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Jean-Paul Chavas

Eleonora Matteazzi

Martina Menon

Federico Perali

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Bargaining in the Family

JEAN-PAUL CHAVAS - jchavas@wisc.edu
Department of Agricultural and Applied Economics, University of Wisconsin

ELEONORA MATTEAZZI - eleonora.matteazzi@univr.it
Department of Economics, University of Verona

MARTINA MENON - martina.menon@univr.it
Department of Economics, University of Verona

FEDERICO PERALI - federico.perali@univr.it
Department of Economics, University of Verona

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Abstract: This paper has the ambition to describe how families bargain to reach an agreement recognizing that the negotiation process is costly. Costs may emerge as the result of cognitive and non-cognitive limitations, information asymmetries and efforts to acquire them, time constraints and uncertain decision environments bounding the rationality of individuals. Family members may therefore act as "satisficers" (Simon, 1979) accepting choices that may not be Pareto efficient, but that, more realistically, satisfy a sufficient level of satisfaction. By introducing the cost of negotiating, we span the whole negotiation space and reproduce in slow motion the progress of the bargaining efforts involving simple iterations reflecting the steps taken during real bargaining sessions. Our novel theory results show that family members can reach inefficient bargaining agreements on the contract curve at relatively low cost or may settle the dispute on an efficient point on the Pareto frontier. The empirical analysis of the bargaining household model shows that the large majority of the family agreements is inefficient, lending empirical support to Simon's hypothesis that rational individuals can be sufficiently satisfied also at inefficient but less conflictual positions on the contract curve. We also investigate the factors affecting agreeableness, the difficulty in reaching an agreement, and how the cost of inefficiency varies across households and affects intra-household inequalities.

Keywords: Bargaining agreements, household efficiency, intrahousehold welfare, threat strategies.
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Bargaining in the Family

1. Introduction

The family is a basic institution in human society. Yet, it has evolved over time and across cultures and countries (Kertzer, 1991; Jonas, 2007). The functioning of the family involves negotiations among household members about intrahousehold resource allocation. The negotiations involve decisions about individual time allocation, the management of household resources, and the distribution of income and purchasing power within the family. However, the negotiations are often complex and deal with conflicts of interest among family members. How these decisions are made has generated much interest (Becker, 1960, 1981). In general, the process of making intrahousehold decisions depends on the bargaining power of each family member.

Most economic models of the family assume a marriage environment favoring the achievement of efficient outcomes of a cooperative bargaining process (Chiappori, 1988; Chiappori, Donni and Komunjer, 2012; Chiappori and Meghir, 2014). In this context, efficient allocations of household resources arise from repeated iterations of the search for an agreement at no interaction costs. This study builds on this literature and extends it by considering explicitly the nature of the intrahousehold negotiation process. Difficulties in negotiations can emerge as the result of cognitive limitations bounding the rationality of individuals, bargaining cost, information asymmetries and time constraints (Simon, 1979; Rubinstein, 1998; Oprea, 2020). Family members may act as "satisficers" (Simon, 1979) accepting choices that may not be Pareto efficient, but that, more realistically, satisfy a sufficient level of "happiness". As Simon notes (1979: 350), below the optimizing surface there is a rich set of properties of the real world where "decision makers can satisfice either by finding optimum solutions for a simplified world, or by finding satisfactory solutions for a more realistic world." In a negotiation context, an agreement may be considered sufficiently satisfactory as compared to an efficient solution. Cognitive and non-cognitive limitations can lead to inefficient outcomes that become parts of observed household behavior.

Family negotiations can lead to conflicts that makes household decisions difficult (e.g., the decision to move to another city because of a better job opportunity for a partner, or the decision to opt for a part-time or a full-time work schedule, or to have a child). Negotiations among family members become part of the household decision making process when there are disagreements about the allocation and distribution of resources within the family. Conflict resolution leading to an agreement is the situation where a common objective is met, and the conflict is quelled. The degree of cooperativeness of a person makes the personality profile more or less accommodating, collaborative or prone to compromising thus affecting the perceived costs associated with a negotiation process, and, ultimately, the likelihood to reach an agreement or an efficient outcome.

To the extent that negotiations among family members play a role in household decision making, it is of interest to examine the process being used describing bargaining efforts. Important questions are: How do family members settle a dispute? Under what conditions would an allocation be an agreement seen as satisfactory to all parties? Importantly, these questions can be addressed without insisting on achieving efficiency. Indeed, in the presence of (implicit or explicit) bargaining cost, it seems reasonable to allow for situations where household bargaining outcomes do not always achieve efficiency. Yet, if an agreement is reached that does not achieve efficiency, it is also important to consider scenarios allowing the bargaining process to continue up to a point where an efficient allocation could possibly be attained.

This paper is about bargaining in the family. Previous literature on this topic has typically been based on the Nash bargaining model presented under the expected utility model (Nash, 1950, 1953). In such models, threat points (representing allocations attained when bargaining fails) underline the bargaining power of each family member and affect the intrahousehold distribution of income and welfare (McElroy, 1990; Lundberg and Pollak, 1993, 1996, 2003; Donni and Ponthieux, 2011; Chiappori, Donni and Komunjer, 2012). Our purpose is to provide a description of the intrahousehold decision process closer to reality. We present a theoretical analysis that extends the original Nash-Harsanyi cardinal representation of bargaining (Nash, 1950; Harsanyi, 1950, 1963, 1977) in two important directions: 1/ it considers bargaining agreements that are not necessarily efficient, and 2/ it applies to ordinal preferences[1] and general household technology.

Our bargaining process is rooted on a measure of the welfare loss perceived by each member facing the prospect of facing a bargaining failure. Building on evolutionary games (e.g., Sandholm, 2010), we propose iterative schemes representing the bargaining process. As in Zeuthen (1930) and Harsanyi (1977), each iteration identifies who is making a concession depending on the perceived adverse effects of facing "threat points" corresponding to bargaining failure. We propose to measure these perceived effects by the individuals' willingness-to-pay to avoid bargaining failure. Using the threat of bargaining failure to identify who is more willing to concede at each iteration, we show how the bargaining process can converge to an agreement. Our analysis allows for bargaining agreements that are not necessarily efficient. In this context, we evaluate the linkages between bargaining situations and income distribution. We show that intrahousehold income distribution depends on the threat points. This reflects the positive aspect of our analysis: income distribution can vary from egalitarian (when all individuals have the same purchasing power at the threat points) to very unequal (when purchasing power varies a lot across household

[1] The Nash-Harsanyi bargaining model relies on cardinal preferences under the expected utility model (Nash, 1950, 1953). Extensions of this model under non-expected utility models have been explored by Rubinstein et al. (1992) and Hanany and Safra (2000). As discussed in Sections 2 and 3, our analysis is more general: it relies on willingness-to-pay measures and applies under general ordinal preferences.

members under bargaining failure). In turn, this stresses the importance of threat points (as influenced by the socio-economic and institutional context) as they affect both bargaining power and the distribution of income. Our analysis allows for inefficient agreements, in which case we identify the associated welfare loss reflecting underlying bargaining cost under bounded rationality (e.g., Oprea, 2020). We also explore situations where the bargaining process leads to an efficient allocation. In this case, the bargaining outcome identifies a unique point on the Pareto utility frontier, illustrating how the bargaining process addresses both efficiency and distribution issues. In addition, we show that bargaining outcomes (whether they are efficient or not) can be represented by a simple maximization problem based on a "generalized Nash-product" applied to willingness-to-pay measures under ordinal preferences.

We illustrate the usefulness of our approach in an empirical application to a sample of Italian households. We explore the empirical content of the bargaining model using the estimates of a family enterprise model (Matteazzi, Menon and Perali, 2017) to obtain the rule governing the intrahousehold resource allocation, the threat points, and the initial bargaining position of each disputant. The empirical analysis of the household bargaining model shows that the large majority of the family agreements is inefficient, lending empirical support to Simon's hypothesis that rational individuals can be sufficiently satisfied also at inefficient but less conflictual positions. We also investigate the factors affecting agreeableness, the difficulty in reaching an agreement, and costs yielded by inefficiency. To the best of our knowledge both the theoretical and empirical results are new contributions to the literature.

2. Household Efficiency

This section establishes the notation. Building on previous literature, it presents a characterization of household efficiency that sets the stage for the analysis given in the rest of the paper. Consider a household composed of n members, with $n \geq 2$. Household members make decisions related to both consumption and production activities. They choose consumption goods, including public consumption market goods $x_0 = (x_{01}, x_{02}, \dots)$ (i.e., purchased goods that affect the welfare of multiple household members, such as having a roof in the house, or someone smoking in the house), private consumption market goods $x_i = (x_{i1}, x_{i2}, \dots), i \in N \equiv \{1, \dots, n\}$ (e.g., clothing and food),[2] and non-market goods $z = (z_1, z_2, \dots)$ (e.g., parental care). The inclusion of public goods x_0 and non-market goods z captures situations where there are externalities across household members (e.g., Ellickson, 2008). The household also chooses production goods, including market goods y used to generate household income (e.g., wage labor or agricultural inputs and outputs in commercial farms). The netput notation is used for the market goods $y = (y_1, y_2, \dots)$ where $y_i \leq 0$ when the i-th market good is an input and $y_j \geq 0$ when the j-th market good is an output.

Preferences of the i-th member of the household are represented by the ordinal utility function $u_i(x_0, x_i, z), i \in N$. Let F be the feasible set for household production, where $(y, z) \in F$ means that (y, z) is feasible. Let X_0 be the feasible set of x_0 and X_i be the feasible set of $x_i, i \in N$, and let $p_0 = (p_{01}, p_{02}, \dots) > 0$ be the prices of x_0, $p_i = (p_{i1}, p_{i2}, \dots) > 0$ be the prices of $x_i, i \in N$, and $q = (q_1, q_2, \dots) > 0$ be the prices of y.

The household receives income $[q\ y] = \sum_j q_j y_j$. Household income can come from three sources: from wage labor; from profit, generated by household production activities involving

[2] Clothing and food are private goods whose consumption is assignable to specific household members. Note that while food items are consumed individually, food purchases are often recorded only at the household level.

marketed goods (e.g., farm profit in farm household); and from non-labor sources (e.g., interests, dividends, and rents). The allocation of time by each household member involves three components: wage labor, treated as a market good and included in the vector y, leisure time included in the vector of private market goods, and work spent on household production activities included in the vector z, where the sum of these three-time components is equal to the total amount of time available for each individual. Note that these time allocation decisions allow for specialization of tasks among individuals covering both intrahousehold work and labor market participation (e.g., Becker, 1960, 1981; Cigno, 1991).

Throughout the paper, we make the following assumptions:

Assumption A1: The utility function of individual *i*, $u_i(x_0, x_i, z)$, is continuous and quasi-concave in $(x_0, x_i, z), i \in N$.

Assumption A2: For each $i \in N$, the utility function $u_i(x_0, x_i, z)$ is non-satiated in x_i, meaning that for any $x_i \in X_i$ and any (x_0, z), there is a $x_i' \in X_i$ satisfying $x_i' \geq x_i$, $x_i' \neq x_i$ and $u_i(x_0, x_i', z) > u_i(x_0, x_i, z)$.

Assumption A3: The sets $X_0, X_1, \dots, X_n$ are closed and convex, each set has a lower bound, and the set F is closed.

In this paper, we examine the economics of household decisions about the allocation (x, y, z), where $x \equiv (x_0, x_1, \dots, x_n) \in X \equiv X_0 \times X_1 \times \dots \times X_n$ and $(y, z) \in F$. Assumptions A1 and A2 are standard (e.g., Deaton and Muellbauer, 1980). For each individual, the utility function $u_i(x_0, x_i, z)$ is ordinal and defined up to a monotonic increasing transformation. Throughout the paper, it means that utilitarian considerations and interpersonal comparison of utilities are superfluous. Note the generality of the approach. First, A3 does not assume that the set F is necessarily convex. This allows for a general household production technology because we do not assume that the technology exhibits constant returns to scale. Second, the allocation (x, y, z) can occur over time, allowing for an intertemporal analysis. In this case, the vector y would include saving/investment (or borrowing if negative) that transfers income across periods and the utility functions would reflect time preferences. Third, the analysis can apply under uncertainty. Representing the uncertainty by states of nature, the elements of the vector (x, y, z) would then be defined as being state-dependent and the utility functions would represent risk preferences (Debreu, 1959; Radner, 1968).

Our analysis will rely in part on the classical concept of Pareto efficiency. We define a household allocation to be efficient if there does not exist another feasible allocation that can make one household member better off without making anyone else worse off. Feasibility includes the household budget constraint

$$p_0 x_0 + \sum_{i \in N} p_i \, x_i \leq q \, y, \tag{1}$$

stating that household expenditures $[p_0 x_0 + \sum_{i \in N} p_i \, x_i]$ cannot exceed household income $[q \, y]$.

Letting $p = (p_0, p_1, \ldots, p_n)$, consider the following maximization problem

$$W(p, q, U) = Max_{x,y,z}\{q \, y - p_0 x_0 - \sum_{i \in N} p_i x_i : u_i(x_0, x_i, z) \geq U_i, i \in N;$$

$$x \in X, (y, z) \in F\}, \tag{2}$$

where U_i is a reference utility level for the i-th household member, $i \in N$, and $U = (U_1, \ldots, U_n)$. Denote the solution to (2) by $x^*(p, q, U), y^*(p, q, U)$ and $z^*(p, q, U)$. The allocation evaluated in equation (2) is conditional on $U = (U_1, \ldots, U_n)$.

We make the additional assumption:

Assumption A4: $U \in \mathcal{U}$ where $\mathcal{U} \equiv \{U : W(p, q, U) \geq 0; [x_i : x_i < x_i^*(p, q, U)] \cap X_i \neq \emptyset, i \in N\}$.

Assumption A4 allows for a wide income distribution, including poverty (where low purchasing power is associated with low consumption of private goods), but it rules out situations of complete destitution where $x_i^*(p, q, U)$ would be located on the lower bound of X_i. This ensures that each individual has enough purchasing power to allow a reduction in consumption of private goods, thus guaranteeing that individual willingness-to-pay remains a valid welfare measure. Assuming that the set $\mathcal{U}$ is not empty, consider the following choice for U:

$$U^* \in \{U: W(p, q, U) = 0, U \in \mathcal{U}\}. \qquad (3)$$

A key representation of household efficiency is presented next. (See the proof in Appendix A).

Proposition 1. Under assumptions A1-A4, a household allocation (x^*, y^*, z^*) is Pareto efficient if and only if it satisfies equations (2) and (3).

