumption $q_j = \hat{q}(x_j, j)$ and labour supply $l_j^g = \hat{l}(x_j, j)$.[5] These in turn imply optimal values of the endogenous state variables $a_{j+1} = \hat{a}(x_j, j)$ and $sb_j^g = \hat{sb}^g(x_j, j)$; using equation (7), we can determine the optimal demand of daycare services $d_j = \hat{d}(x_j, j)$. Moreover taxes and transfers, directly depend on variables such as q_j, l_j^g, a_{j+1} and d_j, which in turn depend on the state vector and age. It is then possible to rewrite $p_j^g = \hat{p}(x_j, j)$, $t_{y,j} = \hat{t}_y(x_j, j)$, $sc_j^g = \hat{sc}^g(x_j, j)$, $tr_{\kappa,j} = \hat{tr}_\kappa(x_j, j)$, $tr_{y,j} = \hat{tr}_y(x_j, j)$.

2.3 Government

In addition to the policy instruments mentioned in describing the budget constraint of equation (9), the government is assumed to finance a per-capita amount $\tilde{G}$ of government consumption. This is meant to account for the value of expenditure, net of revenues, related to programmes that are not explicitly modelled. Under the assumption that the budget is balanced:

$$\tilde{G} = \tilde{T}_y + \tilde{SC} + \tilde{T}_c + \tilde{T}_r - \tilde{P} - \tilde{H} - \tilde{TR}_\kappa - \tilde{TR}_d - \tilde{TR}_y \tag{15}$$

where $\tilde{T}_y$, $\tilde{SC}$, $\tilde{T}_c$ and $\tilde{T}_r$ are respectively per-capita revenues from personal income tax, social contributions, consumption tax and capital income tax; $\tilde{P}$, $\tilde{H}$, $\tilde{TR}_\kappa$, $\tilde{TR}_d$ $\tilde{TR}_y$ are respectively per-capita public expenditure for pensions, health care, child benefits, daycare services and income support. When we calibrate the model as described in Section 3, the balanced budget condition is reached by endogenously adjusting the level of $\tilde{G}$. We specify in Section 4 the policy variable which is adjusted to balance the budget when reforms are performed.

2.4 Recursive competitive equilibrium

We are now ready to define the notion of recursive competitive equilibrium in our model economy. We consider a small open economy, we assume that there is no growth in total factor productivity and we focus on a steady state path.[6]

[5] For household made up by a single male (i.e. $x_j = x_j^m$), we obviously have that $l_j^f = \hat{l}(x_j, j) = 0$; for single female (i.e. $x_j = x_j^f$) it is $l_j^m = \hat{l}(x_j, j) = 0$.

[6] On a steady state path, the growth rate of per capita variables turns out to be zero; aggregate variables growth at a rate equal to n, which is the growth rate of the population.

The definition of equilibrium is entirely standard. Given the world interest rate $\bar{r}$, a small open economy steady state competitive equilibrium is defined as a collection of factor prices w and r, per-capita capital stock $\tilde{K}$ and per-capita labour in efficiency unit $\tilde{L}$, households' distributions χ_j, households' decision rules, accidental bequests per working age, government's revenues and expenditures such that: $r = \bar{r}$, and first order conditions of the firm (2) and (3) holds; market clearing conditions hold; the distribution of households with respect to the state vector is consistent with individual behaviour, i.e, with the transition equations for the state variables implied by the household decision rules; household decision rules are the solution of the dynamic programming problems described by equations (13) and (14); government revenues and expenditures satisfy the government budget constraint (15).

The complexity of the model implies that the solution can only be numerical. A formal definition of the equilibrium and the numerical procedure used for its computation are presented in Appendix C.

3 Calibration

In this section we describe how parameters are set in order to provide a numerical solution to the model. For further details we refer to Bucciol *et al.* (2015).

3.1 Demographics and education

Individuals enter the model when they are 25 years old. One period in the model is equivalent to 5 years. Individuals live up to a maximum of $\bar{J} = 15$ periods (i.e., the maximum age is 100 years). We set $j^R = 9$ (i.e. the retirement age is 65 years) and $J^\kappa = 5$ (i.e., children become independent and leave their parents when they are 25 years old). We set the fraction of men in the population equal to that of women.

A proper calibration of survival probabilities should use cohort life tables rather than period life tables. Of course cohort life tables are incomplete for recent cohorts. Using easily available period life tables, however, would generally lead to an under-estimation of life length because of the well-documented downward trend in mortality. To correctly estimate the survival probability, we then apply the Lee and Carter (1992) model to period life tables, separately by gender: to this end we use the Human Mortality Database for the period 1979-2008. We focus on the cohort of individuals born in 1989. We use these estimates to represent the survival probabilities of retired persons;

as to the survival probabilities of the working age population we assume that they are equal to one: this is a minor simplification (since few people die before the age of 65) but it allows to save computational time.

Based on Eurostat population projections for the period 2013-2040, the annual population growth rate is set equal to 0.4% for France, 0.4% for Italy and 0.8% for Sweden. Note that the annual population growth rate n_a, along with the survival probabilities, affect the age structure of the population, namely the ratio f between the size of the newborn cohorts at $t+1$ and the size of age group with women in their fertility age at t. Under our assumptions on the survival probabilities (which, as said above, are assumed to be 1 up to retirement age) we have: $f = (1 + n_a)^{\iota_F}$ where ι_F is the age at which women are assumed to have children. In Section 2 we assumed that agents have children in the first period they enter the model, i.e. ι_F is between age 25 and 29. Accordingly, if we assume for instance $\iota_F = 25$, the values of n_a set above imply that f is 1.11 for France, 1.11 for Italy and 1.22 for Sweden. If we considered a closed population with no migration flows, such values of f would be inconsistent with the assumptions we have made in Section 2 on the fertility behaviour of households (i.e. the maximum amount of children per household is equal to two and singles cannot have children): indeed such assumptions would imply a value of f below 1. To deal with this problem, we assume that in each period there is an inflow of newborn migrants, such that the size of the new born cohort (natives plus migrants) produces a value of f equal to the value implied by the assumed value of the annual population growth rate n_a. Moreover we assume that such newborn migrants have the same characteristics as the native agents when they become economically active at age 25, namely the same distribution of the demographic variables (i.e. marital status and number of children) and education levels.

Information related to marital status, number of children and education levels are computed for people belonging to the age group 25-50, using EU-SILC 2008 data.

Accordingly, the percentage of households comprising a married couple is set equal to 56.95% in France, 66.59% in Italy and 53.70% in Sweden. To determine the fraction of couples with children (that in our model can be 0 or 2 by assumption), we refer to the fraction of couples with at least one child: 84.21% in France, 82.81% in Italy and the 81.62% in Sweden.

As to education levels, we set them in order to target the proportion of men and women with a college degree and to generate realistic correlations between agents' education within couples. To this end, we use the procedure to generate correlated artificial binary data described in Leisch *et al.* (1998). Accordingly, we set the proportion of men with a college degree equal to: 30.33% in France, 14.37% in Italy and 30.85% in Sweden; the corresponding numbers for women are respectively: 36.80% in France, 18.61% in Italy and 41.64% in Sweden. The correlations of agents' education levels within couples are equal to 0.4309, 0.4130 and 0.3853, respectively for France, Italy and Sweden.

3.2 Preferences and production

We assume that the annual discount factor (β), the inter-temporal and intra-temporal substitution elasticity (γ and ω) are the same in the three countries and we set them respectively equal to 0.99, 0.2 and 0.4. We then choose α so that the fraction of time devoted to market work is about one third[7] in the three countries we consider, which implies α equal to 1.7 for France, 2.3 for Italy, and 2.1 for Sweden.

The equivalence scale parameter, θ, is the square root of the household size (OECD, 2011). The unitary cost p_d of non parental care is set equal to 8.47 euros per hour in France (see Silvera 2008), 4 euros per hour in Italy (our elaborations on the data provided by Istat 2011) and 8.11 euros per hour in Sweden (see Blomquist *et al.* 2010).

For the parameter ν, the share of capital income to total income, we refer to the AMECO database and accordingly set $\nu = 0.3930$ for France, $\nu = 0.4170$ for Italy and $\nu = 0.4840$ for Sweden.[8] The annual depreciation rate is 5% and the annual world return on capital $\bar{r}$ is set equal to 8.1%. The small open economy assumption then implies $r = \bar{r}$. Finally, using equations (2) and (3), the total factor productivity parameter A is chosen in such a way that the wage rate per efficiency units w is normalised to 1 (this calibration procedure implies $A = 1.700$ for France, $A = 1.702$ for Italy and $A = 1.685$ for Sweden).

