A category theory approach to completion-based theorem proving strategies *
(Abstract)

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This work is concerned with the application of category theory to formalize some concepts in automated theorem proving. In his paper "General Logics" [12], J.Meseguer presents an axiomatization of logic in category theory, which focuses on those elements of logic involved in automated theorem proving and logic programming.

This axiomatization provides both the model theoretic view and the proof theoretic view of logic. The model theoretic view is given by the notions of *institution* and *satisfaction relation* $\models$, continuing previous work by J.A.Goguen and R.Burstall [4, 5]. The proof theoretic view is given basically by the notions of *entailment relation* $\vdash$ and *proof calculus*, where entailment states the existence of a proof, $\Gamma \vdash \varphi$ meaning that φ is derivable from Γ, and a proof calculus defines the structure of proofs.

For instance, a proof calculus for equational logic given in [12] associates to an equational theory E on signature Σ the category whose objects are Σ-terms and whose morphisms are equational proofs, that is chains of elementary steps of equational replacement by equations in E. The properties of equality as a congruence relation become properties of morphisms in this category. The categorical approach to equational logic was started by F.W.Lawvere in [9] and is described in [11] and [13]. Brief explanations of basic ideas can be found in [7] and [12].

Our research aims at exploring how to extend the categorical approach in [12] from a description of logic for computer science to a description of theorem proving methods. More precisely, we are interested in *Knuth-Bendix type completion procedures* for theorem proving [8, 6, 1, 2].

A *completion procedure* is given by a set of *inference rules* and a *search plan*. The inference rules determine what consequences can be derived from a given set of sentences. The search plan decides which inference rule to apply to which sentences at each step of the computation. An inference rule is an *expansion* inference rule if it expands a given set of sentences by deriving new sentences, it is a *contraction* inference rule if it contracts a given set of sentences by deleting some

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sentences and possibly replacing them by others. The properties that the inference rules and the search plan of a completion procedure are required to satisfy are given in [2, 3].

Clearly, completeness of a theorem proving strategy can be guaranteed by the expansion inference rules only. However, the contraction inference rules and the search plan determine the efficiency of a theorem proving method and therefore its practical applicability.

The search plan is actually what turns a set of inference rules into a procedure. However, most theorem proving strategies are simply presented by giving a set of inference rules and the task of designing a suitable search plan is left to the implementation phase. This is not satisfactory, since the actual performance of the prover depends heavily on the search plan.

No notion of search plan appears in the categorical approach in [12]. Also, the notion of proof calculus seems to assume expansion inference rules only. In other words, the two most interesting aspects of completion-based theorem proving are not yet covered in the categorical framework.

We plan to investigate how to include contraction inference rules and search plans in the categorical approach. We expect that the application of category theory leads to a better understanding of these problems and especially to a systematic study of the notion of search plan, with formal definitions of the requirements for a search plan for theorem proving.

References

[1] L.Bachmair, N.Dershowitz, J.Hsiang, Orderings for Equational Proofs, in *Proceedings of the First Annual IEEE Symposium on Logic in Computer Science*, 346–357, Cambridge, Massachussets, June 1986.

[2] M.P.Bonacina, J.Hsiang, Completion Procedures as Semidecision Procedures, to appear in M.Okada, S.Kaplan (eds.), *Proceedings of the Second International Workshop on Conditional and Typed Rewriting Systems*, Montreal, Canada, June 1990.

[3] M.P.Bonacina and J.Hsiang, "On fairness of completion-based theorem proving strategies", in R.V.Book (ed.), *Proceedings of the Fourth International Conference on Rewriting Techniques and Applications*, Como, Italy, April 1991.

[4] J.A.Goguen, R.Burstall, Introducing institutions, in E.Clarke, D.Kozen (eds.) *Logic of Programs*, Lecture Notes in Computer Science, Vol. 164, 221–256, Springer Verlag, 1984.

[5] J.A.Goguen, R.Burstall, Institutions: Abstract Model Theory for Computer Science, Technical Report CSLI-85-30, Center for the Study of Language and Information, Stanford University, 1985.

[6] G.Huet, A Complete Proof of Correctness of the Knuth-Bendix Completion Algorithm, *Journal of Computer and System Sciences*, Vol. 23, 11–21, 1981.

[7] G.Huet, Deduction and Computation, INRIA, Rapport de Recherche No. 513, April 1986.

[8] D.E.Knuth, P.Bendix, Simple Word Problems in Universal Algebras, in J.Leech (ed.), *Proceedings of the Conference on Computational Problems in Abstract Algebras*, Oxford, England, 1967, Pergamon Press, Oxford, 263–298, 1970.

[9] F.W.Lawvere, Functorial semantics of algebraic theories, in *Proceedings of the National Academy of Sciences*, 50, 1963, Summary of Ph.D. Thesis, Columbia University.

[10] S.MacLane, *Categories for the working mathematician*, Springer, 1971.

[11] E.G.Manes, *Algebraic Theories*, Springer Verlag 1976.

[12] J. Meseguer, General Logics, in H.-D. Ebbinghaus et al. (eds.) *Proceedings Logic Colloquium 1987*, North Holland, 1989.

[13] M.E.Szabo, *Algebra of Proofs*, North Holland 1978.