For a given household welfare distribution $U = (U_1, \dots, U_n)$, equation (2) associates an efficient allocation with the maximization of household net income $[q\ y - p_0 x_0 - \sum_{i \in N} p_i\, x_i]$. It defines $W(p, q, U)$ as the maximized household net income conditional on $U = (U_1, \dots, U_n)$. Equation (3) states that the distribution of welfare $U = (U_1, \dots, U_n)$ is such that the maximized net income $W(p, q, U)$ is entirely redistributed among the n household members. To see that (3) is an integral part of an efficient allocation, note that having $[q\ y] < [p_0 x_0 + \sum_{i \in N} p_i\, x_i]$ would not be feasible (as it would not satisfy the budget constraint (1)) while having $[q\ y] > [p_0 x_0 + \sum_{i \in N} p_i\, x_i]$ would be inefficient (under assumption A2), implying the household budget constraint (1) must be binding under efficiency.

A useful interpretation of equation (3) is that choosing $U = (U_1, \dots, U_n)$ such that $W(p, q, U) = 0$ defines the household utility frontier. For example, when $n = 2$, solving $W(p, q, U_1, U_2) = 0$ for U_2 gives the household utility frontier $U_2(p, q, U_1)$, which traces out the utilities $\{U_1\ , U_2(p, q, U_1)\}$ that can be attained under an efficient household allocation. Thus, when

$n \geq 2$, equations (2) and (3) identify the set of efficient allocations along the household utility frontier. It makes it clear that efficient allocations are not unique (as they change depending on the values taken by U_1 and $U_2(p, q, U_1)$). Which point on the utility frontier is actually obtained depends on the distribution of welfare within the household. In turn, since x^*, y^* and z^* depend on U^*, the welfare levels obtained by each household member will affect behavior. In that sense, both efficiency and distribution issues are relevant in the analysis of household behavior.

Next, we establish linkages between Proposition 1 and previous literature on household economics. Proposition 1 can alternatively be written as follows.

Corollary 1. Under assumptions A1-A4, a household allocation (x^*, y^*, z^*) is Pareto efficient if and only if it satisfies equation (3), along with

$$e_i(p_i, x_0, z, U_i) = \min_{x_i}\{p_i x_i : u_i(x_0, x_i, z) \geq U_i, x_i \in X_i\}, i \in N, \tag{4}$$

$$E(p, z, U) = \min_{x_0}\{p_0 x_0 + \textstyle\sum_{i \in N} e_i(p_i, x_0, z, U_i) : x_o \in X_0\}, \tag{5}$$

$$\pi(q, z) = \max_y\{q\, y : (y, z) \in F\}, \tag{6}$$

and

$$W(p, q, U) = \max_z\{\pi(q, z) - E(p, z, U)\}. \tag{7}$$

Equations (4)-(7) follow directly from a stage-wise decomposition of the optimization problem in (2). Conditional on (x_0, z), equation (4) is a standard expenditure minimization problem. For the i-th member of the household, it generates the private expenditure function $e_i(p_i, x_0, z, U_i)$. Denoting by w_i the part of household income received by the i-th individual, a dual representation of (4) is given by the standard utility maximization problem $V_i(p_i, x_0, z, w_i) = \max_{x_i}\{u_i(x_0, x_i, z) : p_i x_i \leq w_i, x_i \in X_i\}$, where $V_i(p_i, x_0, z, w_i)$ is an indirect utility function satisfying $w_i = e_i(p_i, x_0, z, V_i(p_i, x_0, z, w_i)), i \in N$ (Deaton and Muellbauer, 1980, p. 38). While z are non-market goods, they have shadow prices. Under differentiability, $\partial e_i / \partial z$ measures the shadow prices of z for each $i \in N$. Equation (5) is an optimization problem involving the choice of x_0 conditional on z (Pollak and Wachter, 1975). It generates the household expenditure function $E(p, z, U)$. Equation (6) is a standard profit maximization problem, yielding the profit or household income function $\pi(q, z)$. Under differentiability, $\partial\pi/\partial z$ measures the shadow cost of the non-market goods z. Finally, equation (7) states that the efficient choice of non-market goods z is consistent with the maximization of net aggregate household income $[\pi(q, z) - E(p, z, U)]$. Note that the choices for (x_0, y, z) given in (5)-(7) include the case of efficient management of externalities across individuals within the household (e.g., Coase, 1960; Ellickson, 2008; Chavas, 2015).

From equation (7), feasibility means that the household budget constraint can be written as $E(p, z, U) \leq \pi(q, z)$ or $W(p, q, U) \geq 0$. From equation (3), Pareto efficiency means that $E(p, z, U) = \pi(q, z)$, implying that the household budget constraint is necessarily binding. If $W(p, q, U) \geq 0$ under feasibility and $W(p, q, U) = 0$ under Pareto efficiency, it follows that $W(p, q, U) > 0$ is necessarily associated with inefficiency. The inefficiency can come from three sources: 1/ the household does not maximize profit, thus lowering its purchasing power; 2/ the consumers do not minimize their expenditures; 3/ some of the income is not used, with adverse effects on expenditures and household welfare.

The efficiency considerations presented in Proposition 1 and Corollary 1 are commonly assumed in the analysis of economic behavior (e.g., Deaton and Muellbauer, 1980). However, empirical research has presented evidence of situations where families are unable to reach efficient resource allocations (Apps and Rees, 2010). For example, using data from Burkina Faso, Udry (1996) finds that among plots planted with the same crop in the same year within a given household, those controlled by women produce lower yields than the men's plots. Also, Duflo and

Udry (2004) show that expenditure patterns of households in Côte d'Ivoire are inconsistent with a Pareto efficient allocation of household resources. Next, we investigate how the intra-household bargaining process can affect both efficiency (or inefficiency) and distribution issues in household allocations.

3. Intra-household Bargaining

How to choose a point U^* on the Pareto utility frontier, as given in equation (3)? Many efficient points exist along the Pareto utility frontier. Given $w_i = e_i(p_i, x_0, z, V_i(p_i, x_0, z, w_i))$, moving along the Pareto utility frontier is associated with changing the distributions of income $(w_1, \dots, w_n)$ within the household. In egalitarian households, we would have $w_1 = w_2 = \cdots = w_n$. However, efficiency can also be associated with unequal distribution of income. Under extreme inequality, it is possible to have an efficient allocation where one individual receives most of the household income. The sharing of income within the household can be measured by the i-th individual sharing rule $s_i = w_i / \sum_{i \in N} w_i$, $i \in N$. A very unequal income distribution would occur when s_i is large and close to 1 for one individual. Although this would likely imply a very unequal intra-household distribution of welfare, this situation can be consistent with Pareto efficiency. For example, it could happen when an individual acquires enough power within the household to shift the welfare distribution in her/his favor. This indicates a need to go beyond Pareto efficiency and to examine the exercise of power and its implications for income and welfare distribution within the household.

This section investigates intrahousehold distribution as the outcome of a bargaining process among the n members of the household, which could lead to either efficient or inefficient outcomes. The analysis is inspired by seminal work on bargaining by Nash (1950) and Harsanyi

(1950, 1963, 1977). As noted in previous literature, the Nash-Harsanyi approach is limiting as it was presented under the expected utility model where utilities are defined up to a positive linear transformation. We extend the Nash-Harsanyi approach by presenting our analysis under ordinal preferences (where each utility $u_i(x_0, x_i, z)$ is defined up to a positive monotonic transformation).

The analysis starts with threat points. Threat points are defined as feasible allocations representing outcomes that would arise in case of bargaining failure among the *n* household members. We denote the threat allocation for the i-th individual by $(x_{0i}^t, x_i^t, y_i^t, z_i^t) \in X_0 \times X_i \times F, i \in N$. Denote by $U_i^t = u_i(x_{0i}^t, x_i^t, z_i^t)$ the utility obtained by individual $i \in N$ if no agreement is reached,[3] with threat utilities given by $U^t = (U_1^t, \ldots, U_n^t)$. In this section, we start with the simple situations where the threat points are treated as given. How bargaining can fail and how the threat points get determined will be discussed in Section 6.

Using equation (4), $e_i(p_i, x_0, z, U_i)$ measures the private expenditure (or purchasing power) of the i-th individual conditional on (x_0, z, U_i). Let

$$\Delta_i(p_i, x_o, x_i, z, U_i^t) = e_i(p_i, x_0, z, u_i(x_0, x_i, z)) - e_i(p_i, x_0, z, U_i^t), i \in N. \quad (8)$$

The term $\Delta_i(p_i, x_o, x_i, z, U_i^t)$ in (8) is a willingness-to-pay measure for the i-th individual facing a choice between (x_i, x_0, z) and the threat point $U_i^t = u_i(x_0^t, x_i^t, z^t), i \in N$. For a given (x_i, x_0, z), it is a money-metric measure of the welfare loss perceived by the i-th individual facing the prospect of reaching the breakdown position: the greater $\Delta_i(p_i, x_o, x_i, z, U_i^t)$, the greater the reduction in her/his purchasing power, and the greater the associated welfare loss. Given $U_i^t =$

[3] Some household bargaining models take the threat points to be the utility that each spouse would achieve if the couple divorced (e.g., Manser and Brown 1980; McElroy and Honey, 1981). Other authors identify the threat points with the solution of noncooperative Nash equilibrium (e.g., Lundberg and Pollak, 1993, 1996; Ulph, 1988; Woolley, 1988). See Section 6 for a discussion of these issues.

$u_i(x_{0i}^t, x_i^t, z_i^t)$, note that the individual willingness-to-pay measure $\Delta_i(p_i, x_o, x_i, z, U_i^t)$ is invariant to a positive monotonic transformation of u_i.

To the extent that the threat outcomes are feasible and always available, no individual will not be willing to accept any payoff less favorable than his/her threat point. Thus, we restrict our bargaining analysis to the allocations $(x, y, z) \in X \times F$ satisfying $u_i(x_0, x_i, z) \geq U_i^t, i \in N$. It follows from (8) that $\Delta_i(p_i, x_0, x_i, z, U_i^t) \geq 0, i \in N$, stating that the willingness-to-pay to avoid the threat points is non-negative. It also implies that $W(p, q, U^t) \geq 0$, with $W(p, q, U^t)$ providing a measure of the aggregate welfare loss associated with bargaining failure.

We propose to use $\Delta_i(x_i, x_0, z)$ to evaluate the bargaining position of the i-th individual, $i \in N$.

Definition 1: The i-th individual facing (x_0, x_i, z) is less willing to accept a bargaining failure than individual i' if

$$\Delta_i(p_i, x_0, x_i, z, U_i^t) > \Delta_{i'}(p_{i'}, x_0, x_{i'}, z, U_{i'}^t), i \in N, i' \in N - i. \quad (9)$$

Definition 1 states that a move from an allocation $(x, y, z) \in X \times F$ to the threat points would decrease the purchasing power more for individual i than individual i'. It asserts that the i-th individual is less willing to face a bargaining failure (compared to i') when a bargaining failure (represented by a move from (x_0, x_i, z) to (x_{0i}^t, x_i^t, z_i^t)) would reduce his/her purchasing power more than individual i'. For a given (x_0, x_i, z), the greater $\Delta_i(p_i, x_0, x_i, z, U_i^t)$ is, the less the individual is willing to accept a bargaining failure.

Definition 2: A feasible allocation $(x, y, z) \in X \times F$ satisfying equation (1) is a bargaining agreement for all n individuals if

$$\Delta_i(p_i, x_0, x_i, z, U_i^t) = M(p, x, z, U^t) \text{ for all } i \in N, \quad (10a)$$

where

$$M(p, x, z, U^t) = \max_i \{\Delta_i(p_i, x_0, x_i, z, U_i^t) : i \in N\}. \quad (10b)$$

Definition 2 states that a feasible allocation (x, y, z) is a bargaining agreement when all n individuals are equally willing to accept a bargaining failure. How can household members proceed to reach a bargaining agreement? Building on evolutionary games (e.g., Sandholm, 2010), we investigate below alternative representations of the bargaining process.

First, we consider the following iterative scheme (as stressed below, this scheme does not necessarily yield efficient agreements).

<u>Bargaining scheme S</u>:

- Step S1: Start at iteration $k = 1$. Propose a feasible allocation $(x^k, y^k, z^k) \in X \times F$ satisfying equation (1), with corresponding utilities $U_i^k = u_i(x_0^k, x_i^k, z^k), i \in N$. Let $\Delta_i^k = [e_i(p_i, x_0^k, z^k, U_i^k) - e_i(p_i, x_0^k, z^k, U_i^t)], i \in N$, and $M^k = \max_i \{\Delta_i^k : i \in N\}$.
- Step S2: Find the set N^k of individuals being least willing to face a bargaining failure

$$N^k = \{i : \Delta_i^k = M^k, i \in N\}. \quad (11)$$

- Step S3:
  - step S3a: If $N^k = N$, then stop the bargaining process at $k^\# = k$ and announce $(x^{k^\#}, y^{k^\#}, z^{k^\#})$ as a bargaining agreement.
  - step S3b: If $N^k \neq N$, propose a feasible allocation$(x^{k+1}, y^{k+1}, z^{k+1}) \in X \times F$ satisfying equation (1),

$$\Delta_i^{k+1} < \Delta_i^k, i \in N^k, \quad (12a)$$

and

$$\Delta_i^{k+1} \leq M^k, i \in N - N^k, \quad (12b)$$

where $U_i^{k+1} = u_i(x_0^{k+1}, x_i^{k+1}, z^{k+1}), \Delta_i^{k+1} = [e_i(p_i, x_0^{k+1}, z^{k+1}, U_i^{k+1}) - e_i(p_i, x_0^{k+1}, z^{k+1}, U_i^t)]$, $i \in N$, and $M^{k+1} = \max_i \{\Delta_i^{k+1}: i \in N\}$. Then, let $k = k + 1$ and go to step S2.

The bargaining scheme S involves identifying the individuals who are the least willing to face a bargaining failure. At the k-th iteration, the set of these individuals is denoted by N^k in equation (11) in step S2. In a way consistent with Definition 2, step S3a identifies a bargaining agreement occurring when $N^k = N$, i.e. when all n individuals are equally willing to face a bargaining failure. In step S3b, equations (12) state that the individuals in N^k are the ones making a concession at the k-th iteration, where making a concession means reducing their willingness to face a bargaining failure. Note that, as in Zeuthen (1930), the individuals making concessions are the ones least willing to face a bargaining failure (Zeuthen, 1930; Harsanyi, 1950, 1977). As stated next, upon convergence, the process of making concessions in S leads to a bargaining agreement (see the proof in Appendix A).

Proposition 2: Upon convergence, the bargaining scheme S identifies a bargaining agreement $(x^{k^\#}, y^{k^\#}, z^{k^\#})$.

Proposition 2 identifies a bargaining agreement. Note that Proposition 2 does not say that a bargaining agreement is unique. In general, it allows for multiple bargaining agreements. From Definition 2, a bargaining agreement occurs when all household members are equally willing to face a bargaining failure.

This raises two questions. First, what are the relationships between a bargaining agreement and bargaining power? Second, what are the linkages between bargaining agreements and Pareto efficiency? These two questions are addressed in the next sections.