3.3 Efficiency units

In the model, efficiency units $e^g_{j,h,ms,\kappa,\zeta}$ deterministically depend on age, gender, education level, marital status and the number of children, but are also subject to an idiosyncratic shock following a discrete Markov process.

Note that, since the wage rate per efficiency unit is normalized to one (see Section 3.2), efficiency units are equivalent to hourly wages. To determine efficiency units we therefore estimate, separately for each country and for college and non-college graduates, the wage profile for individual i at age j

[7]This is broadly consistent with data on time use for prime age workers (computations based on the Harmonized European Time Use data, https://www.h5.scb.se/tus/tus/)

[8]For these statistics we refer to the average over the period 1960-2016.

as the combination of deterministic and stochastic components:

$$\ln(\bar{e}_{ij}) = \bar{x}'_{ij}\beta + \epsilon_{ij}, \tag{16}$$

with $\bar{e}_{ij}$ to denote gross hourly wage, $\bar{x}_{ij}$ explanatory variables (i.e. a set of age dummies, a dummy for gender, one for marital status and one for the presence of children in the household), β parameters and ϵ_{ij} the error term. We further split ϵ_{ij} in persistent and transitory components, respectively η_{ij} and u_{ij}:

$$\epsilon_{ij} = \eta_{ij} + u_{ij}. \tag{17}$$

The persistent component follows an AR(1) process,

$$\eta_{ij} = \rho\eta_{ij-1} + v_{ij}. \tag{18}$$

Both errors u_{ij} and v_{ij} are assumed iid and mutually independent, with mean 0 and variance σ^2_U and σ^2_V, respectively.

The estimation procedure, applied to the EU-SILC panel dataset (waves 2004-2011)[9], is made of two steps: first, we estimate the parameters of the deterministic component; second, we estimate the parameters of the stochastic components. The parameters estimated on the continuous model can be used to generate shocks for the stochastic wage component. We finally discretise the continuous shock process in a four states-state markov chain using the Tauchen (1986) method, in order to get ζ^g and $p(\zeta^g_{j+1}|\zeta^g_j)$. For further details on the estimation procedure, see Appendix B.

3.4 Tax and expenditure programmes

Here we provide basic information on the modelling of public policies for the three countries considered. The interested reader can find a detailed description in Bucciol *et al.* (2015). Despite our goal to provide a coverage of public programmes as wide as possible, we exclude some among those of limited size for practical reasons. The list of the programmes modelled for each country is reported in Table 1. In modelling policies, the following general rules have been applied. First, for the definition of the different rules of tax and expenditure programmes we refer to year 2013. In very few cases, information on 2013 was not available. In that case, we referred to the latest year for which the information could be retrieved. For pensions, we apply to the whole working population the most recently defined set of rules, which are typically those relevant for younger cohorts. Second, we only consider those features of the programmes such that the definition of eligibility an/or

[9] We only consider employees, thus excluding self-employed workers.

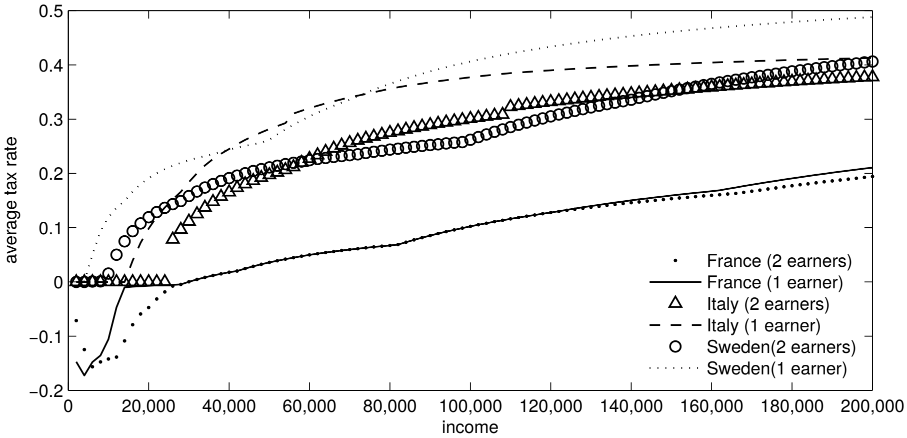


Figure 1: Average tax rate of personal income tax, as a function of family income for a couple with two children

of the amount due/received depends on characteristics accounted for in the model. This means that, for example, we do not consider tax credits related to expenditure on specific goods, because we have a single consumption good in the model. Third, only mandatory public programmes are covered by our analysis and for the pension system we only explicitly include the pay-as-you go component.

3.4.1 Taxes

The three countries we consider are quite similar in terms of overall size of public intervention, but are different in terms of institutional features of expenditures and revenues. Concerning taxation, a key difference is that the tax unit for personal income taxation is the family in France, whereas it is the individual in Italy and Sweden. Figure 1 illustrates the implications of the choice of the tax unit on the average tax rate for a family with two spouses and two children. For families with two earners total income is assumed to be equally split between them. For Italy and Sweden the combination of the individual as tax unit and the progressivity of the schedule implies that splitting income between the two earners substantially reduces the average tax rate. This is not the case for France. However, the figure also shows that average tax rates are lower in France than in Italy and Sweden, which is consistent with aggregate data on revenues over GDP. Finally, only in France the tax debt can be negative. This is the result of the introduction in 2009 of the '*Prime pour l'emploi*', a tax credit specifically designed to provide an

Table 1: List of expenditure and taxation programmes modelled

	FRANCE	ITALY	SWEDEN
Taxes			
Personal income tax	Personal income tax	Personal income tax (central and local)	Personal income tax (central and local)
Social contributions	'*contribution sociale généralisée*' and other SC related to mandatory pensions, health insurance, family benefits	SC related to mandatory pensions and family benefits	SC related to mandatory pensions and health insurance
Consumption taxation	implicit tax rate on consumption	implicit tax rate on consumption	implicit tax rate on consumption
Capital taxation	implicit tax rate on capital and business income	implicit tax rate on capital and business income	implicit tax rate on capital and business income
Transfers			
Pensions	Earnings related and mandatory occupational pension; survivors' pension	Notional Defined Contribution public pension; survivors' pension	Notional Defined Contribution public pension
Minimum pension	*Minimum contributif*	-	*Garantipension*
Children: in-kind	calibrated subsidy on childcare (see model description)	calibrated subsidy on childcare (see model description)	calibrated subsidy on childcare (see model description)
Children: cash	Family allowance; mean-tested young children allowance; baby bonus; mean-tested education related family benefit	Mean-tested child benefit	Child-benefit (non-mean tested)
Low income support	basic 'solidarity labour income' (RSA), activity RSA	'*assegno sociale*' for persons aged 65 and over	–
Health care	average public health expenditure by age	average public health expenditure by age	average public health expenditure by age

incentive to increase labour supply for low income individuals. It is worth mentioning that for France we ignore the option that tax payers have to pay taxes on capital income through the system of personal income taxation. This is to avoid double-counting, given that our capital income tax rate is the implicit one.

For consumption taxation (τ_q) and capital income taxation (τ_r) we use implicit tax rates for 2012 (Eurostat, 2014); accordingly we set τ_q equal to 19.8%, 17.7%, 26.5% and τ_r equal to 25.7%, 26.5%, 23.2%, respectively for France, Italy and Sweden.

3.4.2 Public expenditure

In order to estimate per-capita public and total health care expenditure we use data from de la Maisonneuve and Martins (2013) to estimate the age profiles of public health expenditure. Data are provided as percentage of GDP per capita for age classes (5 years). These are converted into monetary amounts using data on GDP per capita (reference year 2013) from the national institutes of statistics of the three countries. These values are subsequently rescaled in order to match data from OECD Health Statistics 2014[10] on public health care expenditure over GDP. Since the same data are not available for total (public and private) health expenditure, we assume the same age profile for the private as for the public component. Country specific data on the percentage of private health care expenditure are taken from OECD Health Statistics 2014. The data do not allow to let per-capita health care expenditure depend on characteristics other than age.

As to daycare, the subsidy provided by the public sector (τ_d) is calibrated so that the overall amount of public expenditure over GDP matches the data. According to the OECD family database,[11] the ratio between public expenditure and GDP in 2009 was 0.4% in France, 0.2% in Italy, 0.9% in Sweden.[12]

Concerning pension systems, those of Italy and Sweden are conceptually very similar, both being '*notional defined contribution*' schemes. There is, however, a large difference concerning the size of the programme. According to Eurostat data, in 2012 total public pension expenditure as a ratio of

[10] Available at http://www.oecd.org/els/health-systems/oecd-health-statistics-2014-frequently-requested-data.htm.

[11] http://www.oecd.org/els/family/database.htm.