4. Bargaining Position and Income Distribution

From equations (8)-(10), our definition of a bargaining agreement relies on the i-th individual willingness-to-pay measure $\Delta_i(p_i, x_0, x_i, z, U_i^t) \equiv [e_i(p_i, x_0, z, u_i(x_0, x_i, z)) - e_i(p_i, x_0, z, U_i^t)], i \in N$. This measure depends on individual preferences $u_i(x_0, x_i, z)$ as well as the threat utility $U_i^t = u_i(x_0^t, x_i^t, z^t), i \in N$. Our analysis indicates that any factor that contributes to increasing (decreasing) $\Delta_i(p_i, x_0, x_i, z, u_i(x_0^t, x_i^t, z^t))$ would increase (decrease) the i-th individual's willingness to make a concession and thus weaken (strengthen) his/her bargaining position within the household. This section discusses the implications of these arguments.

First, consider the role of preferences and their effects on bargaining outcomes. An example is the role of patience in situations where (x, y, z) represents a multi-period allocation over some future planning horizon. Different individual may discount the future at different rates, with "more patient" individuals having a lower discount rate (Rubinstein, 1982). A result we obtain from our analysis is that greater patience would weaken the bargaining position of individual i when it induces an increase in $\Delta_i(p_i, x_0, x_i, z, u_i(x_0^t, x_i^t, z^t))$, as this would make individual i more willing to make a concession. This would occur when bargaining failure has large adverse effects on future benefits, thus associating greater patience with a rise in *ex ante* willingness to pay to avoid a bargaining failure. Note that this result differs from the one obtained by Rubinstein (1982), reflecting that our evaluation of bargaining is relative (all allocations being compared to the threat points).

Another example involves the effects of risk preferences when benefits are uncertain. How would change in risk aversion affect the outcome of the bargaining process (e.g., Murnighan et al., 1988)? From our analysis, the willingness-to-pay $\Delta_i(p_i, x_0, x_i, z, u_i(x_0^t, x_i^t, z^t))$ is an *ex ante* measure. For risk averse individuals, this willingness-to-pay would decline when benefits become more uncertain. A prediction from our analysis is the i-th individual becoming more risk averse would weaken his/her bargaining position when this change raises $\Delta_i(p_i, x_0, x_i, z, u_i(x_0^t, x_i^t, z^t))$, thus increasing his/her willingness to make a concession. This would occur when bargaining failure tends to increase individual risk exposure, thus associating higher risk aversion with a rise in *ex ante* willingness to pay to avoid a bargaining failure.

Perhaps more importantly, our analysis provides useful insights on the role of threat points (represented by $U_i^t = u_i(x_o^t, x_i^t, z^t), i \in N$) and their effects on intra-household bargaining and income distribution. For the i-th individual, the expenditure function $e_i(p_i, x_0, z, U_i)$ reflects his/her purchasing power under utility U_i and can be used to evaluate how income is distributed within the household. To see that, consider the following expenditure share associated with the i-th household member

$$\sigma_i(p_i, x_0, z) = \frac{e_i(p_i, x_0, z, u_i(x_0, x_i, z))}{\sum_{i' \in N} e_{i'}(p_{i'}, x_0, z, u_{i'}(x_0, x_{i'}, z))} \in [0, 1], i \in N. \tag{13}$$

Equation (13) measures the proportion of expenditures on private goods captured by the i-th individual. This measure has several attractive features. First, it reflects the role of relative bargaining power within the household. Intuitively, seeing a family member receiving a larger (smaller) expenditure share would indicate that this individual has a greater (weaker) bargaining power. Second, it is money metric measure of how income is shared within the household, thus providing useful insights into intra-household welfare distribution. Finally, as illustrated below, $\sigma_i(p_i, x_0, z)$ in equation (13) is empirically tractable.

What are the linkages between $\sigma_i(p_i, x, z)$ in equation (13) and a bargaining agreement? To answer this question, let (x^b, y^b, z^b) be a bargaining agreement satisfying Definition 2. It follows from equations (8)-(10) that $e_i\left(p_i, x_0^b, z^b, u_i\left(x_0^b, x_i^b, z^b\right)\right) - e_i\left(p_i, x_0^b, z^b, U_i^t\right) = k, i \in N$, for some constant k, implying that

$$e_i\left(p_i, x_0^b, z^b, u_i\left(x_0^b, x_i^b, z^b\right)\right) - e_i\left(p_i, x_0^b, z^b, U_i^t\right) =$$

$$\sum_{i \in N}\left[e_i\left(p_i, x_0^b, z^b, u_i\left(x_0^b, x_i^b, z^b\right)\right) - e_i\left(p_i, x_0^b, z^b, U_i^t\right)\right]/n,$$

or, using equation (13),

$$\sigma_i\left(p_i, x_0^b, z^b\right) = \frac{1}{n} + \frac{e_i(p_i, x_0^b, z^b, U_i^t) - [\sum_{i' \in N} e_{i'}(p_{i'}, x_0^b, z^b, U_{i'}^t)/n]}{\sum_{i' \in N} e_{i'}\left(p_{i'}, x_0^b, z^b, u_{i'}\left(x_0^b, x_{i'}^b, z^b\right)\right)}. \quad (14)$$

Equation (14) shows how the expenditure share of the i-th individual depends on the threat points $(U_1^t, \ldots, U_n^t)$. It indicates that a *ceteris paribus* increase in U_i^t would increase both $e_i\left(p_i, x_0^b, z^b, U_i^t\right)$ and $\sigma_i\left(p_i, x_0^b, z^b\right)$. Thus, an individual would increase his/her expenditure share when he/she can make a bargaining failure less threatening. Alternatively, an individual would receive a lower expenditure share when faced with more threatening bargaining failures. Associating the threat utility U_i^t with the strength of the i-th individual's bargaining power, we obtain the intuitive result: a family member with stronger bargaining power would capture a larger share of household expenditures.

Equation (14) implies that, under a bargaining agreement, $\sigma_i\left(p_i, x_0^b, z^b\right)\begin{Bmatrix}>\\=\\<\end{Bmatrix} 1/n$ when $e_i\left(p_i, x_0^b, z^b, U_i^t\right)\begin{Bmatrix}>\\=\\<\end{Bmatrix}\sum_{i' \in N} e_{i'}(p_{i'}, x_0^b, z^b, U_{i'}^t)/n$. It follows that all individuals would have equal expenditure share (with $\sigma_i = 1/n$) when they have the same purchasing power at the threat points (when bargaining fails). This corresponds to an egalitarian distribution of income (where each household member receives the same individual income). Our analysis also allows for unequal income distribution. Indeed, equation (14) can generate a very unequal distribution of household income when purchasing power varies a lot across household members under bargaining failure. In this case, individuals facing damaging threats (with low $e_i(p_i, x_0^b, z^b, U_i^t)$) would receive lower income, while individuals with high $e_i(p_i, x_0^b, z^b, U_i^t)$ would receive a larger share of household income. These results have two implications. First, they illustrate that our analysis is positive (and not normative). As such, our approach can provide useful insights into intra-household welfare inequality and distribution issues (including the case of domestic violence). Second, our analysis stresses the importance of threat points as they affect both bargaining power and the distribution of income. The socio-economic and political factors affecting threat strategies are discussed in Section 6 below.

5. Bargaining and Efficiency

Note that Proposition 2 does not require the bargaining agreement $(x^{k^\#}, y^{k^\#}, z^{k^\#})$ to be Pareto efficient. Thus, it allows for inefficient bargaining outcomes. This seems useful to the extent that assessing household efficiency typically requires a lot of information about all aspects of household allocations. When obtaining and processing this information proves difficult, identifying and implementing efficient allocations may be problematic in household bargaining.

Alternatively, if the bargaining outcome $(x^{k^\#}, y^{k^\#}, z^{k^\#})$ is not Pareto efficient, it means the possibility of untapped efficiency gains. Letting $U^\# = \left(U_1^{k^\#}, \dots, U_n^{k^\#}\right)$, the corresponding aggregate efficiency gain would be $W(p, q, U^\#) > 0$. This raises the question: could the bargaining scheme S be modified to converge to an efficient allocation? A positive answer to this question is presented next.

<u>Bargaining scheme S^E (Efficient scheme)</u>: The scheme <u>S^E</u> is the same as S except that step S3 is replaced by <u>S^E3</u>:

- Step S^E3
  - step S^E3a: If $N^k = N$ and $W(p,q,U^k) = 0$, then stop the bargaining process at $k^* = k$ and announce $(x^{k^*}, y^{k^*}, z^{k^*})$ as a bargaining agreement.
  - step S^E3b: If $N^k \neq N$ or $W(p,q,U^k) > 0$, propose a feasible allocation $(x^{k+1}, y^{k+1}, z^{k+1})$ satisfying equations (12a), (12b) and

$$W(p,q,U^{k+1}) < W(p,q,U^k) \text{ if } W(p,q,U^k) > 0 \tag{15}$$

where $U_i^{k+1} = u_i\big(x_0^{k+1}, x_i^{k+1}, z^{k+1}, d\big)$, $U^{k+1} = (U_1^{k+1}, \dots, U_n^{k+1})$. $\Delta_i^{k+1} = [e_i(p_i, x_0^{k+1}, z^{k+1}, U_i^{k+1}) - e_i(p_i, x_0^{k+1}, z^{k+1}, U_i^t)]$, $i \in N$, and $M^{k+1} = \max_i \{\Delta_i^{k+1}: i \in N\}$. Then, let $k = k + 1$ and go to step S2.

Scheme S^E differs from S in two ways. First, $W(p,q,U^k) = 0$ has been added in step S^E3a. From Proposition 1, this corresponds to Pareto efficiency. Second, when $W(p,q,U^k) > 0$, equation (15) has been added to the iterative scheme. Equation (15) states that the bargaining process must reduce the distance to the Pareto utility frontier across iterations from k to $k+1$. This implies that the iterative process necessarily moves toward the Pareto utility frontier. Upon convergence, from S^E3a, this leads to an allocation on the Pareto utility frontier.

Building on Proposition 2, we have the following result (see the proof in Appendix A).

Proposition 3: Upon convergence, the bargaining scheme S^E identifies an efficient bargaining agreement $(x^{k^*}, y^{k^*}, z^{k^*})$ that corresponds to a unique point on the Pareto utility frontier.

Proposition 3 states that the bargaining process given in S^E converges to a unique point on the Pareto utility frontier. The bargaining schemes S and S^E have some nice properties. First, they involve simple iterations that may reflect the steps taken during an actual bargaining session. Second, upon convergence, scheme S^E finds a bargaining agreement for an allocation that is Pareto efficient and where all individuals are equally willing to face a bargaining failure. Third, the bargaining process S^E identifies a unique point on the Pareto utility frontier. This unique point is linked directly with the threat of bargaining failure, thus stressing the importance of threat strategies.

Note that the identification of a unique point on the Pareto utility frontier was obtained from an intra-household bargaining process. It did not require the specification of a household preference function. Yet, a household preference function can always be found to rationalize observed behavior. For example, if the Pareto utility frontier is concave, then there exists a hyperplane tangent to the utility frontier at some evaluation point. In this case, the slopes of this hyperplane can be treated as Bergsonian weights in a household utility function. But changing these Bergsonian weights can rationalize any point of the utility frontier, meaning that a household preference function does not really help identifying the factors affecting distribution issues. In contrast, our bargaining approach and its reliance on threat points do provide the additional information we can use in the investigation of income and welfare distribution within the household.

This raises the question: Is there a simple representation of the outcome of the bargaining process S^E? The answer is given in the following proposition (see the proof in Appendix A).

Proposition 4. Consider the following optimization problem

$$\underset{x_0,y,z,U}{\text{Max}}\{\prod_{i\in N}[e_i(p_i,x_0,z,U_i)-e_i(p_i,x_0,z_0,U_i^t)]:$$

$$\textstyle\sum_{i\in N}e_i(p_i,x_0,z,U_i)+p_0x_0\le qy,$$

$$e_i(p_i,x_0,z,U_i)\ge e_i(p_i,x_0,z_0,U_i^t), i\in N; x_0\in X_0, (y,z)\in F\}, \tag{16}$$

which has for solution (x_0^e,y^e,z^e,U^e). The allocation (x_0^e,y^e,z^e,U^e) is an efficient bargaining agreement.

Proposition 4 is an extension of Nash bargaining under ordinal preferences (Nash, 1950).[4] As such, the term $\prod_{i \in N}[e_i(p_i, x_0, z, U_i) - e_i(p_i, x_0, z, U_i^t)]$ in (16) is a generalized "Nash product". The maximization problem in (16) subject to two sets of constraints: the budget constraint $\sum_{i \in N} e_i(p_i, x_0, z, U_i) + p_0 x_0 \leq qy$; and the incentive compatibility constraints $e_i(p_i, x_0, z, U_i) \geq e_i(p_i, x_0, z, U_i^t), i \in N$. When combined with Proposition 3, Proposition 4 states that the constrained maximization of the generalized Nash product in problem (16) picks a unique point on the Pareto utility frontier that is also a bargaining agreement (as identified in scheme S^E).

While Proposition 4 identifies scenarios where bargaining leads to efficient allocations (under bargaining scheme S^E), we also considered allocations associated with bargaining agreements that may be inefficient. In general, the bargaining scheme S can generate bargaining outcomes that are inefficient. Clearly, inefficiency could arise only if the individuals involved in the bargaining process fail to identify the benefits from efficiency gains. This can happen if the individuals have difficulties in identifying these benefits, for example due to cognitive limitations bounding the rationality of the individual, the endowment of noncognitive abilities favoring cooperation and willingness to solve conflicts, and/or information cost. In this case, bargaining agreements can arise that are not necessarily efficient. In this context, it is useful to examine what are the efficiency cost of these agreements.

Consider a bargaining agreement (x^a, y^a, z^a) associated with scheme S. How does it compare with an efficient bargaining agreement (x^e, y^e, z^e) associated with scheme S^E? Let $U^a =$

[4] The Nash bargaining model was first proposed by Nash (1950, 1953). McElroy and Horney (1981) and McElroy (1990), Notburga (1992) initially applied the Nash model to household economics. Extensions of the Nash bargaining model using ordinal preferences have been explored by Rubinstein et al. (1992) and Hanany and Safra (2000). Our approach also applies under ordinal preferences, but it goes beyond Rubinstein et al. (1992) or Hanany and Safra (2000) in the sense that we do not require probability assessments.

$\{(U_1^a, \ldots, U_n^a): U_i^a = u_i(x_0^a, x_i^a, z^a), i \in N\}$. Let $U^e = \{(U_1^e, \ldots, U_n^e): U_i^e = u_i(x_0^e, x_i^e, z^e), i \in N\}$. From Proposition 1, note that $W(p, q, U^e) = 0$ under efficiency, while $W(p, q, U^a) \geq 0$. Then the efficiency cost of a bargaining agreement (x^a, y^a, z^a) can be measured as

$$\Delta W^a = W(p, q, U^a) \geq 0. \tag{17}$$

Equation (17) gives as ΔW^a as a welfare measure of the distance between the bargaining agreement (x^a, y^a, z^a) and the Pareto utility frontier. When bargaining leads to efficient outcomes then $\Delta W^a = 0$ (from Proposition 1). But when bargaining agreements lead to allocations that fall short of efficiency, then $\Delta W^a > 0$. In this case, equation (17) provides a basis to evaluate the welfare cost of imperfect bargaining.