[12] The calibrated values of τ_d are 76%, 50% and 91%, respectively for France, Italy and Sweden. It should be remarked that the ratios of public daycare expenditures to GDP and accordingly the calibrated τ_d, reflect both the percentage of the cost which is publicly financed and the the number of households using daycare.

GDP ranges from 11.9% in Sweden to 16.6% in Italy. Of course this also means that social contribution rates are quite different. The standard total rate (including the parts due by both the employer and the employee) of pension related social contributions[13] is around 17% in Sweden and around 33% in Italy. Moreover, the Italian system is much more generous in its survivors component and social contributions in Italy fund almost exclusively pensions, whereas they also fund other programmes in France and Sweden. The characteristics of the French system are slightly different.[14] The system is based on two tiers. The first is an earnings-related public pension, where the pension benefit depends on the average of the 25 highest annual salaries indexed to prices; reductions are applied when the number of quarters of contribution is less than 166[15]. The second tier is a mandatory occupational pension scheme with a defined benefit component. In this case the benefit is based on a points system.

As to the remaining part of social expenditure, public intervention is far less extensive in Italy than in France and Sweden. This is basically true for all ages. For example, Italy is the only country with no transfer made to families with dependent children irrespective of economic conditions, and with no low income support programme accessible irrespective of age.[16] In Italy there exists also no programme to guarantee a minimum pension.

3.5 Data matching

The model seems to be able to match reasonably well some important aggregate and distributional features of the three countries. Table 2 shows the ratios between the main tax and expenditure programmes and GDP generated by the model and their empirical counterpart computed as an average

[13]Note that in all countries there exist thresholds above and below which the rate changes and may even go to zero.

[14]In France, there are non negligible differences between the rules for public and private employees. Since this is not a dimension of heterogeneity in our model, we refer to the rules that apply to the private sector.

[15]In our model one period is equal to 5 years. Therefore it is not possible to model directly this feature of the pension system. To indirectly take it into account, we calibrate the liquidation rate of pension benefits in order to match the replacement rate of the public pension system. To this end we use the projections for 2060 reported in AWG (2012).

[16]Only people older than 65 are eligible to the only programme targeted to poverty ('*assegno sociale*'). In Sweden, a similar programme of limited size exists (approximately 0.3% of GDP). However, we do not model it (see Table 1), because ownership (or not) of a house is a key determinant of the eligibility in reality, but it is not accounted for in our model.

Table 2: Main Taxes and Expenditures (% of GDP)

	France		Italy		Sweden	
	Model	Data	Model	Data	Model	Data
Consumption Tax	12.3%	11.0%	9.7%	10.6%	11.8%	12.8%
Capital Income tax	8.5%	5.5%	6.4%	7.8%	6.7%	5.1%
Personal Income Tax	5.1%	7.9%	10.2%	11.4%	13.0%	16.5%
Social security contributions	17.3%	16.6%	16.0%	13.1%	8.1%	8.5%
Pensions (old age)	12.9%	12.3%	16.7%	11.8%	7.0%	6.6%
Government Consumption	18.6%	16.2%	16.9%	20.6%	22.8%	24.2%

of the available data for years 2005-2012.[17] In the model, only public day-care expenditures and public health care expenditure as a share of GDP are calibrated to match their empirical counterpart. Hence, there is no guarantee that the other public finance programmes are able to match real data. However, we can see that the model is able to reproduce reasonably well the data for all the three countries.[18]

Finally, we look at the ability of our model economy to reproduce some distributional features of the economies of France, Italy and Sweden.[19]

Table 3 compares the value of the Gini coefficient generated by the model with the data, in terms of gross income and capital income. The model performs well in reproducing the Gini coefficient of gross income during working

[17]Data concerning revenues are taken from Eurostat (2014). As to the expenditures on pensions, we consider the ageing working group (AWG) projections for 2060 (AWG 2012).

[18]It is noteworthy that the difference between "model" and "data" for the personal income tax in France (2.8%) corresponds, with opposite sign, to the difference on capital income taxation. This is due to the fact French tax payers have an option to pay taxes on capital income through PIT, which is not accounted for in the model, for the reasons that were previously explained. We can also note that a likely explanation of the difference between "model" and "data" for social contributions and pension benefits in Italy, could be a higher degree of contribution evasion in Italy than in France and Sweden. To the best of our knowledge there are no comparable estimates of contribution evasion for the three countries. However there are data pointing to a greater importance of the whole shadow economy for Italy than for France and Sweden (Schneider *et al.*, 2010) and contribution evasion is arguably correlated with the level of the shadow economy.

[19]Data on the distribution of income are computed using EU-SILC 2008. These computations have been made on a sample which excludes households with self employed workers (in order to be consistent with the sample used for the estimation of individual labour productivity; see Section 3.3) and people younger than 65 but retired (to be consistent with the model, which does not allow retirement before that age).

Table 3: Statistics: Income (Gini)

	France		Italy		Sweden	
	Model	Data	Model	Data	Model	Data
Gross income (working age)	0.295	0.307	0.260	0.303	0.242	0.291
Gross income	0.324	0.418	0.321	0.438	0.271	0.420
Capital income	0.520	0.820	0.526	0.752	0.499	0.832

Table 4: Statistics: Earnings (Working Age)

	France		Italy		Sweden	
	Model	Data	Model	Data	Model	Data
Gross earnings: Gini	0.307	0.295	0.269	0.301	0.284	0.289
Gross earnings: P90/P10	4.762	4.069	3.596	4.249	4.096	3.995
Gross earnings: P90/P50	1.969	1.845	1.868	1.839	1.818	1.753
Gross earnings: P75/P25	2.415	1.846	2.025	1.993	2.076	1.914
Gross earnings: P10/P50	0.414	0.453	0.520	0.433	0.444	0.439

age (first row of Table 3), while it does a poorer job concerning gross income of the whole population (second row of Table 3). As can be understood looking at the last row of the table, the reason is that the distribution of capital income in the data is more concentrated than the distribution of capital income produced by the model. On the other hand, as shown in Table 4, the model generates a realistic distribution of earnings.

As stressed by De Nardi (2015), many quantitative models used for policy analysis produce a distribution of capital income that is far less concentrated than in the data. At the moment, there is no consensus on the appropriate savings mechanism that can be used to correctly reproduce the distribution of capital income and the mechanisms that have been proposed have quite different policy implications. Among the competing savings theory, an important role is played by those theories pointing to the importance of intergenerational transfers of wealth. We view the introduction of intergenerational transfers as a possible extension of the model presented in this paper.

4 Numerical experiments

The model we described in the previous sections can be used for several purposes: a wide range of policy reforms can be simulated in order to assess their impact along several dimensions. In particular, it is possible to simulate both marginal reforms, where policy parameters are changed by a small amount, and more radical reforms. In performing these policy experiments either a positive or a normative approach can be adopted.

In this Section, we focus on radical reforms with a positive approach. In particular, we examine the redistributive properties of public policies with respect to annual and life cycle inequality. With annual inequality we mean inequality in the distribution of equivalent disposable income in a given period of time.[20] We define disposable income as: gross cash income (labour and capital income) minus direct taxes (personal income tax and capital income tax) plus cash transfers (pension benefits, child benefits, income support transfers) and in-kind transfers to which we have assigned a monetary value (health care and day care transfers).[21] With life cycle inequality we mean inequality in the distribution of the present value, at age 1, of equivalent disposable incomes along the entire life cycle.[22]

To assess the redistributive effects of a public policy, we compare the steady state Gini coefficient of equivalent disposable income obtained including all programmes, with the one obtained including all programmes but the one under evaluation. When an expenditure programme is removed to assess its redistributive properties, also social contributions, if any, that are explicitly used to finance those benefits, are removed. We also study the impact of removing simultaneously all the policies currently in place in the

[20]Thus the term annual redistribution is actually used to denote redistribution in a given period of time. We use this terminology even though in our case one period does not correspond to one year for consistency with the existing literature.

[21]In the individual budget constraint (9) health care and day care programmes, that are in-kind transfers, are represented as monetary subsidies, whose value is then assigned according to a procedure specified in Section 3.4.2. We point out that, our definition of disposable income does not include the consumption tax. However we have also performed the analysis presented in this Section using an alternative definition of disposable income that takes consumption taxation into account (details are available upon requests). The general conclusions we draw from the analysis are preserved.

[22]There is a debate concerning whether life cycle equivalent income should be annualised, dividing it by the lifespan of the agent. We choose to not annualise equivalent life cycle incomes. Nelissen (1996) presents the pros and cons of annualisation and suggests that not annualising equivalent life cycle income might be a better strategy. He also reports a quantitative analysis that shows how annualising or not equivalent life cycle incomes does not have sizeable impacts on the strength of redistributive effects of public policies, with the only exception of the pension system.

three countries considered. Specifically, we study the redistributive impact of:

- personal income taxation
- capital income taxation
- consumption taxation
- subsidy to daycare
- child benefits
- health care
- low income support
- pensions
- all the above together

It should be stressed that the effect of removing a policy obviously depends on how the government budget constraint (15) is adjusted to keep it balanced. For instance, when the reform removes a tax, it makes a difference for the impact of the reform itself, if the government budget is balanced raising another tax, reducing a transfer, or reducing the government consumption $\tilde{G}$. Our approach is to use a proportional tax (subsidy) on disposable income and to set its rate so that the value of the exogenous revenue requirement, $\tilde{G}$, is the same as before removing the programme.

Table 5: Redistributive impact of public policies

	France			Italy			Sweden		
	Annual	Life cycle	Ratio	Annual	Life cycle	Ratio	Annual	Life cycle	Ratio
Personal income tax	0.01895	0.01373	0.72434	0.02866	0.02302	0.80327	0.04147	0.02351	0.56687
Capital tax	0.01362	0.00608	0.44660	0.01720	0.00871	0.50629	0.00917	0.00683	0.74487
Consumption tax	-0.00213	-0.00800	3.75107	-0.00078	-0.00028	0.35938	0.00614	0.00182	0.29587
Daycare subsidy	0.00911	-0.00830	-0.91116	0.01124	0.00122	0.10859	0.02453	0.00468	0.19085
Child benefit	0.00397	0.00334	0.84249	-0.00038	-0.00094	2.44126	0.00204	-0.00070	-0.34196
Health care expenditure	0.00729	0.00711	0.97594	0.00484	0.00251	0.51791	0.00968	0.00338	0.34947
Low income support	-0.01034	-0.00867	0.83856	-0.00216	-0.00218	1.00796	-	-	-
Pension system	0.00652	-0.00230	-0.35324	0.01156	0.00179	0.15451	0.00564	-0.00471	-0.83483
All	0.04808	0.02369	0.49275	0.05127	0.03624	0.70692	0.06705	0.03554	0.53010

Table 5 reports the difference between the steady-state Gini coefficient of disposable income in the steady state after and before the policy reform. Recalling that in these experiments reforms correspond to the removal of different programmes, a positive (negative) number means that the presence of a specific programme reduces (increases) inequality in disposable income. The same results are illustrated in Figure 2.

In general, the results shows that the differences between annual and life cycle redistribution may be large, thus suggesting that policy makers wishing to undertake reforms to reduce inequality should be aware of which dimension of inequality is targeted. This will affect the selection of the most effective policy tool.[23]

In all the three countries, the most redistributive programme (on both annual and life cycle basis) is personal income taxation. Looking at annual redistribution, we note that the ranking between the other programmes is very similar for France and Italy, whereas Sweden shows some relevant differences. For example, the very large redistributive impact of daycare subsidization in Sweden on an annual basis is noteworthy and it makes it the second most redistributive programme on an annual basis for that country. Other peculiarities of Sweden when compared with the other two countries include a comparatively large impact of consumption taxation and a small one for capital taxation and pensions. Concerning pensions, the size of the programme is likely to play a major role here: Sweden has by far the lowest pension expenditure over GDP (see Table 2) among the three countries. A final consideration is that Sweden is the only country where, on an annual basis, all programmes reduce inequality in disposable income.

Moving to the life cycle perspective, we note that the difference with respect to annual redistribution is, as expected, higher for programmes that are targeted to very specific phases of the life cycle, such as pensions, daycare and child benefits. For some of these programmes the sign of the redistributive impact may even become negative in switching from annual to life cycle redistribution. This is the case for daycare subsidization in France, child benefits in Sweden and of the pension system both in France and in Sweden. For day care and for the child subsidy it is not surprising that our measure of life cycle redistribution could be negative: indeed there is no obvious relationship between having children and life cycle disposable income. As to the impact of the pension system on life cycle inequality, it is noteworthy that France and Sweden share more similarities than Italy, notwithstanding the fact that both Italy and Sweden have a notional defined contribution system.

[23]When interpreting the results presented in Tables 5 and 6, it should be kept in mind that differences in the redistributive effects of public policies across countries are not only driven by differences in the institutional features of policies but also by differences in the wage process specified in Section 3.3. If one would capture the impact of policies only, it would be possible to assess the redistributive effects of policies using the same wage process for the three countries we consider, namely a wage process estimated pooling together the data for the three countries. Details are available upon request.

This finding suggests that the impact of a policy not only depends on the features of that specific policy, but also on the size of the programme and the interaction with other policies.

The combination of the above characteristics is such that, looking at the overall redistributive impact of public policies (last row of Table 5), redistribution on annual basis is higher in Sweden than in France and in Italy. This is essentially the results of two features that have already been mentioned: the fact that, unlike for the other two countries, all programmes reduce inequality on an annual basis, and the large, positive impact on redistribution of personal income taxation. On the other hand, if life cycle redistribution is considered, the ranking of overall redistribution changes: Italy is redistributing more than Sweden and France. Indeed the difference between annual and life cycle redistribution is higher in the last two countries than in Italy. According to Table 5 and Figure 2, a key reason for the comparatively poor redistributive performance of France, especially in the life cycle perspective, seems to be related to a less important role played by the single programme that redistributes more for all the three countries, i.e. personal income taxation. This is mainly due to the smaller tax revenue relative to GDP (see also Table 2). Moreover, transfers targeted to low income support, which are far more important than in Italy and Sweden, show a negative redistributive impact.

This result concerning low income support transfer might seem counterintuitive. To better understand this result and more generally how public programs affect redistribution, it is useful to note that the impact on disposable income of the removal of a policy can be decomposed in three components:

1. *Mechanic*: when a policy is removed it obviously no longer appears in the computation of disposable income and the other taxes and transfers should be accordingly adjusted; for instance, when the pension system is removed, pensions no longer contribute to income, the personal income tax debt of the elderly is reduced and, possibly, they become eligible to an income support transfer. This component abstracts form changes in individual choices and thus in gross incomes.

2. *Behavioural*: this takes into account that agents, facing the new individual budget constraints (those where the policy under consideration has been removed), modify their choices and thus gross incomes also change.

3. *Budget balancing*: the impacts described in the previous two points imply changes in the amounts paid and received by the public sector

through the different programmes and therefore a need to re-adjust the budget to make it balanced. As explained above, this is achieved by adjusting the tax rate of a proportional tax (subsidy) on disposable income. Such a tax (subsidy) will obviously have its own feedback impacts on individual behaviour and therefore on gross incomes: through this channel it eventually affects the distribution of disposable income.

We stress that the tax (subsidy) on disposable income used to balance the government budget has an impact on the distribution of disposable income only when changes of individual choices and thus of gross incomes are taken into account. Accordingly we can say that both the second and the third component mentioned above hinges upon changes in gross incomes while the first impact abstract from any change in gross incomes.

Table 6: Redistributive impact of public policies when changes in gross incomes are not taken into account

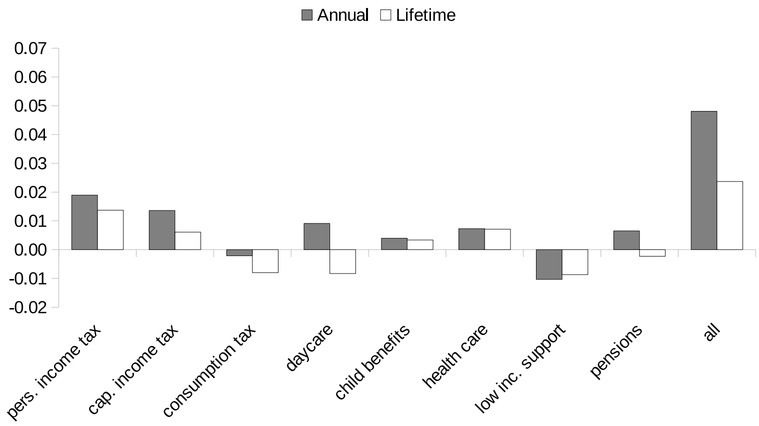
	France			Italy			Sweden		
	Annual	Life cycle	Ratio	Annual	Life cycle	Ratio	Annual	Life cycle	Ratio
Personal income tax	0.01353	0.01128	0.83400	0.02058	0.01565	0.76065	0.02388	0.00988	0.41388
Capital tax	0.01634	0.01338	0.81839	0.01605	0.00947	0.58972	0.01091	0.00737	0.67528
Consumption tax	0.00000	0.00000	-	0.00000	0.00000	-	0.00000	0.00000	-
Daycare subsidy	0.00759	-0.00438	-0.57715	0.01065	0.00333	0.31291	0.01614	0.00316	0.19579
Child benefit	0.00658	0.00380	0.57776	0.00904	0.00492	0.54371	0.00441	0.00001	0.00290
Health care expenditure	0.01576	0.01537	0.97534	0.00642	0.00810	1.26305	0.01343	0.00568	0.42323
Low income support	0.00797	0.01088	1.36533	0.00026	0.00004	0.17055	-	-	-
Pension system	0.03548	0.00910	0.25652	0.04537	0.00184	0.04055	0.02320	-0.00292	-0.12606
All	0.11882	0.06232	0.52450	0.12556	0.04209	0.33521	0.09583	0.03067	0.32010