Note that there are many ways for a household to be inefficient. From Corollary 1, inefficiency can arise when x_0 does not satisfy (5), when y does not satisfy (6), when z does not satisfy (7) and/or when the household budget constraint (1) is not binding. Given this complexity, is there a simple way to represent inefficient agreements? As indicated in Oprea (2020), the answer is affirmative. Consider the case where imperfections in household bargaining can be represented by a bargaining cost c. This bargaining cost can provide a rational for inefficient agreements: the household would stop bargaining when $\Delta W^a < c$, i.e., when the benefit of bargaining is less than its cost. This suggests introducing bargaining cost in the evaluation of bargaining agreements. This can be done by modifying Proposition 4 as stated next. (The proof is similar to Proposition 4 and is omitted).

<u>Proposition 5:</u>

a/ if $c \geq W(p, q, d, U^t)$, there is no bargaining agreement, and the household allocation is the threat point (x_0^t, y^t, z^t, U^t). (18a)

b/ if $0 \leq c < W(p, q, d, U^t)$, consider the optimization problem

$$\underset{x_0,y,z,U}{\text{Max}}\{\prod_{i\in N}[e_i(p_i,x_0,z,U_i)-e_i(p_i,x_0,z,U_i^t)]:$$

$$\sum_{i\in N} e_i(p_i,x_0,z,U_i)+p_0x_0\leq qy-c,$$

$$e_i(p_i,x_0,z,U_i)\geq e_i(p_i,x_0,z,U_i^t), i\in N; x_0\in X_0, (y,z)\in F;\}, \tag{18b}$$

which has for solution $\left(x_0^b, y^b, z^b, U^b\right)$. Then, the allocation $\left(x_0^b, y^b, z^b, U^b\right)$ is a bargaining agreement which is efficient if $c = 0$ but inefficient if $c > 0$.

Proposition 5 proposes a simple representation of inefficient bargaining agreements, conditional on c. Equation (18a) states that household bargaining fails (generating the threat allocation (x_0^t, y^t, z^t, U^t)) when bargaining cost is larger than bargaining benefit: $c \geq W(p,q,U^t)$. And equation (18b) generalizes (16) by introducing cost c in the budget constraint. In a way similar to (16), the solution to (18b) is a bargaining agreement (see the proof in Proposition 4). When $c = 0$, (18b) reduces to (16), which identifies an efficient bargaining agreement (from Proposition 4). But when $0 \leq c < W(p,q,U^t)$, (18b) provides useful information on inefficient bargaining agreements. In general, the solution $\left(x_0^b, y^b, z^b, U^b\right)$ to (18b) varies with c. Evaluating (18b) for different values of $c \geq 0$ generates a set of bargaining agreements going from an efficient agreement (when $c = 0$), to inefficient agreements (when $0 < c < W(p,q,U^t)$ and to no agreement (when $c \geq W(p,q,U^t)$). In this context, following Oprea (2020), c can be interpreted as a measure of cost that reflects both the degree of complexity and the degree of inefficiency: the agreements represented by (18b) become increasing inefficient as c increases from 0 to $W(p,q,U^t)$.

This is illustrated in Figure 1 in the case where $n = 2$. Figure 1 shows the range of utilities (U_1, U_2) that can be obtained under alternative allocations. The Pareto utility frontier is given by the line AEC, representing the set of utilities (U_1, U_2) where $W(p, q, U_1, U_2) = 0$. The set of utilities obtained under bargaining agreements represented by (18a)-(18b) is given by the line TBE, that we label bargaining or contract curve. This line starts at point T, representing the threat points (U_1^t, U_2^t) obtained when bargaining fails, i.e., when $c \geq W(p, q, U^t)$. As c decreases from $W(p, q, U^t)$ toward 0, the line TBE represents inefficient bargaining agreements as one moves up toward the Pareto utility frontier. When $c = 0$, the line TBE crosses the Pareto utility AEC at point E, identifying point E as an efficient agreement. As illustrated in Figure 1, any move up the line TBE corresponds to a Pareto improvement (as it would make both individuals better off). And point E is unique: it is the only point (U_1, U_2) that is both efficient and a bargaining agreement. This illustrates a key result from Propositions 3 and 4, the bargaining scheme S^E generates a unique point on the Pareto utility frontier. This is one of our important results: combining bargaining and efficiency settles the issue of how welfare is distributed within the household. But scheme S opens the possibility that bargaining does not always lead to efficient outcomes. Figure 1 shows that any point along the line TBE is a possible bargaining agreement. Point B represents one of those agreements. Point B is inefficient (as it is below the Pareto utility frontier AEC). In this case, the extent of inefficiency is given by the distance between point B and point E. From equation (17), a welfare measure of the corresponding efficiency loss is given by $W\left(p, q; U_1^b, U_2^b\right) \geq 0$.

6. Threat Strategies

What determines the threat strategies $(x_{0i}^t, x_i^t, y_i^t, z_i^t)$, $i \in N$? In this section, we discuss alternative approaches to the investigation of threat strategies. In general, the higher is the initial level of utility at the threat point, the higher is the final utility at the agreement point because higher is the initial bargaining power. The credibility of the threat affects the location of the equilibrium on the contract curve and the frontier (Donni and Ponthieux, 2011).

The first approach involves a situation where the threats imply breaking up the households into n parts, each household member managing his/her resources independently. In this case, the threat points would correspond to the exit options available to each individual. An example would be the case of divorce applied to a married couple, which would represent an *extrema ratio* solution (Manser and Brown 1980; McElroy and Honey, 1981). The threat to break-up the sentimental relation is probably the most powerful among the possible threats. It corresponds to the level of utility, net of transaction or emotional costs, that would be achieved if living alone. Compared to a threat based on a non-cooperating decision, this one has the practical advantage of defining the maximum size of the negotiation set. In this context, the threat points $(x_{0i}^t, x_i^t, y_i^t, z_i^t) \in X_0 \times X_i \times F$ would be obtained from applying the analysis presented above to each individual, $i \in N$. Interestingly, Boto-García and Perali (2020) find that this exit option is more frequent than one may expect. Around 44 percent of their sample has seriously considered to end the relationship with their current partner in the past. The intention to break-up is more frequent among those who score low in marital locus of control, males, low-income earners, individuals with university studies and couples without children. This is the threat strategy adopted in the present empirical application, assuming that individual preferences are not affected by marital status.

The second approach would consist in associating the threats to non-cooperative behavior within the household (Lundberg and Pollak, 1993, 1996; Ulph, 1988; Woolley, 1988). In this case, the threat points are internal to the marriage, not external as in situations that consider divorce as a credible threat (Lundberg and Pollak, 1993, 1996). These disagreement situations would be identified as the outcomes of a non-cooperative game played by the n household members. For example, such outcomes would be equilibrium points in non-cooperative, voluntary contribution Nash equilibrium (Nash, 1951; Rosen, 1965). As noted by Lundberg and Pollak (1996:147) “noncooperative marriage in which the spouses receive some benefits due to joint consumption of public goods may be a more plausible threat in day-to-day marital bargaining.” Deciding to live non-cooperatively in separate spheres within the same house is also a strong threat especially if sustained through time. It is probably a strategy that is not commonly adopted in ordinary conflicts. By reinterpreting an inefficient bargaining agreement as a non-cooperative threat with respect to a movement towards the efficient agreement, in the sense that the couple decides not to cooperate to do an extra effort to move closer to the Pareto frontier, as we do in our study, we are also adopting a milder version of the separate sphere noncooperative threat.

A third approach would involve the role of social rules and their effects on the resolution of conflicts within the household. Such social rules can guide and constrain socio-economic behavior of household members. As discussed in Ellickson (2008), these rules can be formal (e.g., explicit contracts) as well as informal (e.g., implicit contracts developed and managed within the household). Social rules play a role when conflicts arise among household members. For example, conflict resolution mechanisms can help regulate bargaining failure and thus affect the threat points used in our analysis. This includes the role of the Courts in settling disputes (including divorce settlements). These arguments illustrate that the determinants of threat points are often complex and can vary with the socio-political environment of each household.

7. Empirical Application of the Household Bargaining Model

In this section we take Propositions 2, 3 and 5 to the data. While Proposition 2 identifies a set of bargaining agreements using the bargaining scheme S, Proposition 3 identifies efficient bargaining outcomes employing the efficient bargaining scheme S^E. Both schemes involve an *iterative* process to reach an agreement. Proposition 5 generates a set of bargaining agreements using the notion of bargaining cost associated with the individual willingness-to-pay to avoid the breakdown position. Unlike schemes S and S^E, Proposition 5 involves an *optimization* process.

7.1. Data, Individual Preference Structure and Household Technology

The analysis is carried out using a nationwide survey on socioeconomic characteristics of Italian rural households undertaken in 1995 by the Italian Institute for Agricultural Markets (ISMEA). The farm-household survey combines information about household and farm characteristics, farm production and profits, stylized time use, off-farm labor income, governmental and intra-household transfers, consumption, and information on the degree of autonomy in decision making by household members. A relevant feature of the ISMEA survey is that it records information about the private consumption of assignable goods, such as clothing for women and men, which is sufficient to identify the bargaining power as defined in equation (13) that is a concept analogous to the sharing rule governing the intra-household allocation of resources (Chiappori 1988, 1992; Chiappori and Ekeland, 2009).

The functional forms and associated parameters used to implement the bargaining processes are those econometrically estimated by Matteazzi, Menon and Perali (2017), and Magnani, Matteazzi and Perali (2007) to carry out individual welfare analyses.[5] As shown in Appendix B, it is assumed a Translog specification for the cost function of both the marketable production and the non-marketable domestic production, and an AIDS demand system for the individual budget shares. Using the estimated parameters and sample mean values for the variables of the production costs, we predict household profits from the marketable agricultural production

[5] Estimated parameters used in the simulation analysis are available upon request from the authors.

and the endogenous price of the nonmarketable domestic good. Predicted profit level, added up to the sample mean value of household non-labor income, contributes to define the household income ($\pi = qy$) over which spouses negotiate the redistribution to afford private consumptions.

7.2. Implementation of the Bargaining Schemes

When we apply the bargaining algorithms, we assume that two individuals ($n = 2$), the wife ($i = 1$) and the husband ($i = 2$), bargain over household resources. As assumed in Section 3, the threat point is taken as given. For each spouse the breakdown position is defined as his/her purchasing power when bargaining fails, that is we take the threat points as the level of private expenditure of a person living alone, under the assumption that individual preferences are not affected by marital status. To investigate how the threat point influences the bargaining process and the final location on the utility Pareto frontier, we replicate the iterative procedure for different values of the threat point, which results are discussed below.

In the iterative schemes S and S^E, the spouses negotiate the redistribution of household resources conditional on the household optimal production decisions. They bargain over the amount of total household income that is available for the individual private consumptions of leisure, clothing, the domestic produced good, food and other goods. We mimic the iteration processes in S and S^E using a stochastic simulation to approximate our theoretical analysis. At each iteration, the individual demands for these private goods, with the except of clothing, are randomly generated between a lower and an upper bound corresponding to the minimum and maximum consumption value observed in the dataset. For the individual demand of clothing, the simulation randomly chooses a value between a lower and an upper bound defined by equations (12a)[6] and

[6] For the household member with the highest willingness-to-pay to avoid the threat allocation, equation (12a) is $\Delta_i^{k+1} < \Delta_i^k$. Given $\Delta_i^k = e_i^k(p_i, x_0, z, u_i^k(x_0, x_i^k, z)) - e_i(p_i, x_0, z, U_i^t)$, and $e_i(p_i, x_0, z, U_i^t)$ is treated as given, from equation (12a) it follows $e_i^{k+1}\left(p_i, x_0, z, u_i^{k+1}(x_0, x_i^{k+1}, z)\right) < e_i^k\left(p_i, x_0, z, u_i^k(x_0, x_i^k, z)\right)$. Hence, for the member with the

(12b)[7] for scheme S, and equations (12a), (12b) and (15)[8] for scheme S^E.[9] Therefore, at each iteration k for each spouse, the private expenditure $e_i^k\left(p_i, x_0, z, u_i^k\left(x_0, x_i^k, z\right)\right)$ is randomly generated to ensure feasibility of the household budget and, only for the bargaining scheme S^E, possibly efficiency. At each iteration k, we calculate the willingness-to-pay $\Delta_i^k = e_i^k(p_i, x_0, z, u_i^k(x_0, x_i^k, z)) - e_i(p_i, x_0, z, U_i^t)$ of each spouse i. If the difference between the spouses' willingness-to-pay is sufficiently small,[10] spouses will reach an agreement, otherwise, they will continue to negotiate until they reach the contract curve. We randomly generate a total of 1,000,000 feasible allocations.[11]

Table 1 shows the simulations of the bargaining schemes S and S^E for the average family of our sample. Panel A refers to the bargaining scheme S, while Panel B to the efficiency scheme S^E. Each row of the table represents a k-th iteration of the negotiation process. In both panels, the first iteration, $k = 1$, is the starting point of the negotiation processes, which corresponds to the average sample values. As previously mentioned, the threat point is defined as the level of private expenditure of a single person household.[12] In our analysis, it amounts to 1622€ for the wife and

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highest willingness-to-pay to avoid a bargaining failure, individual expenditure at iteration k represents an upper bound for her private expenditure at iteration $k + 1$.

[7] For the household member with the lowest willingness-to-pay to avoid the threat allocation, equation (12b) implies that $e_i^{k+1}\left(p_i, x_0, z, u_i^{k+1}\left(x_0, x_i^{k+1}, z\right)\right) \leq e_j^k\left(p_j, x_0, z, u_j^k\left(x_0, x_j^k, z\right)\right) - e_j(p_j, x_0, z, U_j^t) + e_i(p_i, x_0, z, U_i^t)$.

[8] In presence of untapped efficiency gains at iteration k, equation (15) sets an increase in household total private expenditure at iteration $k + 1$ entailing a reduction in household welfare losses. Operationally, for the member with the lowest willingness-to-pay to avoid the threat allocation, equation (15) implies that $e_i^{k+1}\left(p_i, x_0, z, u_i^{k+1}\left(x_0, x_i^{k+1}, z\right)\right) \geq e_i^k\left(p_j, x_0, z, u_i^k\left(x_0, x_i^k, z\right)\right) + e_j^k\left(p_j, x_0, z, u_j^k\left(x_0, x_j^k, z\right)\right) - e_j^{k+1}\left(p_j, x_0, z, u_j^{k+1}\left(x_0, x_j^{k+1}, z\right)\right)$.