Table 5 and Figure 2 discussed above assess the redistributive impact of public policies taking all the components mentioned above into account. We now want to capture the mechanic effect, thus neglecting any change in gross incomes. The results are presented in Table 6.[24] This analysis is useful, for instance, to understand one of the results in Table 5 that could seem surprising and counter-intuitive at first glance: the transfer targeted to poor people, that is commonly thought to redistribute from "the rich to the poor", according to Table 5 is actually redistributing in the opposite direction. Such a "puzzling" finding disappears when we only take into account the mechanical impact of the policy (see Table 6 and the equivalent Figure 3): the income support programme is, as expected, reducing disposable income inequality. Indeed the result of Table 5 depends on poor agents raising their

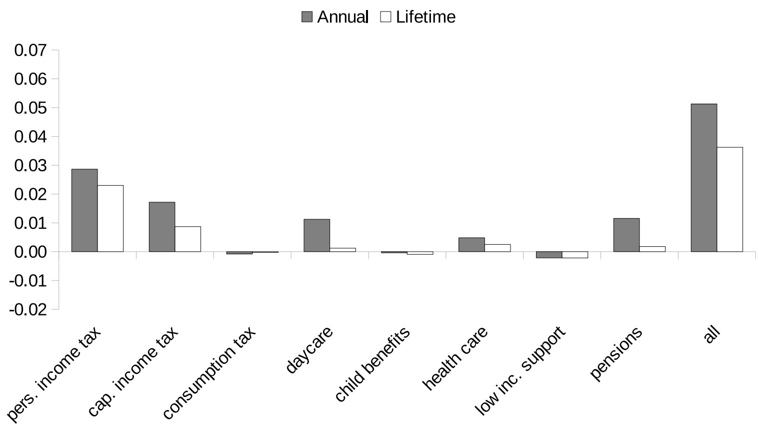
[24]Note that the redistributive effect of the consumption tax is zero in Table 6 in which changes in gross incomes are not taken into account. This is due to the fact that, as already explained, our definition of disposable income does not include the consumption tax. In other terms, the impact of the consumption tax on redistribution in Table 5 entirely relies on changes in gross incomes.

labour supply after the removal of the policy targeted to them: accordingly the distribution of gross labour income is more equal. This effect turns out to be strong enough to also make the distribution of disposable income more equal, even if the programme targeted to poor people has been removed. More generally, the comparison between Table 5 and 6, allows to understand how important changes in gross incomes might be for a proper assessment of the redistributive impact of the different public policies.

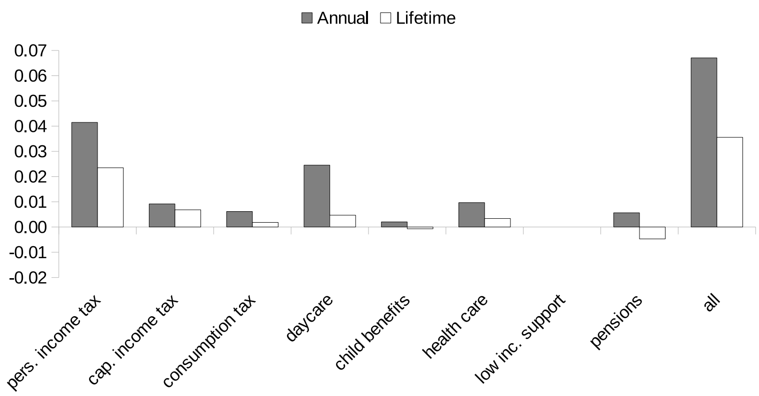
To conclude the analysis of results a caveat should be mentioned. For the sake of tractability, we take as exogenous the decisions on education, retirement and private health care expenditure, as well as whether to get married or not and whether to have children or not. These decisions are likely to be affected by specific programmes. For instance, the decision to have children or not may depend on transfers targeted to families with children and on the amount of daycare subsidies. To the extent that the impact of specific programmes on these decisions, which our model neglects, depends on the position of a household in the distribution of disposable income, our estimates of the redistributive impact of a policy will be partial.



(a) France

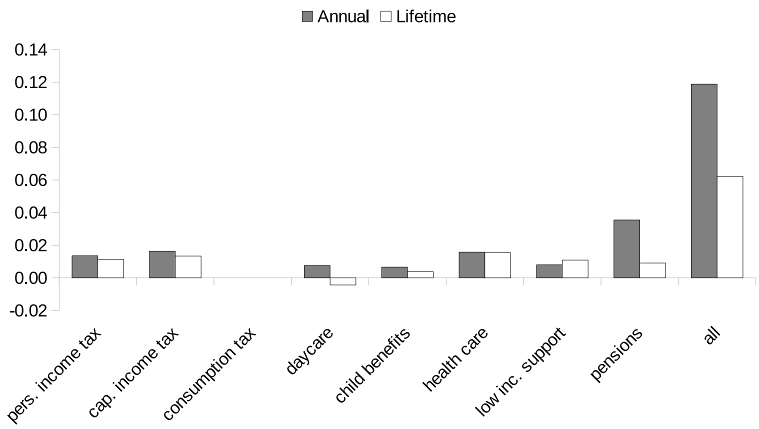


(b) Italy.

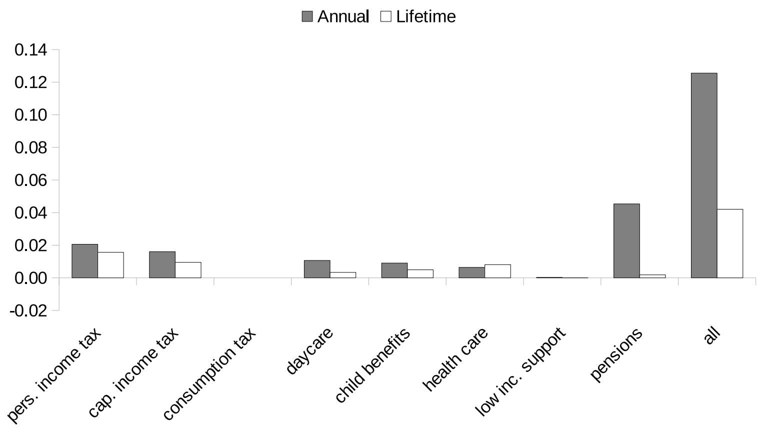


(c) Sweden.

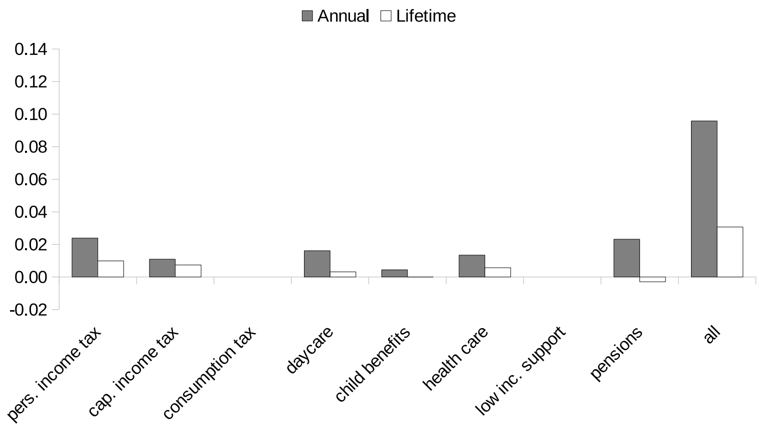
Figure 2: Redistributive impact of public policies



(a) France



(b) Italy.



(c) Sweden.

Figure 3: Redistributive impact of public policies when changes in gross incomes are not taken into account

5 Conclusion

This paper uses a large scale overlapping generation model to asses the impact of the main tax and expenditure programmes on inequality. The model is applied to three countries – France, Italy and Sweden – differing in the composition and some institutional features of social expenditure. The analysis may be valuable for policy makers in choosing the most appropriate tool to achieve specific redistributive goals.

Specifically, we compare our model economies featuring the current set of public policies implemented in the three countries, with alternative economies where some or all the public finance programmes are absent. The comparison is made in terms of both annual and life cycle redistribution. Our results confirm that not only annual and life cycle redistribution are conceptually separated, but also the level of redistribution achieved by a specific programme is largely different depending on which type of redistribution is considered. This is obviously true for programmes that are targeted to very specific phases of the life cycle, such as pensions, daycare and child benefits. However the difference between annual and life cycle redistribution can also be sizeable for other programmes (e.g. the personal income tax). Our results also suggest that behavioural responses play a major role when assessing the redistributive impact of a policy and should be carefully taken into account: for instance, policies such as the low income support subsidy that are redistributive in the absence of behavioural responses, could raise inequality when agents' reactions to incentives are considered.

In the paper, we focus on the long run effects of policy reforms, therefore abstracting from the transition dynamics, whose incorporation into the model is a natural extension. This development is particularly important, because it will allow to extend the benefits of our comprehensive analysis of public intervention in other directions. For example, this would allow for a full characterisation of who benefits and who loses from a reform, depending on the age at the time when the reform takes place, in addition to the other state variables that determine gains or losses in the current version of the model. This will be essential for an analysis of reforms from a political economy perspective. The main advantage in comparison with more standard approaches to the analysis of reforms from this perspective is the opportunity to assess political platforms rather than reforms of single programmes of taxation or expenditure, while accounting for the interaction among them.

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A List of key variables

Table A1: Definition of main notation

Symbol	Definition
J^R	Retirement age
$\overline{J}$	Maximum age
h^g	Education level for gender g
k_j	N. children at age j
$\psi_j(g)$	Probability of surviving up to age j for gender g
q	Household consumption
c_j	Individual consumption at age j
l_j	Labor supply at age j
z_j	Leisure at age j
d_j	Demand for day care services at age j
a_j	Assets at age j
$\hat{B}$	accidental bequests
r	Interest rate
τ_r	Tax rate on capital income
τ_q	Tax rate on consumption
τ_d	Subsidy to purchase of day care services
y_j^g	Income at age j for gender g
$e_{j,h,ms,\kappa,\zeta}^g$	Efficiency unit at age j for gender g and education h
ζ_j^g	Value of the idiosyncratic productivity shock
sc_j^g	Social contributions at age j for gender g
hs_j^g	Public health expenditure at age j for gender g
$\overline{hs}_j^g$	Total health expenditure at age j for gender g
sb_j^g	Accumulated pension rights at age j for gender g
p_d	Resource cost of day care services
$t_{y,j}$	Personal income tax at age j
$tr_{k,j}$	Child subsidy at age j
$tr_{y,j}$	Income support transfer at age j

B Estimation of the the wage process

Here we report the details of the estimation of the wage process characterized by equations (16), (17) and (18).

B.1 Deterministic component

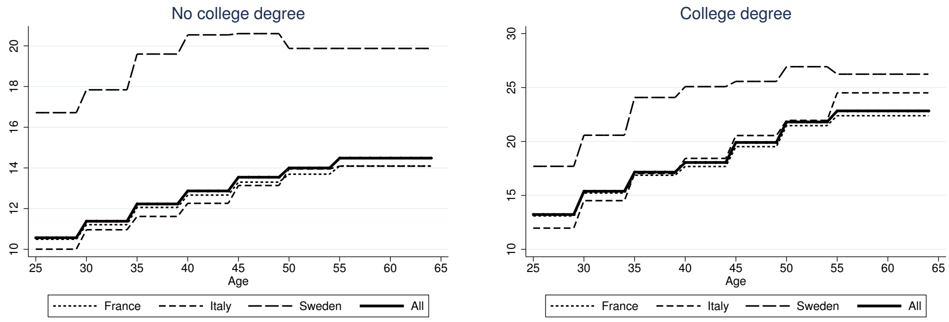

Figure 4: Average wage profile

We estimate the parameters β of the deterministic component from a random-effect GLS regression of equation (16) to deal with panel data. The random-effect model is preferred to a fixed-effect model because we need estimates of coefficients on time-invariant variables such as gender.

The specification includes a set of 5-year range age binary variables (chosen to be consistent with the model structure; the reference category is the age range 55-59), and binary variables on having children in age 0-5, in age 6-18, being male, and being married. These variables capture the heterogeneity of individual characteristics in the model. Finally, the specification controls for time effects by means of year binary variables.

Table B1 and Table B2 show the results of the estimates, separately for non-college and college graduates; similarly, Figure 4 plots the predicted age-wage profile for an individual with average characteristics, from the regression of Table B1 and Table B2. We notice that wages grow following a roughly linear trend.[25] Sweden shows generally higher wages, which also grow more quickly over the years: the average growth rate is 1.8% per year for college graduates as opposed to 1.1% per year for non-college graduates.

[25]In fact, estimates are similar if we replace the age binary variables with one age linear trend.

Table B1: Deterministic profile, no college degree

	(1) France	(2) Italy	(3) Sweden	(4) Overall
age 25-29	-0.295***	-0.342***	-0.173***	-0.315***
	(0.014)	(0.010)	(0.015)	(0.008)
age 30-34	-0.229***	-0.251***	-0.108***	-0.241***
	(0.012)	(0.009)	(0.013)	(0.007)
age 35-39	-0.156***	-0.193***	-0.014	-0.169***
	(0.011)	(0.009)	(0.013)	(0.006)
age 40-44	-0.107***	-0.139***	0.033***	-0.118***
	(0.011)	(0.008)	(0.012)	(0.006)
age 45-49	-0.058***	-0.070***	0.036***	-0.067***
	(0.010)	(0.008)	(0.012)	(0.006)
age 50-54	-0.029***	-0.010	-0.000	-0.034***
	(0.008)	(0.007)	(0.010)	(0.005)
n. children 0-5	-0.028***	0.008*	-0.112***	-0.021***
	(0.006)	(0.005)	(0.007)	(0.003)
n. children 6-24	0.001	0.007**	0.021***	0.004*
	(0.004)	(0.003)	(0.005)	(0.002)
male	0.240***	0.102***	0.176***	0.213***
	(0.007)	(0.005)	(0.007)	(0.004)
married	0.016**	0.019***	0.052***	0.013***
	(0.008)	(0.006)	(0.008)	(0.005)
year 2003			-0.312***	-0.027
			(0.017)	(0.035)
year 2004	-0.014		-0.271***	-0.005
	(0.227)		(0.011)	(0.021)
year 2005	-0.026***		-0.252***	-0.033***
	(0.006)		(0.008)	(0.004)
year 2006	-0.011**	-0.005	-0.221***	-0.017***
	(0.005)	(0.003)	(0.007)	(0.003)
year 2008	-0.009*	-0.018***	-0.040***	-0.012***
	(0.005)	(0.003)	(0.007)	(0.003)
year 2009	0.003	-0.022***	-0.124***	-0.007**
	(0.006)	(0.004)	(0.008)	(0.003)
year 2010	0.023***	-0.011***	-0.008	0.016***
	(0.006)	(0.004)	(0.009)	(0.004)
year 2011	0.026***	-0.018***	-0.127***	0.011**
	(0.009)	(0.006)	(0.012)	(0.005)
Constant	2.330***	2.436***	2.723***	2.374***
	(0.011)	(0.010)	(0.012)	(0.007)
Households	26,611	35,337	15,713	77,661
Observations	77,556	89,057	42,871	209,484

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table B2: Deterministic profile, college degree

	(1) France	(2) Italy	(3) Sweden	(4) Overall
age 25-29	-0.536***	-0.717***	-0.394***	-0.546***
	(0.016)	(0.021)	(0.020)	(0.011)
age 30-34	-0.386***	-0.524***	-0.243***	-0.395***
	(0.015)	(0.018)	(0.019)	(0.010)
age 35-39	-0.283***	-0.364***	-0.086***	-0.286***
	(0.015)	(0.018)	(0.019)	(0.010)
age 40-44	-0.236***	-0.285***	-0.045**	-0.235***
	(0.015)	(0.017)	(0.020)	(0.010)
age 45-49	-0.137***	-0.176***	-0.026	-0.137***
	(0.015)	(0.017)	(0.019)	(0.010)
age 50-54	-0.042***	-0.110***	0.026	-0.046***
	(0.014)	(0.016)	(0.017)	(0.009)
n children 0-5	0.017***	0.007	-0.154***	0.009**
	(0.006)	(0.010)	(0.009)	(0.004)
n children 6-24	0.022***	0.047***	0.028***	0.024***
	(0.005)	(0.007)	(0.007)	(0.004)
male	0.166***	0.141***	0.305***	0.171***
	(0.008)	(0.011)	(0.011)	(0.006)
married	0.072***	0.097***	0.055***	0.074***
	(0.009)	(0.013)	(0.012)	(0.006)
year 2003			-0.293***	-0.050
			(0.028)	(0.054)
year 2004			-0.292***	-0.061*
			(0.017)	(0.031)
year 2005	-0.003		-0.278***	-0.016***
	(0.008)		(0.013)	(0.006)
year 2006	-0.009	0.012	-0.230***	-0.019***
	(0.007)	(0.008)	(0.011)	(0.005)
year 2008	-0.001	-0.006	-0.067***	-0.005
	(0.007)	(0.008)	(0.011)	(0.005)
year 2009	0.000	-0.008	-0.139***	-0.007
	(0.007)	(0.009)	(0.012)	(0.005)
year 2010	0.018**	-0.070***	-0.010	0.010*
	(0.008)	(0.010)	(0.013)	(0.006)
year 2011	0.032***	-0.096***	-0.134***	0.013*
	(0.011)	(0.012)	(0.016)	(0.008)
Constant	2.805***	2.912***	2.932***	2.822***
	(0.015)	(0.019)	(0.019)	(0.010)
Households	14,148	7,703	9,846	31,697
Observations	41,634	19,053	27,068	87,755