[9] Given that the individual willingness-to-pay to avoid the threat allocation must be non-negative, the private expenditure associated with the threat allocation represents a lower bound for spouses' expenditure on private goods. This means that, at each iteration and for all household members, $e_i^k(p_i, x_0, z, u_i^k(x_0, x_i^k, z)) \geq e_i(p_i, x_0, z, U_i^t)$.

[10] In the analysis, the difference $\Delta_i^k - \Delta_j^k$ is assumed to be sufficiently small if it is between −5€ and +5€, which corresponds to about 0.06 percent of the average household income.

[11] About 5 percent of the generated allocations are not feasible because total household expenditures exceed total household income. This event is especially frequent for households with low levels of household income.

[12] The estimation of the cost of living of a single is based on the notion of indifference scales. Let us define the indifference scale as the index $I_i^h = e_i^s(p_i, x_0, z, U_i^T)/\pi(q, z)$ that adjusts the income of member i of household h to

1533€ for the husband. Note that in our analysis, the spouses have a similar purchasing power at the threat point. The equality of the individual purchasing power translates into an egalitarian distribution of resources between the spouses (at the threat point $\sigma_1 = 0.51$ and $\sigma_2 = 0.49$).

In Panel A of Table 1, at the sample mean and given the chosen threat point, the willingness-to-pay Δ_1^1 of the wife to avoid the threat allocation is greater than the husband willingness-to-pay Δ_2^1 (695€ and 657€, respectively). The wife is thus less willing to accept a bargaining failure than her husband, and she will be willing to make a concession at $k = 2$ by reducing her willingness to face a bargaining failure. Comparing the result obtained at $k = 1$ with those at $k = 2$, we see a reduction in the private expenditure e_1^k of the wife, while the husband private expenditure e_2^k slightly increases. At $k = 2$ the wife willingness-to-pay Δ_1^2 decreases and the willingness-to-pay of the husband Δ_2^2 increases. Because the spouses have different willingness-to-pay, the negotiation continues until they reach an agreement. For the scheme S, a bargaining agreement is reached at $k = 39$, where the difference between the spouse willingness-to-pay is sufficiently small, $\Delta_1^{39} - \Delta_2^{39}= 4.4$.[13] Note that the maximized net income (W) is not entirely allocated among the spouses, the family saves 362€, and this agreement is not Pareto efficient. As shown in Proposition 2, the bargaining scheme S does not require the bargaining agreement to be Pareto efficient.

reach the same indifference curve as when living alone, where $e_i^s(p_i, x_0, z, U_i^T)$ is the income that member i would need if living alone (that is at the threat point) to attain the same level of utility, and $\pi(q, z)$ is the household income (Lewbel 2003; Chiappori 2016). The individual income at the threat point for member i is obtained from the knowledge of household expenditure $e^s(p, x_0, z, U^T)$ multiplied by the ratio of the actual share of resources within the couple s_i with respect to an equal division, that is $e_i^s(p_i, x_0, z, U_i^T) = e^s(p, x_0, z, U^T) * (s_i/0.5)$. An example may help. Suppose that the cost of living of a reference couple is 1000€. Then, the cost of living of an equivalent adult is 500€. Using the estimates in Perali (1999) and Menon and Perali (2010) for Italy, the cost of living of a person living alone is 70 percent of the cost of living of a couple, then the cost of living of the single is 700€, which is 1.4 times the cost of an adult equivalent. Assume further that $s_1 = 0.6$ and $s_2 = 1 - s_1 = 0.4$, then e_1^s and e_2^s amount to 840€ and 560€, respectively.

[13] The spouse willingness-to-pay Δ_i is expressed in Euro.

The bargaining scheme S^E adds to the scheme S the efficient condition (15), which allows the bargaining process to move toward a unique point on the utility Pareto frontier. In general, a Pareto efficient agreement requires more negotiations, as shown in Panel B where for our average family the efficient agreement is reached at $k = 136{,}526$. The difference between spouses' willingness-to-pay is sufficiently small and W is almost 0.

Figures 2 and 3 provide a graphical representation of the bargaining processes S and S^E, respectively. The figures trace the feasible negotiation set that provides spouses with a utility level equal to or greater than the individual utility associated with the threat point. The grey line, starting from the threat point and ending onto the Pareto frontier, is the set of bargaining agreements defining the bargaining curve. It corresponds to the TBE blue line in Figure 1. Only the point on the Pareto frontier is an efficient agreement. Only in the 1.5 percent of randomly generated allocations (or 14,142 out of 1,000,000) spouses reach a bargaining agreement and in most of the cases the bargaining agreement is far from being a Pareto efficient outcome. In line with our theoretical results, the process of scheme S^E moves the allocations closer to the utility Pareto frontier (Figures 3). The set of bargaining agreements with a sufficiently small value of W represents the 0.2 percent of the cases in the bargaining scheme S^E and only 0.05 percent of the cases in the scheme S. These figures suggest that there exists significant inefficiency within the family. Efficient allocations are clearly desirable outcomes, but, more realistically, family life tends to accept sufficient levels of satisfaction even though inefficient.

Proposition 5 shows how to identify the set of bargaining agreements (both efficient and inefficient) conditional on a bargaining cost c.[14] We solve the optimization model of equations

[14] The empirical implementation of Proposition 5 involves solving the following set of equations

(18a) and (18b) for different values of the bargaining cost c. Table 2 shows the results for different values of c. When $c = 0$, the spouses are on the Pareto frontier with a utility level equal to 7.7 for the wife and 10.9 for the husband, which corresponds to the efficient allocation of iteration k = 136,526 in Table 1, Panel B. If the bargaining cost is set to 2666€, corresponding to the welfare loss associated with the threat allocation, we obtain the utility levels corresponding to the threat point, that is 6.4 for the wife and 9.6 for the husband. For values of c between 0 and 2666€, the optimal solutions of the program correspond to the bargaining agreements but inefficient (grey curve in Figure 3). The finding that Proposition 5 and 3 lead to the same solution is evidence of the stability and robustness of the proposed algorithms.

7.3. Threat Point, Bargaining and Intra-Household Inequality

This section shows: 1/ how the bargaining process S^E generates a Pareto-improving redistribution of resources within the couple, and 2/ how the threat point affects both the location of the bargaining agreement on the Pareto frontier and the intra-household inequality. This is described in Figure 4. In Panel A, the average husband and wife of our sample have similar purchasing powers at the threat point ($\sigma_2 = 0.486$ and $\sigma_1 = 1 - \sigma_2 = 0.514$). In Panel B, we simulate a stronger purchasing power for the wife ($\sigma_1 > \sigma_2$). At the threat point, the private expenditure of the wife is 2085€, while the private expenditure of the husband is 657€, with an expenditure share σ_2 equal to 0.240 ($\sigma_1 = 0.760$), suggesting that the husband faces a relatively more damaging

$$\frac{1}{e_1\left(p_1, x_0, z, u_1\left(x_0, x_1, z\right)\right) - e_1(p_1, x_0, z, U_1^t)} = \lambda,$$

$$\frac{1}{e_2\left(p_2, x_0, z, u_2\left(x_0, x_2, z\right)\right) - e_2(p_2, x_0, z, U_2^t)} = \lambda,$$

$$e_1\big(p_1, x_0, z, u_1(x_0, x_1, z)\big) + e_2\big(p_2, x_0, z, u_2(x_0, x_2, z)\big) = y - c,$$

where y is the fixed household income, c is the bargaining cost, and λ is the Lagrangean multiplier associated with family budget constraints. The specification of the individual expenditure functions is presented in Appendix B.

threat than his wife. In Panel C, the husband is favored at the threat point. His purchasing power is 1751€, against 927€ for his wife, with an expenditure share σ_2 equal to 0.654.

Note that moving upward along the bargaining curve, from the threat point towards the Pareto frontier, equity improves. When the couple achieves a bargaining agreement, the intra-household allocation of resources is more equal than the one associated with the threat point, showing that the bargaining process within the family is Pareto and equity improving. In line with Section 4, the bargaining agreement maintains the ordering of the initial conditions.

Figure 4 also shows, as is well known, that the degree of equity of process S^E depends on the threat point. If at the threat point resources are (almost) equally allocated within the couple, then equality is maintained at the final location on the Pareto utility (Panel A). On the other hand, Panel B and C show that if resources are unequally allocated between the spouses at the threat point, then the final location on the Pareto utility frontier remains unequal, though less so. This is evidence that the bargaining process may be unable to fully adjust for equity, even when efficiency is reached. In any event, the allocation at the efficient bargaining agreement is more equal than the one at the threat point, suggesting that the bargaining process is equity improving, as measured by the Shannon entropy index $-\sum_{i=1}^{2}\sigma_i \ln \sigma_i$ developed by Chavas et al. (2018) and presented in Panel D. The inequality index takes values in the range $(0,1)$ and has an inverted U-shape relationship. For egalitarian allocations ($\sigma_i = 0.5$, $i = 1,2$),[15] the Shannon index takes its highest value of 0.693. The lower the index, the higher the degree of inequality within the couple. As shown in Panel D, at the Pareto frontier, the value of the Shannon index is equal to or higher than its value at the threat point and closer to 0.693, implying that the negotiation process improves equity within the couple. When individuals have similar purchasing powers at the threat point

[15] In Appendix C, Figure C1 shows the bargaining process S^E starting at an egalitarian threat point.

(Panel A), the Shannon index is close to its maximum value of 0.639 and remains unchanged at the efficient bargaining agreement. The size of the equity-improving effect of the bargaining process is bigger when at the threat point resources are in favor of one spouse (Panel B and C).

To describe the factors affecting the realization of the many possible agreements along the contract curve, we devote the next section to the analysis of the determinants of the bargaining outcomes.

8. Determinants of Bargaining Outcomes

This section addresses the following questions: a) What are the factors affecting agreeableness? b) How difficult is it to reach an agreement? and c) How does the cost of inefficiency vary across households?

Unlike the analysis developed in Section 7, here we base our estimations on a database generated by replicating the efficient bargaining scheme S^E for each household of the sample. For each family, we randomly generate $k = 2000$ feasible allocations.[16] At each iteration the family can either fail to reach an agreement, be on the contract curve, or move toward the contract curve up to an efficient point on the Pareto utility frontier. In our sample, within our selected range of k iterations, 93 percent of families reaches at least a bargaining agreement, 40 percent reaches an efficient allocation and only 9 percent simultaneously reaches both a bargaining agreement and an efficient allocation.

8.1 What are the factors affecting agreeableness?

[16] This choice is based on the trade-off between computational burden and robustness. We find that at the ceiling of about 2000 allocations, the number of families reaching an agreement, efficient or inefficient, remains almost unchanged, and the number of families reaching an efficient bargaining agreement slightly increases (Table C1 in Appendix C).

We study the determinants of the likelihood of occurrence of the bargaining outcomes by estimating three logit models: 1) the probability of reaching a bargaining agreement (93 percent), 2) the probability of reaching Pareto efficiency (40 percent), and 3) the probability of simultaneously reaching a bargaining agreement and an efficient allocation (9 percent). The set of explanatory variables includes a set of proxies measuring the husband's intra-household bargaining power of the husband. One proxy is a dummy equal to one if his share of total household expenditure at the threat point is larger than 0.5. This variable measures the extent of the husband's outside options in case of a bargaining failure. The husband-wife wage ratio captures the husband bargaining power. We also construct an ordinal index capturing the husband's decision-making power by summarizing four decision-making domains: decisions related to business, the family, off-farm labor, and household finance. For each of these decisional spheres, we define a variable taking value 2 if the husband is the sole decision-maker, 1 if the husband and the wife decide together, and 0 if the wife is the sole decision-maker. Summing over the four domains, we associate a score in the [0,8] range to the husband decision-making power index. A value of 0 means that the husband does not exert power in any of the four domains, while a value of 8 indicates that the husband acts as "a dictator." We also control for the husband's bargaining power using a dummy equal to one if he inherited the farm activity.

Other variables included in the analysis are the educational level (a dummy equal to one if both spouses have at least a lower secondary education), the age classes of the husband (19-34, 35-60, and 61-65, with the former as reference category),[17] farmer's propensity to take risks in the agricultural activity (low, medium, and high, with the former as reference category), macro-regions (North, South, and Center, with the latter as reference category), and the number of wage

[17] We do not control for wife's age classes because spouses' ages are highly correlated (0.92 significant at 1% level).

earners in the family, as a proxy for family income. Descriptive statistics are in Table C2 of Appendix C.

Table 3 shows the predicted probabilities and average marginal effects of the three logit models. On average, a household with a threat allocation more favorable to the husband has a probability of reaching a bargaining agreement 8 percentage points lower than a family where the wife has better outside options than her spouse (column 1). This suggests that households where the husband has the most to lose from a failed negotiation are less likely to come to an agreement. A possible explanation can be a lower attitude of men to compromise by making concessions. The allocation at the threat point does not matter for the other two outcomes (column 2 and 3). On the other hand, an increase in the decision-making power of the husband results in a reduction of the probability of reaching an efficient allocation (column 2). In a household where the husband has little decision-making power, the predicted probability of attaining efficiency is 55 percent, against 31 percent when he acts as a "dictator". These results suggest that inefficiency tends to be lower when wives are more involved in the household decision process.

An increase of one standard deviation in the wage ratio results in a 2.5 percentage points reduction in the probability of achieving an efficient bargaining agreement (column 3). The variable indicating whether the husband inherited or not the agricultural activity is not statistically significant in the three models. Age maturity raises the probability of reaching a bargaining agreement (column 1), while it does not impact significantly on the other two outcomes (column 2 and 3). In addition, a high number of wage earners within the family positively affects the likelihood of attaining a bargaining agreement (column 1). On the other hand, many wage earners reduce the probability of reaching an efficient outcome (column 2). A higher number of wage earners within the family is associated with a relatively larger earnings capacity of the family, and a higher probability that a share of household income would be saved. Further, results show statistically significant regional differences in the likelihood of attaining efficiency.

8.2 How difficult it is to reach an agreement?

Descriptive statistics shows that families are highly heterogeneous with respect to the speed with which they reach an agreement or efficiency (Table C2 in Appendix C). Reaching an efficient (inefficient) outcome may take a short or long negotiation process. We refer to families where the outcome is reached within the first 50 iterations as *fast* families as opposed to *slow* families, which attain an agreement or efficiency after the 50-th iteration.[18] Here, the time notion is associated with the sequence of iterations. Table 4 shows the estimation results of a multinomial logit model explaining the different bargaining behavior.[19]

In line with the logit estimates, more favorable outside options to the husband are associated with a higher probability of a bargaining failure (column 1). As for the other controls, results show that *fast* families (column 3) are somehow different from the other two household types. For instance, age, risk behavior and being located in the North of Italy are generally associated with a higher probability of being a *fast* family, while decreasing the likelihood of being both a family that succeeds in reaching a bargaining agreement, or a family that experiences a bargaining failure.

Focusing on efficient allocations, *fast* and *slow* families (column 5 and 6) appear to have similar bargaining behavior, except for the role of the decision-making power of the husband. An increase in his decision-making power increases the probability of failure to achieve efficiency by 3.2 percentage points (column 4), while decreasing by a similar amount the probability of being a

[18] We cannot undertake this analysis for the process of reaching an efficient bargaining agreement because of the low number of such outcomes.

[19] Multinomial logit model relies on the IIA assumption. The Hausman-McFadden test does not reject the null that the IIA assumption is not violated.

slow family (column 5). However, it does not significantly affect the probability of being a *fast* family. The predicted probability of being a *fast* family is equal to 16 percent when the husband has no decision-making power and 20 percent when he acts as a dictator. Hence, the impact of the husband's decision-making power on the probability of being a *fast* family is negligible. On the other hand, the probability of failure to reach efficiency is 43 percent when the husband has no decision-making power and 69 percent when he acts as a dictator. These probabilities are 41 percent and 11 percent, respectively, for *slow* families, demonstrating that the effect of the decision-making power of the husband is a significant determinant.

8.3 How does the cost of inefficiency vary across households?

We also examine the determinants of the cost associated with inefficient outcomes. To do so, for each randomly generated allocation, we compute the level of private expenditure $e_i\big(p_i, x_0, z, u_i(x_i, x_0, z)\big)$ of the wife and the husband. To derive the cost of inefficiency, we measure the difference between total household income (π) and the sum of the spouses' private expenditures, $W = \pi - \sum_{i=1}^{2} e_i\big(p_i, x_0, z, u_i(x_i, x_0, z)\big)$. The cost W of imperfect bargaining varies across initial allocations. To account for household heterogeneity, we estimate a linear mixed-effect model with random intercepts, which vary across households.

Table 5 shows the linear-mixed model estimations. The dependent variable in column 1 is the bargaining cost. The coefficient associated with the husband decision-making power is positive and significant, suggesting that an increase in the decision-making power of the husband raises the cost of imperfect bargaining. The estimated cost for a family whose husband has no decision power amounts to 3067€, while it increases up to 5395€ for a family whose husband acts as a "dictator". This finding is strictly related to the evidence in Table 3, where the index for the husband's decision-making power negatively affects the probability of reaching efficiency.

The efficiency cost is also explained by the education of the wife, the age of the husband and the risk propensity. Families with a medium-highly educated wife have an efficiency cost higher by almost 1400€ than families with lower educated wife. A possible explanation is an income effect because the earnings of a medium-highly educated woman are likely higher than those of a low-educated one. This would contribute to increase household income, and in turn leaving the possibility that a share of income would be saved, generating inefficiency.

A bargaining agreement is generally associated with a lower degree of inefficiency. The closer the bargaining agreement to the frontier, the higher the gain from marriage. The efficiency cost associated with a bargaining agreement is, on average, lower by an amount of 91€ as compared to allocations that are not a bargaining agreement. However, the effect is small, suggesting that the achievement of efficiency and the achievement of a bargaining agreement are quite independent processes. The result suggests that the inefficient bargaining agreements, drawn on the grey path in Figure 3, on average are characterized by lower welfare costs than the other (inefficient) allocations of the negotiation set (that is the grey points in Figure 3), but the difference in the efficiency cost is small.

In line with the logit and multinomial logit estimates, the inefficiency cost increases with the number of family wage earners. To check the robustness of our results, in column 3 we estimate the same model, but the dependent variable is the per-capita bargaining cost. The results are in line with those previously discussed.

9. Conclusion

This study introduces new theoretical elements to the Nash-Harsanyi bargaining model making it closer to the day-to-day functioning of a family and a more realistic representation of the decision process. Our approach recognizes that negotiation comes at a cost explaining why family members may accept Pareto inefficient bargaining agreements that make the family disputants sufficiently happy (Simon 1979). While our framework allows for pushing the negotiations until reaching the Pareto utility frontier, it allows for inefficient bargaining agreements reflecting transaction costs and/or cognitive difficulties in achieving efficiency.

We extend the Nash-Harsanyi cardinal representation of the bargaining process to ordinal preferences. We further introduce a money-metric measure of the welfare loss perceived by each family member corresponding to the willingness-to-pay to avoid a breakdown situation. Then building on Zeuthen's original insight, we propose that a disaccord would be settled when each family member is equally willing to accept a bargaining failure (as measured by individual willingness-to-pay). We also characterize the algorithm that identifies the efficient bargaining agreement in correspondence of the unique Pareto equilibrium at the intersection of the contract curve with the Pareto frontier.

The theoretical mechanisms designed to reach bargaining agreements are taken to the data in a natural way revealing the many interesting and meaningful traits of a previously unexplored empirical content of the bargaining model. Given the assumptions made about preferences and threats, the empirical implementation closely reproduces the theory results. The outcomes of the bargaining schemes show that most of the agreements is not efficient, lending empirical support to Simon's hypothesis that rational individuals can be sufficiently satisfied also at inefficient but less conflictual positions on the contract curve. Interestingly, the bargaining process normally reduces intra-household inequalities but does not necessarily lead to an equal allocation when efficiency is reached, depending on how unequal the distribution of resources is at the threat point. The higher the initial level of utility at the threat point, the higher the final utility at the agreement point. An improvement in the outside options of a spouse translates into a higher bargaining power and a more favorable outcome.

The study of the determinants of the bargaining agreements is also very informative. In general, a household with a threat allocation more favorable to the husband has a lower probability of reaching a bargaining agreement than a family where the wife has better outside options. This suggests that households where husbands have the most to lose from a failed negotiation are less likely to come to an agreement. In a household where the husband has little decision-making power, the probability of attaining efficiency is significantly higher as compared to a situation where the husband holds most of the power. Inefficiency tends to be lower when wives are more involved in the household decision process. The probability of attaining an efficient bargaining agreement is significantly higher as the wife's earnings increase. Older couples have a higher probability of reaching a bargaining agreement. A high number of wage earners in the family positively affects the likelihood of attaining a bargaining agreement but efficient outcomes are less likely to occur.

Families differ substantially also in the speed in reaching an agreement. Families that are *fast* in reaching a bargaining agreement are relatively older and less risk averse. On the other hand, *fast* and *slow* families appear to have similar bargaining behavior when engaged in reaching efficient agreements, except for the role of the bargaining power of the husband. In *slow* families, preponderant husbands are less influential. Welfare costs associated with bargaining are in general positively related with the bargaining power of the husband, his age and risk propensity and the education level of the wife.

While we have presented a general conceptual approach to intrahousehold bargaining, there is a need to extent its applications to specific economic issues. This includes studies of labor supply choices, the harmonization of work and family duties, task specialization within the household, fertility choices, and antisocial behavior (e.g., the case of domestic violence). It would also be useful to explore the linkages between intrahousehold behavior and economic policy. Addressing these issues seem to be good directions for future research.

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Figures

Figure 1: Efficiency and Bargaining

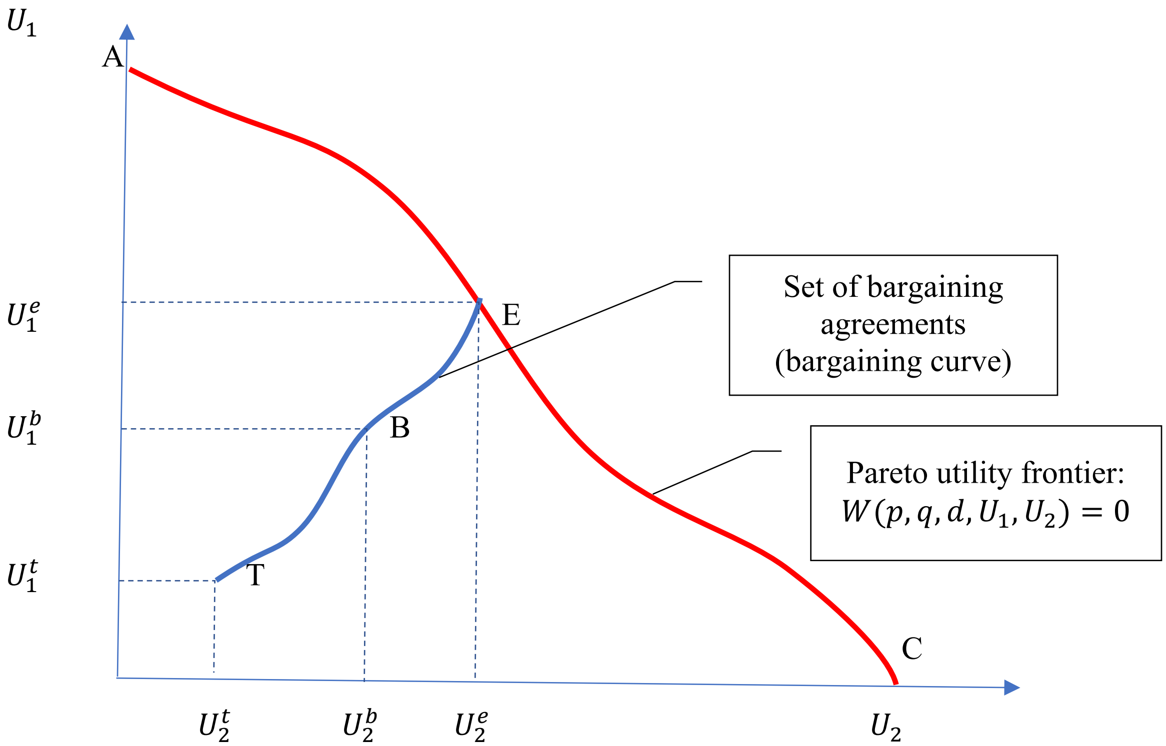
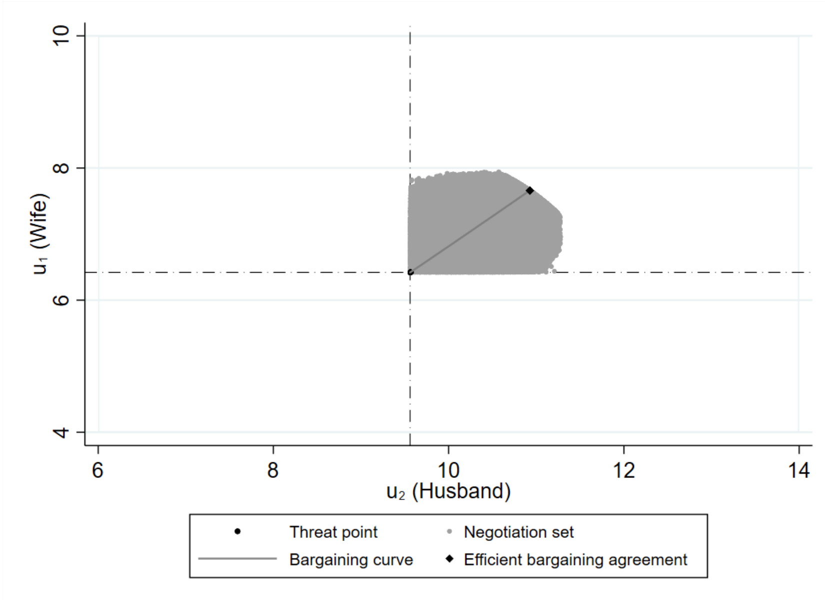
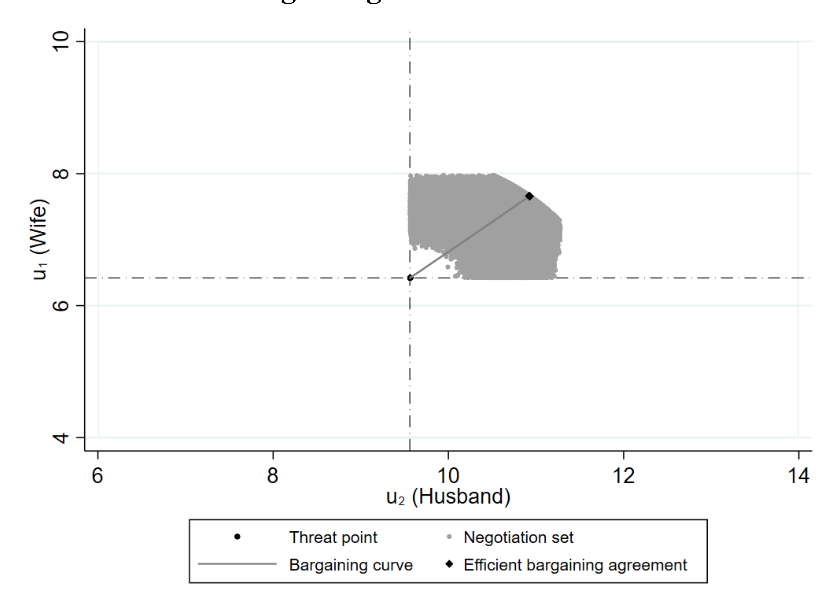


Figure 2: Representation of the Bargaining Scheme S



Notes: Individual utility at the threat point: $u_1^t = 6.4$, $u_2^t = 9.6$, individual utility on the Pareto frontier: $u_1^E = 7.7$, $u_2^E = 10.9$.

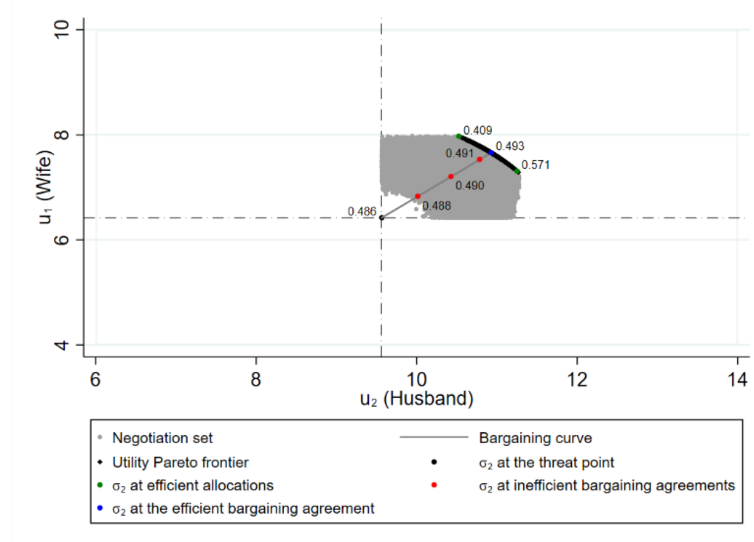
Figure 3: Representation of the Bargaining Scheme S^E



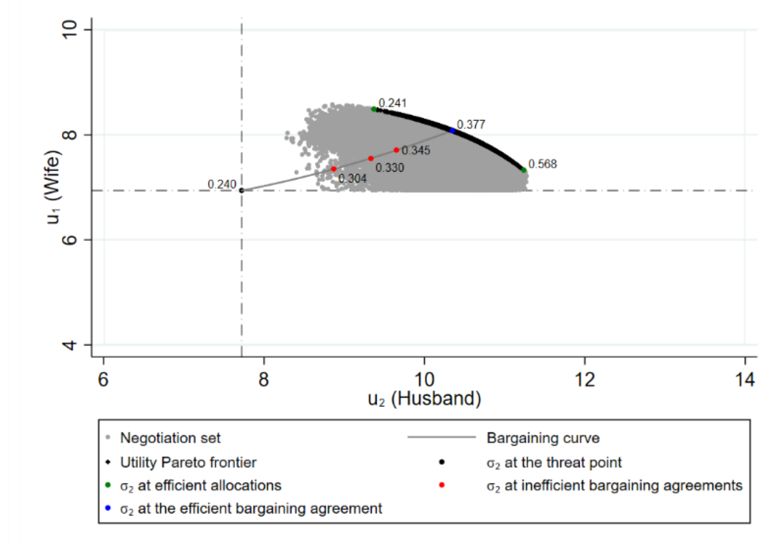
Notes: Individual utility at the threat point: $u_1^t = 6.4$, $u_2^t = 9.6$, individual utility on the Pareto frontier: $u_1^E = 7.7$, $u_2^E = 10.9$.