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

B.2 Stochastic component

We exploit the fact that

$$m_k = E\left[\epsilon_{it}\epsilon_{it-k}\right] = \begin{cases} \sigma_U^2 + \sigma_H^2 & k = 0 \\ \rho^k \sigma_H^2 & k > 0 \end{cases} \tag{B.1}$$

to estimate the stochastic parameters $\theta = \left[\begin{array}{ccc} \rho & \sigma_H^2 & \sigma_U^2 \end{array}\right]'$ where $\sigma_H^2 = \frac{\sigma_V^2}{1-\rho^2}$ using a minimum distance approach from the minimisation of the moment condition[26]

$$M(\theta) = \left[\begin{array}{c} \sigma_U^2 + \sigma_H^2 \\ \rho\sigma_H^2 \\ \rho \end{array}\right] - \left[\begin{array}{c} \widehat{m}_0 \\ \widehat{m}_1 \\ \frac{\widehat{m}_2}{\widehat{m}_1} \end{array}\right] \tag{B.2}$$

where $\widehat{m}_k$ is the sample estimate of m_k based on the residuals of the regression in the first step.

The variance of the estimated vector $\widehat{\theta}$ is given by

$$V\left(\widehat{\theta}\right) = \left(D'D\right)^{-1} D'HD\left(D'D\right)^{-1} \tag{B.3}$$

where

$$D = \frac{\partial M(\theta)}{\partial \theta} = \left[\begin{array}{ccc} 0 & 1 & 1 \\ \sigma_H^2 & \rho & 0 \\ 1 & 0 & 0 \end{array}\right] \tag{B.4}$$

$$H = V\left(\sqrt{N} M(\theta)\right)^{-1} \tag{B.5}$$

with N denoting the number of observations, that we replace with the sample counterpart.

Table B3 and Table B4 report the estimates of the stochastic component parameters, separately for non-college and college graduates. The two tables also report the variance $\widetilde{\sigma}^2$ of the residuals $\widehat{\epsilon}_{it}$ in the age range 25-29; this information is used to generate an initial distribution of wages at the beginning of adult age.

The AR(1) coefficient ρ is well below 1, especially in Sweden, indicating that shocks produce effects over several years but they are not permanent. The dispersion of wages at the beginning of the career is also higher in Sweden and lower in Italy.

[26] We consider an equally-weighted estimator.

Table B3: Stochastic component parameters, no college degree

	(1)	(2)	(3)	(4)
	France	Italy	Sweden	Overall
ρ	0.861	0.848	0.714	0.864
	(0.005)	(0.014)	(0.012)	(0.003)
σ_H^2	0.250	0.200	0.192	0.243
	(0.006)	(0.018)	(0.018)	(0.004)
σ_U^2	0.123	0.058	0.099	0.112
	(0.007)	(0.022)	(0.020)	(0.005)
$\widetilde{\sigma}^2$	0.354	0.284	0.445	0.349
Households	26,611	35,337	15,713	77,661
Observations	77,556	89,057	42,871	209,484

Table B4: Stochastic component parameters, college degree

	(1)	(2)	(3)	(4)
	France	Italy	Sweden	Overall
ρ	0.893	0.838	0.697	0.875
	(0.014)	(0.030)	(0.016)	(0.010)
σ_H^2	0.170	0.194	0.240	0.177
	(0.018)	(0.046)	(0.023)	(0.013)
σ_U^2	0.149	0.071	0.165	0.144
	(0.018)	(0.058)	(0.025)	(0.013)
$\widetilde{\sigma}^2$	0.341	0.374	0.562	0.356
Households	14,148	7,703	9,846	31,697
Observations	41,634	19,053	27,068	87,755

B.3 Use of the estimates in the model

Estimates of the β parameters generate the deterministic wage component as a function of the observable characteristics; estimates of the $\theta = \begin{bmatrix} \rho & \sigma_H^2 & \sigma_U^2 \end{bmatrix}'$ parameters generate shocks for the stochastic wage component. We then discretise the continuous shock process in a four-state markov chain using the method suggested by Tauchen (1986).

A concern is that our estimates are obtained from data observed at the annual frequency, while the model is scaled on a five-year time span. This is not an issue for the deterministic component – since the specification makes use of five-year age dummies – but gives rise to potential inconsistency on the stochastic component.

To overcome this problem, the AR(1) process of the stochastic component in the code actually involves

$$\begin{aligned} \eta_{it} &= \rho\eta_{it-1} + v_{it} \\ &= \rho\left(\eta_{it-2} + v_{it-1}\right) + v_{it} \\ &= \ldots \\ &= \rho^5\eta_{it-5} + \left(v_{it} + \rho v_{it-1} + \rho^2 v_{it-2} + \rho^3 v_{it-3} + \rho^4 v_{it-4}\right) \\ &= \rho^5\eta_{it-5} + \widetilde{v}_{it}. \end{aligned} \tag{B.6}$$

In practice, this means that in the code: i) the AR(1) coefficient is not ρ, but ρ^5; ii) the variance of the persistent shock is not σ_V^2, but $\left(1 + \rho^2 + \rho^4 + \rho^6 + \rho^8\right)\sigma_V^2$.

C Recursive competitive equilibrium and computational procedure

In this Appendix we provide a more formal definition of the notion of steady state recursive competitive equilibrium introduced in Section 2.4 and some details on its numerical computation.

C.1 Recursive competitive equilibrium

Given the world interest rate $\bar{r}$, a small open economy steady state competitive equilibrium is defined as a collection of factor prices w and r, per-capita capital $\tilde{K}$ and per-capita labour in efficiency unit $\tilde{L}$, a distribution of households χ_j over the individual state vectors x_j, household decision rules, accidental bequests per working age agent and per-capita government revenues and expenditures such that:

1. $r = \bar{r}$, and the first order conditions for the firm's optimisation problem (equations (2) and (3)) hold.

2. household decision rules for household consumption $q_j = \hat{q}(x_j, j)$ and labour supply $l_j^g = \hat{l}(x_j, j)$ solve the dynamic programming problems described by equations (13) and (14). Then using the appropriate transition equations[27], endogenous state variables $a_{j+1} = \hat{a}(x_j, j)$ and $sb_j^g = \hat{sb}^g(x_j, j)$ can be computed. Using equation (7), we can also determine the demand of day care services $d_j = \hat{d}(x_j, j)$. Finally the values of taxes and transfers as a function of the state vector and of age can be calculated[28]: $p_j^g = \hat{p}(x_j, j)$, $t_{y,j} = \hat{t}_y(x_j, j)$,$sc_j^g = \hat{sc}^g(x_j, j)$, $tr_{\kappa,j} = \hat{tr}_\kappa(x_j, j)$, $tr_{y,j} = \hat{tr}_y(x_j, j)$.

3. distributions χ_j are consistent with individual behaviour, i.e. if $x_{j+1} = (h^m, h^f, \kappa_{j+1}, \zeta_{j+1}^m, \zeta_{j+1}^f, a_{j+1}, sb_{j+1}^m, sb_{j+1}^f)$ is a specific value of the state vector at age $j + 1$:

$$\chi_{j+1}(x_{j+1}) = \int_X \Pi(x_{j+1}, x_j) d\chi_j(x_j) \tag{C.1}$$

[27]The transition equation for asset is equation (9); the transition equation for pension rights depends on the legal rules of the pension systems of the three countries we consider (for a description of this rules see Section 3.4).