Figure 4: Threat Point, Bargaining and Intra-Household Inequality

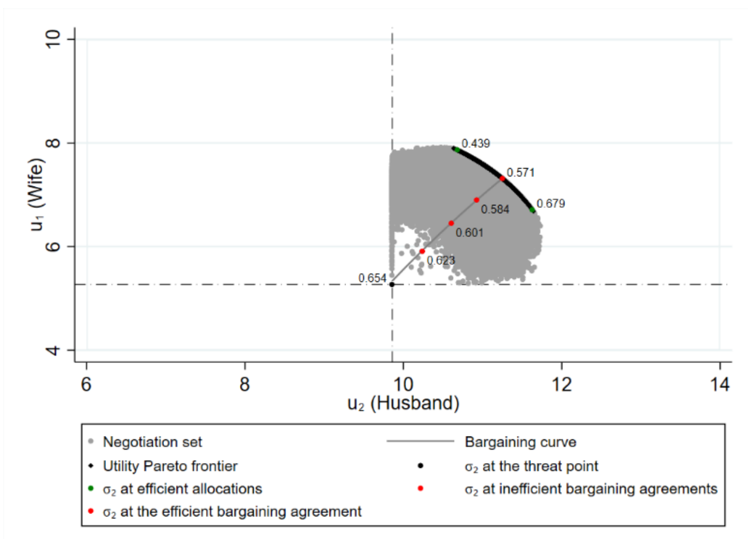
Panel A. Quasi-Egalitarian Threat Point



Panel B. Threat Point More Favorable to the Wife



Panel C. Threat Point More Favorable to the Husband



Panel D. Intra-Household Inequality

	Threat point		Pareto frontier	
	σ_2	Shannon index	σ_2	Shannon index
Panel A	0.486	0.693	0.493	0.693
Panel B	0.240	0.551	0.377	0.661
Panel C	0.654	0.645	0.571	0.683

Note: σ_2 is the husband resource share. Shannon index $= -\sum_{i=1}^{2} \sigma_i \ln\sigma_i$. It takes values in the range (0,1) and has an inverted U-shape relationship. When resources are equally allocated between the spouses ($\sigma_1 = \sigma_2 = 0.5$), the associated Shannon index is 0.693.

Tables

Table 1: Empirical Implementation of the Bargaining Schemes S and S^E (Iterative Processes)

	Wife						Husband						W	σ_2
	Expenditure for:			e_1^k	U_1^k	Δ_1^k	Expenditure for:			e_2^k	U_2^k	Δ_2^k		
K	Leisure	Other goods	Clothing				Leisure	Other goods	Clothing					
PANEL A - BARGAINING SCHEME S														
1	1037.10	1272.81	7.28	2317.19	7.16	695.41	909.70	1272.81	6.84	2189.34	10.34	656.86	1313.87	0.49
2	230.21	1842.48	152.34	2225.03	7.08	603.26	185.59	1842.48	168.97	2197.05	10.35	664.56	1398.32	0.50
3	1165.07	839.24	33.75	2038.07	6.89	416.29	259.52	839.24	987.34	2086.11	10.24	553.62	1696.23	0.51
4	857.54	1840.82	0.00	2698.36	7.47	1076.59	230.53	1840.82	0.83	2072.18	10.22	539.70	1049.86	0.43
5	1432.12	988.80	245.56	2666.48	7.45	1044.70	860.87	988.80	153.20	2002.88	10.15	470.39	1151.05	0.43
…														
39	358.02	1712.71	700.79	2771.51	7.53	1149.74	876.02	1712.71	97.56	2686.29	10.79	1153.80	362.60	0.49
PANEL B - BARGAINING SCHEME S^E														
1	1037.10	1272.81	7.28	2317.19	7.16	695.41	909.70	1272.81	6.84	2189.34	10.34	656.86	1313.87	0.49
2	230.21	1842.48	152.34	2225.03	7.08	603.26	185.59	1842.48	199.82	2227.89	10.38	695.41	1367.48	0.50
3	1165.07	839.24	312.87	2317.19	7.16	695.41	259.52	839.24	1021.78	2120.55	10.27	588.06	1382.67	0.48
4	396.84	1840.82	9.52	2247.18	7.10	625.40	1092.84	1840.82	0.00	2933.66	10.98	1401.18	639.55	0.57
5	1432.12	988.80	33.80	2454.72	7.28	832.94	860.87	988.80	1018.79	2868.46	10.93	1335.98	497.22	0.54
…														
136526	523.60	1148.80	1280.94	2953.34	7.66	1331.56	321.74	1148.80	1396.41	2866.94	10.93	1334.46	0.12	0.49

Notes: e_i^k is the total private expenditure of member i at iteration k, U_i^k is the associated utility level, Δ_i^k is the individual willingness-to-pay to avoid the disagreement point allocation, W is the measure of inefficiency, and σ_2 is the husband resource share.

Table 2: Bargaining Costs and Individual Utility Levels

Bargaining cost - c	U_1	U_2
0	7.7	10.9
298.3	7.6	10.8
565.4	7.5	10.7
1589.6	7.0	10.2
1948.4	6.8	10.0
2666.1	6.4	9.6

Notes: Bargaining costs are in euro per month.

Table 3: Probabilities and Average Marginal Effects of Logit Model (726 Families)

	Outcome		
	Bargaining agreement	Efficient allocation	Efficient bargaining agreement
	(1)	(2)	(3)
Predicted probability	0.923***	0.401***	0.079***
	(0.010)	(0.037)	(0.010)
Average marginal effects			
Husband's share at the threat point > 0.5	−0.073**	0.033	0.045
	(0.034)	(0.046)	(0.033)
Husband's market wage/Wife's market wage	−0.120	0.131	−0.575**
	(0.239)	(0.423)	(0.242)
Husband's decision-making power	0.006	−0.030**	−0.001
	(0.007)	(0.012)	(0.007)
Husband inherited the farm	0.017	0.046	−0.002
	(0.022)	(0.037)	(0.021)
Medium-highly educated husband	0.027	−0.058	−0.024
	(0.024)	(0.042)	(0.023)
Medium-highly educated wife	−0.005	−0.095	−0.035
	(0.039)	(0.060)	(0.028)
Husband aged 35-59	0.078*	−0.045	−0.059
	(0.044)	(0.062)	(0.044)
Husband aged 60 and over	0.057**	0.007	−0.041
	(0.026)	(0.076)	(0.031)
Medium propensity to take risks	0.022	−0.063	0.005
	(0.022)	(0.042)	(0.026)
High propensity to take risks	0.014	−0.072*	0.003
	(0.022)	(0.041)	(0.024)
North	0.037	−0.297***	−0.084***
	(0.026)	(0.043)	(0.024)
South	0.012	−0.114***	0.003
	(0.027)	(0.043)	(0.026)
Number of wage earners	0.024**	−0.068***	0.001
	(0.011)	(0.017)	(0.009)

Notes: Reference category for husband's age: younger than 35. Reference category for risk behavior: low propensity to take risks. Reference category for macro-regions: Center. Standard errors are in parenthesis. *** p<0.01, ** p<0.05, * p<0.10.

Table 4. Probabilities and Average Marginal Effects of Multinomial Logit Models

	Bargaining agreement			Efficient allocation		
	Failed bargaining	Slow Families	Fast families	Failed efficiency	Slow families	Fast families
	(1)	(2)	(3)	(4)	(5)	(6)
Predicted probabilities	0.077***	0.521***	0.402***	0.599***	0.205***	0.196***
	(0.010)	(0.018)	(0.018)	(0.017)	(0.015)	(0.014)
Average marginal effects						
Husband's share at the threat point > 0.5	0.074**	−0.054	−0.020	−0.035	−0.041	0.076*
	(0.034)	(0.050)	(0.047)	(0.047)	(0.037)	(0.043)
Husband's market wage/Wife's market wage	0.117	−0.889*	0.772*	−0.152	0.335	−0.183
	(0.239)	(0.457)	(0.450)	(0.423)	(0.359)	(0.345)
Husband's decision-making power	−0.006	−0.008	0.014	0.032***	−0.035***	0.003
	(0.007)	(0.013)	(0.013)	(0.012)	(0.011)	(0.010)
Husband inherited the farm activity	−0.017	0.072*	−0.055	−0.047	0.054*	−0.007
	(0.022)	(0.040)	(0.039)	(0.037)	(0.031)	(0.031)
Medium-highly educated husband	−0.028	0.052	−0.025	0.059	−0.061*	0.001
	(0.024)	(0.044)	(0.043)	(0.042)	(0.036)	(0.034)
Medium-highly educated wife	0.006	0.026	−0.033	0.093	−0.035	−0.058
	(0.040)	(0.066)	(0.065)	(0.061)	(0.052)	(0.046)
Husband aged 35-59	−0.078*	−0.051	0.129**	0.043	0.025	−0.068
	(0.044)	(0.066)	(0.062)	(0.063)	(0.054)	(0.055)
Husband aged 60 and over	−0.057**	0.000	0.057	−0.015	0.046	−0.030
	(0.026)	(0.083)	(0.083)	(0.078)	(0.074)	(0.057)
Medium propensity to take risks	−0.022	−0.072	0.094**	0.062	−0.015	−0.048
	(0.022)	(0.045)	(0.045)	(0.042)	(0.036)	(0.033)
High propensity to take risks	−0.014	−0.026	0.040	0.071*	−0.011	−0.060*
	(0.022)	(0.045)	(0.045)	(0.041)	(0.035)	(0.032)
North	−0.037	−0.069	0.106**	0.295***	−0.147***	−0.148***
	(0.026)	(0.052)	(0.051)	(0.043)	(0.036)	(0.035)
South	−0.012	0.012	0.000	0.115***	−0.089**	−0.026
	(0.027)	(0.053)	(0.052)	(0.043)	(0.036)	(0.036)
Number of wage earners	−0.024**	0.009	0.015	0.069***	−0.026*	−0.044***
	(0.011)	(0.017)	(0.017)	(0.017)	(0.014)	(0.015)
Observations	726	726	726	726	726	726

Notes: Reference category for husband’s age: younger than 35. Reference category for risk behavior: low propensity to take risks. Reference category for macro-regions: Center. Standard errors are in parenthesis. *** p<0.01, ** p<0.05, * p<0.10.

Table 5. Linear-Mixed Model with Random Intercepts (726 families)

	Bargaining cost (1)	Per capita bargaining cost (2)
Husband's share at the threat point > 0.5	−0.944*	−0.137
	(0.510)	(0.136)
Husband's market wage/Wife's market wage	2.088	0.533
	(4.737)	(1.263)
Husband's decision-making power	0.291**	0.089**
	(0.136)	(0.036)
Husband inherited the farm activity	−0.218	−0.080
	(0.413)	(0.110)
Medium-highly educated husband	0.477	0.066
	(0.458)	(0.122)
Medium-highly educated wife	1.396**	0.315*
	(0.679)	(0.181)
Husband aged 35-59	1.995***	0.479***
	(0.676)	(0.180)
Husband aged 60 and over	1.057	0.384*
	(0.831)	(0.221)
Medium propensity to take risks	1.135**	0.252**
	(0.472)	(0.126)
High propensity to take risks	0.733	0.274**
	(0.466)	(0.124)
North	3.369***	0.987***
	(0.540)	(0.144)
South	1.148**	0.153
	(0.544)	(0.139)
Number of wage earners	0.912***	
	(0.175)	
Agreement	−0.091***	−0.028***
	(0.003)	(0.001)
Observations	1,255,878	1,255,878
Number of groups	726	726

Notes: In column 1, the bargaining cost divided by 1000. In column 2, the bargaining cost is divided by the number of household members. Reference category for husband's age: younger than 35. Reference category for risk behavior: low propensity to take risks. Reference category for macro-regions: Center. Standard errors are in parenthesis. *** p<0.01, ** p<0.05, * p<0.10.

Appendices

Appendix A: Proofs

Proof of Proposition 1:

Consider a point (x^*, y^*, z^*, U^*) that satisfies equations (2) and (3). Assume that the allocation (x^*, y^*, z^*) is not efficient. It means that there exists another feasible allocation (x^0, y^0, z^0) such that $u_i(x_0^0, x_i^0, z^0) \geq u_i(x_0^*, x_i^*, z^*)$ for all $i \in N$ and $u_j(x_0^0, x_j^0, z^0) > u_j(x_0^*, x_j^*, z^*)$ for some individual $j \in N$. Let $e_i(p_i, x_0, z, U_i^*) \equiv Min_{x_i} \{p_i\, x_i : u_i(x_0, x_i, z) \geq U_i^*, x_i \in X_i\}, i \in N$. Assume that (3) holds. Under assumptions A1 (continuity), A2 (non-satiation) and A4 (no destitution), $u_i(x_0^0, x_i^0, z^0) \geq U_i^*$ implies that $e_i(p_i, x_0^0\,, z^0, U_i^*) \leq p_i\, x_i^0, i \in N$, and $u_j(x_0^0, x_j^0, z^0) > U_j^*$ implies that $e_j\left(p_j, x_0^0\,, z^0, U_j^*\right) < p_j\, x_j^0$. It follows that $q\, y^0 - p_0\, x_0^0 - \sum_{i \in N} p_i\, x_i^0 < q\, y^0 - p_0\, x_0^0 - \sum_{i \in N} e_i\, (p_i, x_0^0, z^0, U_i^*)$. But this contradicts (2). Now assume that (2) holds. The feasibility of (x^0, y^0, z^0) implies that that it satisfies the budget constraint (1): $q\, y^0 - p_0\, x_0^0 - \sum_{i \in N} p_i\, x_i^0 \geq 0$. It follows that $0 \leq q\, y^0 - p_0\, x_0^0 - \sum_{i \in N} p_i\, x_i^0 < q\, y^0 - p_0\, x_0^0 - \sum_{i \in N} e_i\, (p_i, x_0^0, z^0, U_i^*) \leq Max_{x_0, z}\{q\, y - p_0 x_0 - \sum_{i \in N} e_i\, (p_i, x_0, z, U_i^*) : x_0 \in X_0, (y, z) \in F\} = W(p, q, U^*)$, which contradicts (3). Thus, equations (2) and (3) imply efficiency.