[28]In general transfers and taxes depend on individual choices. In turn individual choices are functions of the state vector x_j and of age j. Therefore transfers and taxes themselves can be expressed directly as functions of x_j and j.

where X is the state space and

$$\Pi(x_{j+1}, x_j) = \begin{cases} p(\zeta^m_{j+1}|\zeta^m_j)p(\zeta^f_{j+1}|\zeta^f_j) & \text{if} \quad \kappa_{j+1} = \kappa(\kappa_j, j), a_{j+1} = \hat{a}(x_j, j), \\ & sb^m_{j+1} = \hat{sb}^m(x_j, j), sb^f_{j+1} = \hat{sb}^f(x_j, j) \\ 0 & \text{otherwise} \end{cases} \tag{C.2}$$

4. market clearing conditions hold:

$$F(\tilde{K}, \tilde{L}) + (1-\delta)\tilde{K} = \tilde{G} + \sum_{j=1}^{\bar{J}} \mu_j \int_X \left(\hat{q}(x_j, j) + \hat{a}(x_j, j) + p_d \hat{d}(x_j, j)\right) \, \mathrm{d}\chi_j(x_j) \tag{C.3}$$

where :

$$L = \sum_{j=1}^{\bar{J}} \mu_j \int_X (\hat{l}^m(x_j, j)e^m_{j,h,ms,\kappa,\zeta} + \hat{l}^f(x_j, j)e^f_{j,h,ms,\kappa,\zeta}) \, \mathrm{d}\chi_j(x_j) \tag{C.4}$$

is per-capita labour in efficiency units. Moreover, μ_j is the fraction of age j individuals in the population and it obeys the following equation:

$$\mu_j = \frac{\psi^g_j}{1+n}\mu_{j-1}, \tag{C.5}$$

where μ_1 is normalised in order to make the weights μ_j sum up to 1.

5. Accidental bequests per working age agent $\hat{B}$ are given by:

$$\hat{B} = \frac{\sum_{j=1}^{\bar{J}} \mu_j \int_X \Upsilon_{j+1}(x_j)\hat{a}(x_j, j) d\chi_j(x_j)}{\sum_{j=1}^{JR} \mu_j \int_X (1 - \Upsilon_{j+1}(x_j)) \, d\chi_j(x_j)} \tag{C.6}$$

where:

$$\Upsilon_{j+1}(x_j) = \begin{cases} (1-\psi^m_{j+1})(1-\psi^f_{j+1}) & \text{if} \quad x_j = x^{co}_j \\ (1-\psi^m_{j+1}) & \text{if} \quad x_j = x^m_j \\ (1-\psi^f_{j+1}) & \text{otherwise} \end{cases} \tag{C.7}$$

is the probability that the household will no longer be in our model economy (i.e. both spouses are dead). The numerator of equation (C.6) is the amount of accidental bequests and the denominator represents the working age population.

6. per-capita government revenues and expenditures satisfy the government budget constraint (15), where:

$$\tilde{T}_y = \sum_{j=1}^{\bar{J}} \mu_j \int_X \hat{t}_y(x_j, j) \; \mathtt{d}\chi_j(x_j) \tag{C.8}$$

$$\tilde{SC} = \sum_{j=1}^{\bar{J}} \mu_j \int_X \left(\hat{sc}^m(x_j, j) + \hat{sc}^f(x_j, j)\right) \; \mathtt{d}\chi_j(x_j) \tag{C.9}$$

$$\tilde{T}_c = \sum_{j=1}^{\bar{J}} \mu_j \int_X \hat{q}(x_j, j)\tau_c \; \mathtt{d}\chi_j(x_j) \tag{C.10}$$

$$\tilde{T}_r = \sum_{j=1}^{\bar{J}} \mu_j \int_X r\hat{a}(x_j, j)\tau_r \; \mathtt{d}\chi_j(x_j) \tag{C.11}$$

$$\tilde{P} = \sum_{j=1}^{\bar{J}} \mu_j \int_X \hat{p}(x_j, j) \; \mathtt{d}\chi_j(x_j) \tag{C.12}$$

$$\tilde{H} = \sum_{j=1}^{\bar{J}} \mu_j (hs_j^m + hs_j^f) \tag{C.13}$$

$$\tilde{TR}_\kappa = \sum_{j=1}^{\bar{J}} \mu_j \int_X \hat{tr}_\kappa(x_j, j) \; \mathtt{d}\chi_j(x_j) \tag{C.14}$$

$$\tilde{TR}_d = \sum_{j=1}^{\bar{J}} \mu_j \int_X p_d \hat{d}(x_j, j)\tau_d \; \mathtt{d}\chi_j(x_j) \tag{C.15}$$

$$\tilde{TR}_y = \sum_{j=1}^{\bar{J}} \mu_j \int_X \hat{tr}_y(x_j, j) \; \mathtt{d}\chi_j(x_j) \tag{C.16}$$

C.2 Numerical procedure

The numerical procedure used to compute the steady state recursive competitive equilibrium is the following:

1. Set the capital return r equal to the world interest rate $\bar{r}$; then use equation (3) to compute the ratio K/L and equation (2) to get the wage rate per efficiency unit w.

2. Guess a value for: accidental bequests per working age agent and, if needed[29], for the proportional tax on disposable income.

3. Given the values of the wage rate, and the rate of return on capital, as well as the guesses for accidental bequests per working age agent and, if needed, the tax rate on disposable income, solve the dynamic programming problem and get decision rules and endogenous state variables.

4. Compute accidental bequests per working age agent. If needed, compute the value of the tax rate on disposable income that balances the government budget.

5. Compare the accidental bequest and, if needed, the tax rate on disposable income computed at Step 4 with their initial guesses of Step 2. If the relative difference is sufficiently small stop; otherwise go again to Step 2.

As to step 3, it is performed in the following way:

- first discretise the state space of the endogenous state variables (i.e. assets and pension rights)
- then solve the dynamic programming problem by backward induction:
  - for $j = \bar{J}$ (i.e in the last period of life), use the condition that the propensity to save out of income and labour supply are both zero and compute the value function at age $j = \bar{J}$ for the grid points of the discretised endogenous state variables;
  - for age $j < \bar{J}$ use recursively the value function at age $j + 1$ to solve the consumer optimisation problem and compute decision rules and the value function at age j. To this end, maximisation is performed using the Nelder-Meade simplex algorithm.[30] Value functions off grid points are obtained by linear interpolation of value functions at grid points.

[29]As explained in Section 2.3, when we calibrate the model, the government budget constraint is balanced in a trivial way through government expenditure $\tilde{G}$, which, by assumption, does not affect individual decision. Therefore there is no need to compute the value of $\tilde{G}$ through an iterative procedure. However, as pointed out in Section 4, when reforms are simulated the government budget constraint is balanced through a proportional tax on disposable income. Such a tax obviously affects individual incentives and therefore to find its equilibrium value we use an iterative procedure.

[30]More precisely, the version of this algorithm used is that implemented in the The NAG C Library, The Numerical Algorithms Group (NAG), Oxford, United Kingdom www.nag.com

To perform step 4 we need to compute household distributions χ_j over the individual state vectors x_j for all ages j, ensuring that equation $(C.1)$ holds. We do this by simulating the behaviour of an artificial sample of 40000 individuals. To this end the following procedure is implemented:

- determine (as explained in Section 3) for $j = 1$ the distribution of exogenous state variables (namely gender, education levels, marital status, number of children and productivity shocks) over the artificial sample.
- compute the value of exogenous state variables for $j > 1$:
  - keep education constant over time
  - use transition equation (8) to compute the number of children
  - simulate the markov process of the productivity shocks
  - simulate mortality shocks
  - keep marital status constant, unless one of the two spouses dies
- use the values of exogenous state variables, the value of accidental bequests set in step 2 and decision rules to iterate forward and get for each individual the entire path of his/her control variables and endogenous state variables. Variables off grid points are obtained by linear interpolation of variables at grid points.

Thus we end up with an artificial sample of households whose choices are known along all the life cycle. In a steady state, age j agents at time t are alike age j agents at time $t + j$. Therefore life cycle patterns of our artificial sample can also be used to infer cross sectional information, i.e. information concerning agents of different ages in a given period of time. To build the distribution of all the relevant variables in a given period of time, we simply need to take into account that the size of the generations is different because there is a positive growth rate n of the measure of the newborn cohorts. Using this distribution we can finally compute all the aggregate and per-capita variables.

The simulated behaviour of this artificial sample (for the equilibrium value of accidental bequest and, if any, for the equilibrium value of the tax rate on disposable income) is also used to compute all the distributional features of our model economy used in Section 4, namely the distribution of equivalent disposable income in a given period of time and the distribution of the present value, at age 1, of equivalent disposable incomes along the entire life cycle.