Now consider a feasible allocation (x^a, y^a, z^a) that is efficient. Assume that it does not satisfy equations (2) and (3). This can happen in two ways. First, it can happen if (x^a, y^a, z^a, U^a) satisfies equation (3) but not (2), i.e. if conditional on U^a there exists a feasible allocation $(x^b, y^b, z^b) \neq (x^a, y^a, z^a)$ such that $D \equiv \left[q y^b - \, p_0 x^b - \sum_{i \in N} p_i\, x_i^b\right] - [q y^a - \, p_0 x^a - \sum_{i \in N} p_i\, x_i^a] > 0$. Under assumption A2, redistributing the income $D > 0$ to the n individuals can make at least one person better without making anyone worse off, contradicting efficiency. Second, it can happen if (x^a, y^a, z^a, U^a) satisfies equation (2) but not (3), i.e. if U^a satisfies $W(p, q, U^a) = [q y^a - p_0 x_0^a - \sum_{i \in N} p_i\, x_i^a] \neq 0$. The household budget constraint (1) implies that

$W(p, q, U^a) \geq 0$. But under assumption A2, having $W(p, q, U^a) > 0$ would mean that the income $W(p, q, U^a) > 0$ can be redistributed to the n individuals to make at least one person better off without making anyone worse off, contradicting efficiency. It follows that efficiency implies equations (2) and (3).

Proof of Proposition 2:

When $N^k \neq N$, equations (12a)-(12b) in S3b imply that $M^{k+1} < M^k$, i.e. that the maximum willingness-to-pay to avoid bargaining failure declines across iterations from k to $k + 1$. Thus, the bargaining process given in S moves in the direction of equalizing the willingness to accept a bargaining failure. Upon convergence, from S3a, the iterative scheme in S leads to a point where $N^{k^\#} = N$. From Definition 2, this identifies $(x^{k^\#}, y^{k^\#}, z^{k^\#})$ as a bargaining agreement.

Proof of Proposition 3:

From Proposition 2, $(x^{k^*}, y^{k^*}, z^{k^*})$ is a bargaining agreement. Upon convergence, condition S^E 3a states that $W(p, q, U^{k^*}) = 0$. Under assumptions A1-A4, Proposition 1 implies that U^{k^*} is on the Pareto utility frontier. This implies that the allocation $(x^{k^*}, y^{k^*}, z^{k^*})$ is a bargaining agreement that is Pareto efficient.

We now show that this agreement is a unique point on the Pareto utility frontier. Under assumption A2, note that $W(p, q, U)$ is strictly decreasing in U and $e_i(p_i, x_0, z, U_i)$ is strictly increasing in $U_i, i \in N$. Assume that there are two different agreements on the Pareto utility frontier satisfying $U^a \neq U^b$, with $W(p, q, U^a) = 0$ and $W(p, q, U^b) = 0$. The function $W(p, q, U)$ being strictly decreasing in U implies that there exists some $j \neq i \in N$ such that $U_i^a > U_i^b$ and $U_j^a < U_j^b$. The expenditure function $e_i(p_i, x_0, z, U_i)$ being strictly increasing in U_i, it follows that

$e_i(p_i, x_0, z, U_i^a) > e_i(p_i, x_0, z, U_i^b)$ and $e_j(p_j, x_0, z, U_j^a) < e_j(p_j, x_0, z, U_j^b)$. But this contradicts the definition of a bargaining agreement in equations (8)-(10), implying that only one point on the Pareto utility frontier can be a bargaining agreement.

Proof of Proposition 4.

Under assumption A2, the maximization with respect to $U = (U_1, \dots, U_n)$ implies that the budget constraint in equation (16) is necessarily binding. In turn, this implies equations (3), (5), (6) and (7). From Corollary 1, it follows that the solution to problem (16) is Pareto efficient and that U^e is on the Pareto utility frontier: $U^e \in \{U: W(p, q, U) = 0\}$.

First, consider the case where $W(p, q, U^t) = 0$. Then U^t is on the Pareto utility frontier and $U^e = U^t$. Second, consider the case where $W(p, q, U^t) > 0$. Then, the maximization problem in (16) can be alternatively written as

$$\underset{x_0, y, z, U}{\text{Max}} \{ \sum_{i \in N} \ln [e_i(p_i, x_0, z, U_i) - e_i(p_i, x_0, z_0, U_i^t)]:$$

$$\sum_{i \in N} e_i(p_i, x_0, z, U_i) + p_0 x_0 = qy; \ x_0 \in X_0, (y, z) \in F\}. \qquad (16')$$

Under assumption A2, $e_i(p_i, x_0, z, U_i)$ is strictly increasing in U_i, $i \in N$. Denoting by λ the Lagrange multiplier associated with the budget constraint in (16'), the first-order necessary conditions with respect to U_i in (16') give $\lambda = 1/[e_i(p_i, x_0, z, U_i) - p_i x_i^t]$, $i \in N$. This implies that the solution to problem (16') satisfies the convergence criterion in S^E3a, i.e., that the solution to problem (16) is a bargaining agreement located on the Pareto utility frontier.

Appendix B: Model Specification

This appendix describes the specification of the functional forms adopted in the empirical analysis. They reproduce the empirical specification of the econometric model estimated by Matteazzi, Menon and Perali (2017), whose parameters are used in the iterative and optimization models.

Let the set of market inputs be *v* = {chemicals, materials, hired labor}, *H* = family labor treated as a quasi-fixed factor, and the set of outputs *s* = {crop, livestock, milk, fruits, olives and grapes}. The set of consumption goods is denoted as *k* = {leisure, domestic good, food, clothing, other market goods}. Member 1 is the wife and member 2 the husband.

<u>Cost Function of Marketable Production</u>

Matteazzi, Menon and Perali (2017) estimate the farm production technology from the dual side to account both for the non-homogeneity of family and hired labor and the fact that hired labor is a variable factor with an associated observable market wage, while family labor is a quasi-fixed factor with an associated shadow wage. The total restricted cost function for the agricultural production takes a Translog form with four outputs, three market inputs and the quasi-fixed factor

$$\log VC = \alpha_0 + \sum_s \alpha_s \log q_s + \sum_f \beta_f \log r_f + \chi \log H + \frac{1}{2}\sum_s \sum_u \delta_{su} \log q_s \log q_u +$$

$$+ \frac{1}{2}\sum_f \sum_v \gamma_{fv} \log r_f \log r_v + \sum_s \sum_f \rho_{sf} \log q_s \log r_f + \sum_f \xi_f \log H \log r_f \quad \text{(B1)}$$

where *VC* denotes variable costs of the agricultural production, r_f is the price of market input *f*, *H* is the quasi-fixed factor family labor, and q_s is the levelof the agricultural output *s*. Further, α_0, α_s, β_f, χ, δ_{su}, γ_{fv}, ρ_{sf}, ξ_f are estimated parameters (Matteazzi, Menon and Perali, 2017).

On-farm family labor *H* is a quasi-fixed factor allocatable both across activities, when information is available, and between spouses, as in our case. If entrepreneurs are minimizing costs, the quasi-fixed factor's shadow wage for family labor w_{on}^* can derived by differentiating total costs with respect to the level of quasi-fixed factor

$$w_{on}^{*} = \left(\chi + \sum_{f=1}^{3} \xi_f \log r_f\right) \frac{VC}{H}, \qquad \text{(B2)}$$

where VC are the minimum variable costs in level.

Farm-household profits are

$$\pi = \sum_{s=1}^{4} p_{q_s} q_s - VC + TR. \qquad \text{(B3)}$$

where TR denotes the lump-sum income transfers that farm-households receive from the government.

Cost Function of the Nonmarketable Domestic Production

The total cost function for the domestic production takes a Translog form. The value of the implicit price of the domestic good, p_z^*, is derived as the exponent of a Translog unit cost function (Apps and Rees, 1997)

$$p_z^* = \exp\left(a_0 + \sum_{i=1}^{2} a_i \log w_i + \frac{1}{2} \sum_{i=1}^{2} \sum_{j=1}^{2} a_{ij} \log w_i \log w_j\right), \qquad \text{(B4)}$$

where a_0, a_i, and a_{ij} are estimated parameters (Matteazzi, Menon and Perali, 2017).

Individual Preferences

Matteazzi, Menon and Perali (2017) estimate a linear specification for individual preferences. The budget shares of individual i for good k are

$$\omega_{ik} = \alpha_{ik} + t_{ik}(d) + \sum_n \nu_{ink} \log P_{ik} + \eta_{ik} \log\left(\frac{I_i}{A_i(P_{ik})}\right) \qquad \text{(B5)}$$

where α_0,α_{ik},$t_{ik}(d)$, ν_{ink}, and η_{ik} are estimated parameters (Matteazzi, Menon and Perali, 2017). $t_{ik}(d) =$ is the k-th translating demographic function with d_{im} denoting demographic variables for spouse i, P_{ik} is the set of prices, I_i is individual's full income, and $A_i(P_{ik})$ is a price index taking a Translog form.

Individual full income is measured by the value of individual endowment of time added up to an individual share φ_i of total household nonlabor income, which includes profits from the marketable agricultural production (derived from equation A3) and the nonlabor income y,

$$I_i = w_i(l_i + t_i + L_i) + w_{on}^* h_i + \varphi_i, \tag{B6}$$

where l_i, t_i, L_i and h_i are sample mean values of spouse i's time devoted to leisure, domestic work, market work and on-farm work, respectively. Spouses' shares of total household nonlabor income are defined as

$$\varphi_1 = \psi(w_1, w_2, p_1, p_2, y, d_f)(\pi + y), \tag{B7}$$

$$\varphi_2 = (\pi + y) - \varphi_1, \tag{B8}$$

where the arguments of function ψ are spouses' market wages (w_1, w_2) and prices of clothing (p_1, p_2), non labor income and distribution factors (d_f). The function ψ is specified as a Cobb-Douglas

$$\psi = \left(w_1^{\theta_1} w_2^{\theta_2} p_1^{\theta_3} p_2^{\theta_4} y^{\theta_5} d_f^{\theta_6}\right) \in [0,1] \tag{B9}$$

with $\theta_5 = -\sum_{n=1}^{4} \theta_n$ in order to have individual income shares homogeneous of degree one in monetary variables and the consumption demands satisfying homogeneity of degree zero in prices and nonlabor income. For a detailed econometric characterization of the sharing rule see Matteazzi, Menon and Perali (2017). It is worth noting that the price of the domestic produced good that enters equation A5 is derived as in equation A4.

The individual private expenditure e_i cannot exceed individual full income

$$e_i = \sum_k P_{ik} x_{ik} \leq I_i \tag{B10}$$

where x_{ik} denotes the units of consumption goods. The individual indirect utility function is

$$V_i = \frac{\log e_i - \sum_k \alpha_{ik} \log P_{ik} - \sum_k t_{ik}(d) \log P_{ik} - 0.5 \sum_k \sum_n \nu_{ink} \log P_{in} \log P_{ik}}{\prod_k P_{ik}{}^{\eta_{ik}}}. \tag{B11}$$

Appendix C: Additional Tables and Figures

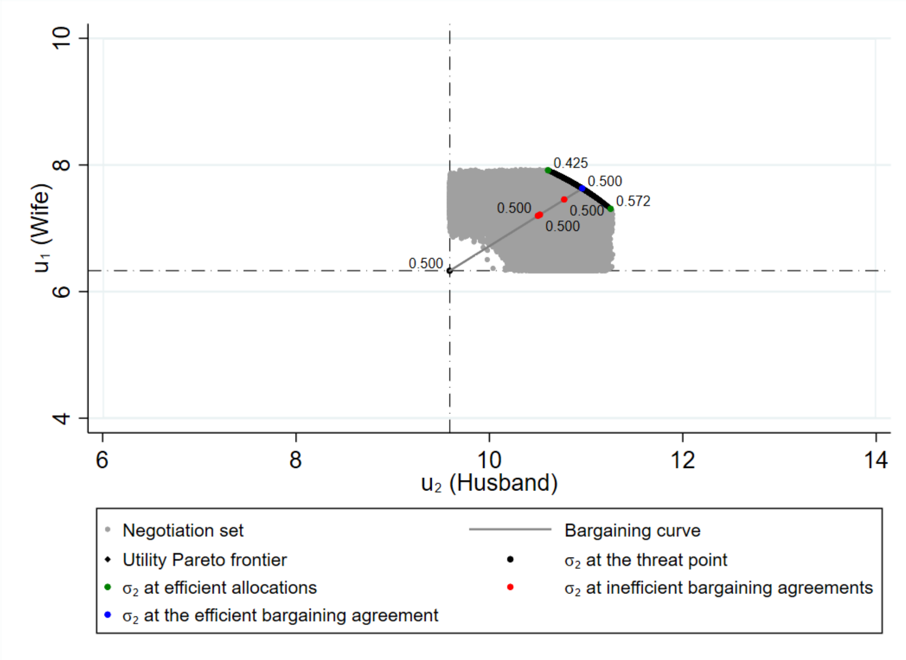
Table C1. Simulated Allocations, Bargaining Agreements, Efficient Allocations and Efficient Bargaining Agreements (726 Families)

Number of randomly generated allocations for each family	Number of families reaching		
	at least one bargaining agreement	at least one efficient allocation	an efficient bargaining agreement
5000	670	303	77
4000	670	301	77
3000	670	300	67
2000	670	291	57
1000	670	283	37
800	670	277	33
500	668	268	22
400	663	262	19
300	645	253	16
200	605	239	11
150	553	224	9
100	466	197	5
50	292	142	2

Table C2. Descriptive Statistics (726 Families)

	Mean	**Std. Dev.**
Outcome variables		
Share of families that reached a bargaining agreement	0.92	0.27
Share of families that reached an efficient allocation	0.40	0.49
Share of families that reached an efficient bargaining agreement	0.08	0.27
Among families that reached a bargaining agreement, share of families that reached the outcome with a number of iterations		
≤ 50 (*fast* families)	0.44	0.50
> 50 (*slow* families)	0.56	0.50
Among families that reached an efficient allocation, share of families that reached the outcome with a number of iterations		
≤ 50 (*fast* families)	0.49	0.50
> 50 (*slow* families)	0.51	0.50
Control variables		
Husband's share at the threat point > 0.5	0.18	0.39
Husband's market wage/Wife's market wage	1.03	0.05
Husband's decision-making power	4.68	1.45
Husband inherited the farm activity	0.66	0.47
Medium-highly educated husband	0.52	0.50
Medium-highly educated wife	0.13	0.33
Husband aged 35-59	0.73	0.44
Husband aged 60 and over	0.17	0.37
Medium propensity to take risks	0.28	0.45
High propensity to take risks	0.31	0.46
North	0.39	0.49
South	0.37	0.48
Number of wage earners	2.57	1.76

Figure C1: Egalitarian Threat Point



Note: σ_2 is husband resource share.